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# Impact of Water Hardness in Instinctive Laundry System Based on Fuzzy Logic Controller

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Abstract. In this paper, we discuss the effects of water types and temperature in automatic washing machine. The automatic washing machines are being used in hard water areas void of useful results because machines could not detect the type of water. Hard water consumes more detergent and washing time for laundry. The proposition of the paper is that the soft water and high temperature should be used in washing machines, it will not only reduce the quantity of detergent but also have positive effects on economy and fabrics. In this way, energy and washing time can be saved. The results are verified by TOPSIS technique of MCDM. The pretending results and the actions of aforesaid device have been done by using MATLABs fuzzy logic toolbox.

AMS (MOS) Subject Classification Codes: 03B52;94D05;62C86. Key Words: Hard Water, pH, Instinctive Laundry System, MATLAB, FIS Editor, Fuzzy Logic Controller (FLC), TOPSIS technique, MCDM.

#### **1. INTRODUCTION AND PRELIMINARIES**

Set theory and logic systems are strongly coupled in the development of modern logic. Classical logic corresponds to the crisp set theory, and fuzzy logic is associated with fuzzy set theory which was proposed by Zadeh in his pioneer work [40]. In crisp set, an element y in the universe Y is either a member of some crisp set B or not. It can be represented mathematically with indicator function:

$$\mu_B(y) = \begin{cases} 1, & y \in B\\ 0, & y \notin B \end{cases}$$

while fuzzy set assigns degree of membership to each element with symbol  $\mu_B(y)$  such that

$$\mu_B(y) \in [0,1]$$

The first fuzzy logic based control experiment was conducted by Mamdani [19] in 1974 who designed the fuzzy logic for a steam engine. After 1980s the use of fuzzy logic control based system became common in washing machines, elevators and metro. Now a days fuzzy logic is used in several fields like electronic control systems, automotive industries, breaking systems, home electronics etc. Everyday many home appliances are being upgraded using fuzzy logic to save time and to conserve electricity [10]. Tiryaki uses fuzzy logic control approach in dish washer [38]. Hatagar [12] has taken three LI's i.e kind of fabric, kind of grime and griminess of fabric while one output i.e wash time and 21 rules were used to attain the output. Khin Thinzar Oo [37] defines FLC for washing machine consisting of three LI's. They are: type of clothes, amount of clothes and amount of dirtiness. While Linguistic outputs are: rinse time, wash time and spin time. He define 27 rules to get outputs.

Fuzzy set theory and its applications in decision making have studied by many researchers (see [1, 3, 13–15, 18, 33, 34, 39]). Fuzzy logic controller with impact of water have studied by many researchers (see [4]-[12]). Malik and Riaz [16, 17] studied action of the modular group on real quadratic field. Riaz and Naeem [20, 21] presented some essential ideas of soft set SS together with soft algebra(SA). They additionally displayed a few utilizations of soft mappings to the decision making problems (DMP). Riaz and Fatima [22] established certain properties soft metric spaces by utilizing soft points, soft elements in the soft sets. Riaz and Hashmi [23–27] investigated certain applications of FPFS-sets, FPFS-topology and FPFS-compact spaces. They also investigated fixed point theorems of fuzzy neutrosophic soft mapping (FNS-mapping) with applications to the decision making problems DMP. Riaz and Tayyba [28, 29] introduced bipolar fuzzy soft topology. Shabir and Naz [30] introduced soft topological spaces. Sezgin [31, 32] *et al.* introduced some operations on soft sets.

Operation System of Washing Machine with Fuzzy Logic Control System have studied by [35–38,41–43].

Demetgul *et al.* [9] has proposed a model having input parameters: amount of dirt, sensitivity of textile, kind of grime, concerning textile. Output parameters: supply of water,

washing time, washing speed, amount of detergent and water hotness. They used a set of 10 rules to obtain the output parameters. Alhanjouri *et al.* [5] using fuzzy logic optimize the wash time of washing gadget and FLC for washing appliance takes two inputs i.e. grime kind and extent of griminess while only one output i.e. washing time. Agarwal is controlling the washing time of washing machine using FLC, including two inputs i.e. kind of grime and griminess of the textile [4]. Kumar [15] also reduces the washing time. Quantity and dirtiness of clothes were taken as input while they used a set of 9 rules to obtain the output parameters. Ahmed [6] has proposed FLC for automatic washing gadget consisting of five LI's. i.e. kind of grime, amount of grime, kind of cloth, quantity of cloth and temperature. Above Linguistic Inputs controls, three Linguistic Outpts i.e. wash time, rinse periods and spin period. Proposed FLC interpretation engine is being arranged by taking two hundred and sixteen rules of wash time, two hundred and sixteen rules for cleanse period and twenty five guideline for twist duration.

Washing machines are commonly used in Pakistan as a cloth washer but which are being used in hard water areas void of useful result, Since machine could not detect the type of water. Water type is categorized on the parameters that represent the absorption of calcium ion  $(Ca^{+2})$  and magnesium ion  $(Mg^{+2})$  in the water. Following table shows the ranges to classify water into hard and soft water type having different amount of calcium and magnesium ions per liter [7].

Classification	Hardness in mg/L	Hardness in mmol/L
Soft	0-60	0-0.60
Moderately hard	61-120	0.61-1.20
Hard	121-180	1.21-1.80
Very Hard	>180	>1.80

The parameter that constitutes the absorption of calcium ion  $(Ca^{+2})$  and magnesium ion  $(Mg^{+2})$  is known as water hardness. It effects the pH [43] of water while pH of water determines the solubility. When detergents [43] (normally have pH between 6 -10) are dissolved in hard water, the solution becomes basic which is harmful for fabrics [7].

Here is the list of some effects on fabric due to the hard water.

- 1) Using hard water, more detergent is required for laundry and washing.
- 2) Fabric wears out 15 percent quicker when it is washed in hard water (Purdue, 1991).
- 3) Laundry re-soiled with greater ameliorate when washed in hard water.
- 4) When fabric washed in hard water, colors fade and whites darken rapidly.

When the water is hard, detergents react with calcium and magnesium salts then less amount of lather formed. This means that there is a less amount of detergents for cleaning. Residents of the region where water is hard, use more detergent than the residents of soft water areas [2].

As compared to the previous studies, in this paper, first time, water type is also taken in account as input, the washing machine discussed in this paper has more inputs and outputs, for example depending on the type of cloth, dirt type on cloth, degree of dirtiness, temperature of water and the type of water. A new Fuzzy logic controller for washing machine is proposed having 576 rules to adjust its washing time and amount of detergent. 1.1. **Instinctive Laundry System.** It means the washing machine in which load is fully treated by the machine without user intervention at any point during the programmed prior to its completion [36].

1.2. Fuzzy Logic Controller. Fuzzy Logic Controller is composed of three sections i.e.

- 1) Fuzzifier
- 2) Interface

3) Defuzzifier.

1.3. **Solubility.** It is defined as the greatest amount of solute that will vanish in unit volume of given solvent at a specific temperature. In maximum substances, temperature affect solubility. Figure shows the dependence of temperature in the solubility of some ionic compounds in water. While detergents are ionic compounds [8].



Dependence of temperature in the solubility of some ionic compounds in water.

1.4. Hard and Soft water. Water ( $H_2O$ ), meaning a liquid that contains one oxygen and two hydrogen atoms in its molecule which are connected by covalent bonds and the parameter that constitutes the concentration of calcium ion (Ca<sup>+2</sup>) and magnesium ion (Mg<sup>+2</sup>) known as water hardness. Water in which calcium and magnesium minerals are in excess called hard water. Hard water is generally can be used for drinking purpose, but bad for fabrics since when it reacts with detergents suds forms and we have to use more detergent for cleaning purpose. The water which contains sodium ions called soft water, Soft water when reacts with detergents, lather is generally formed [42].

1.5. **Defuzzification.** The procedure to convert fuzzy output to a crisp value is said to be defuzzification, of the given fuzzy sets and the correlative membership degree. It is an operation that maps a fuzzy set to a crisp set [35].

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# 2. PROPOSED DESIGN

Since hard water effects washing time so different type of water are taken. Aspire FLC model of automatic washing machine consist of five LI's i.e.

- (1) Fabric type
- (2) Dirt on fabric
- (3) Dirtiness
- (4) Temperature
- (5) Water type

FLC Linguistic Outputs are:

- (1) Washing time
- (2) Amount of detergent

# Inputs

<b>1</b>					
No.	Fabric	Dirt on	Degree of	Temperature	Water Type
		Fabric	Dirtiness		
1	silk	Dust	D1	Moderate	Soft
2	linen	Mud	D2	High	Moderately
					hard
3	satin	Slit	D3	V. High	Hard
4	wool	Oily			V. Hard

Basic approach to Fuzzy Logic Controller is visible in figure 1. Instinctive laundry system based on Fuzzy Logic Controller composed of three sections i.e. Fuzzifier, Interface (fuzzy rule selector) and Defuzzifier [16]. The Membership Function(MF) of fabric type, dirt on fabric, dirtiness, temperature and water type are [0 3], [0 3], [0 2], [1 3] and [0 6]. Membership Function for washing time is between [1 60] and amount of detergent is between [1 10] are crisp values as output.



The aspire FLC inference is designed using 576 rules to select washing time and 576 rules for amount of detergent. Every LIs and LOs has a set of membership functions. **Fuzzy logic rules table** 

Rule No.	Linguistic inputs					Linguistic Outputs			
1100	Fabric	Dirt on Fabric	Dirtness	Temperature	Water Type	Wash Time(min)	Detergent (unit)		
1	0:11-	Durat	D1	Madausta	<u>C</u> - ft	15.0	2.05		
1	Silk	Dust	DI	Moderate	Son	15.8	3.25		
2	Silk	Dust	DI	Moderate	Moderately	23.1	4.38		
3	Silk	Dust	D1	Moderate	Hard	30.5	5.5		
4	Silk	Dust	D1	Moderate	V Hard	37.9	6.63		
5	Silk	Dust	D1	High	Soft	8 37	2.12		
6	Silk	Dust	D1	High	Moderately	15.8	3.25		
-				8	hard				
7	Silk	Dust	D1	High	Hard	23.1	4.38		
8	Silk	Dust	D1	High	V. Hard	30.5	5.5		
9	Silk	Dust	D1	V. High	Soft	3.27	1.35		
10	Silk	Dust	D1	V. High	Moderately hard	8.37	2.12		
11	Silk	Dust	D1	V. High	Hard	15.8	3.25		
12	Silk	Dust	D1	V. High	V. Hard	23.1	4.38		
13	Silk	Dust	D2	Moderate	Soft	23.1	4.38		
14	Silk	Dust	D2	Moderate	Moderately hard	30.5	5.5		
15	Silk	Dust	D2	Moderate	Hard	37.9	6.63		
16	Silk	Dust	D2	Moderate	V. Hard	45.3	7.73		
17	Silk	Dust	D2	High	Soft	15.8	3.25		
18	Silk	Dust	D2	High	Moderately	23.1	4.38		
19	Silk	Dust	D2	High	Hard	30.5	5.5		
20	Silk	Dust	D2	High	V Hard	37.9	6.63		
20	Silk	Dust	D2	V High	Soft	8 37	2.12		
22	Silk	Dust	D2	V. High	Moderately	15.8	3.25		
23	Silk	Dust	D2	V High	Hard	23.1	4 38		
23	Silk	Dust	D2 D2	V High	V Hard	30.5	55		
25	Silk	Dust	D2 D3	Moderate	Soft	30.5	5.5		
26	Silk	Dust	D3	Moderate	Moderately	37.9	6.63		
27	Silk	Dust	D3	Moderate	Hard	453	7 73		
28	Silk	Dust	D3	Moderate	V Hard	52.6	8.88		
29	Silk	Dust	D3	High	Soft	23.1	4 38		
30	Silk	Dust	D3	High	Moderately hard	30.5	5.5		
31	Silk	Dust	D3	High	Hard	37.9	6.63		
32	Silk	Dust	D3	High	V. Hard	45.3	7.73		
33	Silk	Dust	D3	V. High	Soft	15.8	3.25		
34	Silk	Dust	D3	V. High	Moderately	23.1	4.38		
				č	hard				
35	Silk	Dust	D3	V. High	Hard	30.5	5.5		
36	Silk	Dust	D3	V. High	V. Hard	37.9	6.63		
					•				
					•				
573	Wool	Oily	D3	V. High	Soft	15.8	3.25		
574	Wool	Oily	D3	V. High	Moderately hard	23.1	4.38		
575	Wool	Oilv	D3	V. High	Hard	30.5	5.5		
576	Wool	Oily	D3	V. High	V. Hard	37.9	6.63		

576 rules are formed using LIs to obtain Linguistic Output i.e. Wash time, it is analyse in term of conditional statement underneath,

**Rule 1:** IF (kind of fabric is silk) and (dirt kind on fabric is dust) and (griminess is D1) and (temperature is Moderate) and (Water type is soft) THEN (washing time is xxx).

**Rule 2:** IF (kind of fabric is silk) and (dirt kind on fabric is dust) and (griminess is D1) and (temperature is Moderate) and (Water type is moderately hard) THEN (washing time is xxxx).

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Rule 576: IF (type of fabric is wool) and (dirt type on fabric is oily) and (dirtiness is D3) and (tem-

perature is V. High) and (Water type is V. hard) THEN (washing time is xxxxx).

Similarly 576 rules are obtained for amount of detergent by applying conditional statement: **Rule 1:** IF (type of fabric is silk) and (dirt kind is dust) and (dirtiness is D1) and (temperature is Moderate) and (Water type is soft) THEN (amount of detergent is UUU). **Rule 2:** IF (type of fabric is silk) and (dirt type on fabric is dust) and (griminess is D1) and (temper-

ature is Moderate) and (Water type is moderately hard) THEN (amount of detergent is UUUU).

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**Rule 576:** IF (type of fabric is wool) and (dirt type on fabric is oily) and (dirtiness is D3) and (temperature is V. High) and (Water type is V. hard) THEN (amount of detergent is UUUUUU). All the overhead mentioned rules joined using MIN-MAX [15] fuzzy interface approach.

#### 3. DEFUZZIFICATION

The procedure to convert fuzzy output to a crisp value is said to be defuzzification, of the given fuzzy sets and the correlative membership degrees. It is an operation that maps a fuzzy set to a crisp set [35]. The above mentioned quantified results are obtained from fuzzy interface technique. Defuzzification is done using Centroid method.

Wash time=  $\overline{X}$  (Centroid) =  $\frac{\sum_{1}^{60} x\mu(x)}{\sum_{1}^{60} \mu(x)}$ Amount of Detergent =  $\overline{Y}$  (Centroid) =  $\frac{\sum_{1}^{10} y\mu(y)}{\sum_{1}^{10} \mu(y)}$ .

### 4. Results

In fabric 0 assigned to Silk, Dirt on fabric 0 assigned to Dust, Dirtiness 2 assigned to D3 (very large), temperature 1 assigned to moderate, water type 0 assigned to Soft water.

4.1. Effect of Water Type on washing time. 1. Using soft water and moderate temperature for above mentioned inputs washing time is 30.5 min.

2. Using moderate temperature and V.Hard water for above inputs washing time is 52.5 min.

4.2. Effect of Temperature on washing time. 1. Using soft water and high temperature for above mentioned inputs washing time is 15.8 min.

2. Using hard water and high temperature for above mentioned inputs washing time is 37.9 min.

4.3. Effect of Water Type on Amount of Detergent. 1. Using soft water and moderate temperature for above mentioned inputs washing time is 5.5 unit.

2. Using moderate temperature and V.Hard water for above inputs washing time is 8.88 unit.

4.4. Effect of Temperature on Amount of Detergent. 1. Using soft water and high temperature for above mentioned inputs washing time is 3.25 unit.

2. Using hard water and high temperature for above mentioned inputs washing time is 6.63.

# 5. VERIFICATION BY TOPSIS

## Step 1:

First Construct NDM (Normalized Decision Matrix) to transform various attributes dimension into non-dimensional attributes, which allows comparison between the attributes.

$$t_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

Step 2:

Construct the WNDM (Weighted Normalized Decision Matrix). Consider we have a set of weights for all criteria  $q_j$  for j = 1, 2, 3...n. Multiply each column to the normalized decision matrix  $r_{ij}$  associated with its weight. the new element of the matrix is:

$$V_{ij} = q_j t_{ij}$$

Step 3:

Determine Ideal Solution  $A^+ = \{ V_1, \dots, V_n \}$ , where  $V_j^+ = \{ \max(V_{ij}) \text{ if } j \in J; \min(V_{ij}) \text{ if } j \in J^+ \}$   $J^+$  Associated with the criteria having a positive impact. and Negative-Ideal Solution.

 $A^- = \{ V_1, \dots, V_n \}$ , where  $V_j^- = \{ \min(V_{ij}) \text{ if } j \in J; \max(V_{ij}) \text{ if } j \in J^- \}$ 

 $J^-$  Associated with the criteria having a negative impact.

# Step 4:

Calculate separation Measure:

(1) Best Alternative

$$S_i^+ = \sqrt{\sum_{j=1}^n (V_{ij} - V_j^+)^2}$$
  $i = 1, 2, 3, \dots, p$ 

(1) Worst Alternative

$$S_i^- = \sqrt{\sum_{j=1}^n (V_{ij} - V_j^-)^2}$$
  $i = 1, 2, 3, \dots, p$ 

Step 5:

Calculate the Relative Closeness to the Ideal Solution  $S^{-}$ 

 $C_i^*$ 

$$C_{i}^{*} = \frac{B_{i}}{(S_{i}^{+} + S_{i}^{-})}, 0 < C_{i}^{*} < 1, \qquad i = 1, 2, 3, \dots, p.$$

$$C_{i}^{*} = 1, \quad if \ A_{i} = A^{+}$$

$$C_{i}^{*} = 0, \quad if \ A_{i} = A^{-}$$

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# Step 6:

Rank the set of alternatives in preference order. Peterence ranked according to the descending order of  $C_i^\ast.$ 

Step wise verification of proposed model. Step 1:

	Fabric Type		Dirt on I	Fabric	Dirti	ness	Temp	oerature	
Soft water	7		5		3		6		
Hard water	/ Q		3 7		6		8		
woights	0.4	9		/			03		
Step 2:	0.4		0.1		0.2		0.5		
510p 2.	Fabric Type		Dirt on 1	Fabric	Dirti	ness	Temp	oerature	
Soft water	2		2		2		2		
Hard water	0		0		6		0		
normal	2		2		6.324	55532	2		
Step 3:					1				
	Fabric Type		Dirt on I	Fabric	Dirti	ness	Temp	oerature	
Soft water	1	1		1		0.316227766		1	
Hard water	0	0		0		0.948683298		0	
Step 4:									
	Fabric Type		Dirt on 1	Fabric	Dirti	ness	Temp	erature	
Soft water	0.4	0.4		0.1		0.063245553		0.3	
Hard water	0	0		0		0.18973666		0	
ideal	0.4		0.1		0.18973666		0.3		
the worst	0	0		0		0.063245553		0	
Step 5:									
From the ideal									
	Fabric Type		Dirt on	Fabric	Dirti	ness	Temp	berature	
Soft water	0	0		0		0.126491106		0	
Hard water	0.4	0.4		0.1		0		0.3	
from the worst					-				
	Fabric Type		Dirt on 1	Fabric	Dirti	ness	Temp	oerature	
Soft water	0.4	0.4		0.1		0		0.3	
Hard water	0	0		0		0.126491106		0	
Step 6:				-					
	$S_i^+$	$S_i^-$	$C_i^*$		result-		rank		
Soft water	0.126491106	0.50	9902	0.8012	237 1				
Hard water	0.509901951	0.12	6491	0.198763		2			

# 6. CONCLUSION

Results of Fuzzy logic controller of aforesaid inputs clearly shows that choosing high temperature and soft water, amount of detergent and washing time can be saved up to 70 percent.

Water	Temperature	Detergent	Washing
Туре			Time
Soft	Moderate	5.5	30.5
Soft	V.High	3.25	15.8
V.Hard	Moderate	8.88	52.6
V.Hard	V.High	6.63	37.9



So the proposition

of the paper is that the soft water and high temperature should be used in washing machines. It will not only reduce the quantity of detergent but also have positive effects on economy and fabrics. In this way, energy and washing time can be saved, which are the basic objectives of current century. We can attach any small size water filter with washing machine due to which amount of detergent and washing time can be saved, and a box for grey water, so that the water ejected from the machine can be cleaned so comparative study can be done in future.

#### **Conflict of interest**

The authors declare that they have no conflict of interest.

**Competing interest** 

The authors declare that they have no competing interest.

## Authors contributions

The authors contributed to each part of this paper equally. The authors read and approved the final manuscript.

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#### REFERENCES

- S. E. Abbas, M. A. Hebeshi and I. M. Taha, On Upper and Lower Contra-Continuous Fuzzy Multifunctions, Punjab Univ. j. math. 47, No. 1 (2017) 105-117.
- [2] K. Abeliotis, Impact of water hardness on consumers perception of laundry washing result in five European countries, International Journal of Consumer Studies **39**, (2015) 6066.
- [3] M. Akram and G. Shahzadi, Certain Characterization of m-Polar Fuzzy Graphs by Level Graphs, Punjab Univ. j. math. 49, No. 1 (2017) 1-12.
- [4] M. Aggarwal, Fuzzy Logic Controller for Washing Machine, IIT Kharagpur (2011).
- [5] M. Alhanjouri and A. A. Alhaddad, Optimize Wash Time of Washing Machine Using Fuzzy Logic, The Proceedings of The 7th ICTS, Bali, (2013), 77-80.
- [6] T. Ahmed, A. Ahmad and A. Toki, Fuzzy Logic Controller for Washing Machine with Five input and three output, International Journal of Latest Trends in Engineering and Technology (IJLTET) 7, No. 2 (2016) 136-143.
- [7] G. Bowman and R. Mealy, dnr.wi.gov/regulations/labcert/documents/training/basics-genchem.pdf, WSLH (2001) 45-46.
- [8] R. Chang, General chemistry: the essential concepts, Fifth Edition.
- [9] M. Demetgul, O.Ulkir and T. Waqar, Washing Machine Using Fuzzy Logic, Automation, Control and Intelligent Systems 2, No. 3 (2014) 27-32.
- [10] H. Han, Chun-Yi Su and Y. Stepanenko, Adaptive Control of a Class of Non-linear Systems with Non-Linearly Parameterized Fuzzy Approximation, IEEE Transactions on Fuzzy Systems 9, No. 2 (2001) 315-323.
- [11] H. Hans and T. Christoph, *Defuzzification in fuzzy controllers*, Journal of Intelligent and Fuzzy Systems 1, No. 2 (1993), 109 -123.
- [12] S. Hatagar and S. V. Halase, *Three Input One Output Fuzzy logic control of Washing Machine*, International Journal of Scientific Research Engineering and Technology (IJSRET) 4, No. 1 (2015) 57-62.
- [13] H. Kamaci, A. O. Atagun and E. Aygun, Difference Operations of Soft Matrices with Applications in Decision Making, Punjab Univ. j. math. 51, No. 3 (2019) 1-21.
- [14] M. A. Khan and Sumitra, Common Fixed Point Theorems for Converse Commuting and OWC Maps in Fuzzy Metric Spaces, Punjab Univ. j. math. 44, (2016) 57-63.
- [15] D. Kumar and Y. Haider, Fuzzy Logic Based Control System for Washing Machines, International Journal of Computer Science and Technology 4, No. 2 (2013) 198-200.
- [16] M. A. Malik and M. Riaz, *G*-subsets *G*-orbits of  $Q^*(\sqrt{n})$  under action of the Modular Group, Punjab Univ. j. math. **43**, (2011) 75-84.
- [17] M. A. Malik and M. Riaz, Orbits of  $Q^*(\sqrt{k^2m})$  under the action of the Modular Group PSL(2, Z), UPB Scientific Bulletin. Series A: Applied Mathematics and Physics **74**, No. 4 (2012) 109-116.
- [18] T. Mahmood, F. Mehmood and Q. Khan, Some Generalized Aggregation Operators for Cubic Hesitant Fuzzy Sets and Their Applications to Multi Criteria Decision Making, Punjab Univ. j. math. 49, No. 1 (2017) 31-49.
- [19] E. H. Mamdani, Application of Fuzzy Algorithms for Control of Simple Dynamic Plant, Proceedings of the Institution of Electrical Engineers 121, No. 12 (1974) 1585-1588.
- [20] M. Riaz, K. Naeem and M. O. Ahmad, Novel Concepts of Soft Sets with Applications, Annals of Fuzzy Mathematics and Informatics 13, No. 2 (2017) 239251.
- [21] M. Riaz and K. Naeem, Measurable Soft Mappings, Punjab Univ. j. math. 48, No. 2 (2016) 19-34.
- [22] M. Riaz and Z. Fatima, *Certain properties of soft metric spaces*, Journal of Fuzzy Mathematics **25**, No. 3 (2017) 543-560.
- [23] M. Riaz and M. R. Hashmi, Fuzzy parameterized fuzzy soft topology with applications, Annals of Fuzzy Mathematics and Informatics 13, No. 5 (2017) 593-613.

- [24] M. Riaz and M. R. Hashmi, Certain applications of fuzzy parameterized fuzzy soft sets in decision-making problems, International Journal of Algebra and Statistics 5, No. 2 (2016) 135-146.
- [25] M. Riaz and M. R. Hashmi, Fuzzy Parameterized Fuzzy Soft Compact Spaces with Decision-Making, Punjab Univ. j. math. 50, No. 2(2018) 131-145.
- [26] M. Riaz, M. R. Hashmi, *Fixed points of fuzzy neutrosophic soft mapping with decision-making*, Fixed point theory and applications 7, (2018) 1-10.
- [27] M. Riaz and M. R. Hashmi, *Fuzzy parameterized fuzzy soft metric spaces*, Journal of Mathematical Analysis **9**, No. 22 (2018) 25-36.
- [28] M. Riaz and S. T. Tehrim, On Bipolar Fuzzy Soft Topology with Application, Soft Comput. (Submitted) (2018).
- [29] M. Riaz and S. T. Tehrim, Certain Properties of Bipolar Fuzzy Soft Topology Via Q-Neighborhood, Punjab Univ. j. math. 51, No. 3 (2019) 113-131.
- [30] M. Shabir and M. Naz, On soft topological spaces, Comput. Math. Appl. 61, (2011) 1786-1799.
- [31] A. Sezgin and A. O. Atagn, On operations of soft sets, Comput. Math. Appl. 61, No. 5 (2011) 1457-1467.
- [32] A. Sezgin, A. O. Atagn and N. Çağman, Soft intersection near-rings with applications, Neural Compu. Appl. 21, No. 1 (2012) 221-229.
- [33] M. Shakeel, S. Abdullah and A. Fahmi, *Triangular Cubic Power Aggregation Operators and Their Applica*tion to Multiple Attribute Group Decision Making, Punjab Univ. j. math. 50, No. 3 (2018) 75-99.
- [34] M. Shakeel, S. Abdullah, M. S. A. Khan and K. Rahman, Averaging Aggregation Operators with Interval Pythagorean Trapezoidal Fuzzy Numbers and Their Application to Group Decision Making, Punjab Univ. j. math. 50, No. 2 (2018) 147-170.
- [35] T. J. Ross, Fuzzy Logic with Engineering Applications, John Wiley and Sons India (2010), 90-91.
- [36] W. J. Russell, Automatic Control for Washing Machines, Electrical Engineering 61, No. 2 (1942) 89-92.
- [37] K. Thinzar and T. Z. Soe, Operation System of Washing Machine with Fuzzy Logic Control System and Construction of Detergent box, International Journal of Science, Engineering and Technology Research (IJSETR) 5, No. 9 (2016) 2883-2888.
- [38] A. E. Tiryaki and R. Kazan, Bulasik Makinesinin Bulanik Mantik ile Nodellenmesi, Muhendis ve Makina Dergisi, 48, 565.
- [39] M. G. Voskoglou, Application of Fuzzy Numbers to Assessment of Human Skills, Punjab Univ. j. math. 50, No. 2 (2018) 85-96.
- [40] L. A. Zadeh, Fuzzy sets, Information and Control 8, (1965) 338-353.
- [41] W. Ai-Zhen and R. Guo-feng, *The design of neural network fuzzy controller in washing machine*, International Conference on Computing, Measurement, Control and Sensor Network (CMCSN), Shanxi, China, 1, (2012) 136-139.
- [42] https://www.saylor.org/books/, General Chemistry: Principles, Patterns, and Applications, Fifth Edition.
- [43] Soap and Detergent Association (SDA), Soaps and Detergent, (1994) 12-13.