Punjab University
Journal of Mathematics (ISSN 1016-2526)
Vol. 50(3)(2018) pp. 131-144

SEMT Valuations of Disjoint Union of Combs, Stars and Banana Trees

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Received: 18 August, 2017 / Accepted: 02 January, 2018 / Published online: 25 April, 2018

Abstract. A *Graph* $G = (V(G), E(G))$ is the set of points called *vertices* or(nodes) and the lines connecting these points are called *edges*. The number of vertices in a graph is called its *order* and the number of edges is called its *size*, usually denoted as $|V(G)| = n$ (or p) and $|E(G)| = m$ (or q) respectively.
A graph G with p nodes and q lines admits the edge magic total labeling if there exists a one-one, onto map $\psi : V(G) \cup E(G) \rightarrow \{\overline{1, p+q}\} = \{1, 2, 3, \ldots, p+q\}$
s.t weight of every edge is some same constant (say)k, such number k is called the magic constant. If a graph G has an edge magic total labeling $\psi : V(G) \rightarrow \{1, 2, 3, \ldots, p\}$ then ψ is called *super edge magic total*(SEMT) labeling. For graph G, *SEMD* is the number of isolated vertices whose union with G makes the resulting graph SEMT. $\mu_s(G)$, is the minimum non-negative integer n such that $G \cup nK_1$ SEMD will be $+\infty$ if no isolated vertex do this job. In this work SEMT labeling and deficiencies are determined for forests formed by two sided generalized combs, stars, combs and banana trees.

AMS (MOS) Subject Classification Codes: 05C78
Key Words: SEMT graph, SEMD, banana tree, two sided generalized comb.

1. Definitions, Notations and Results

In this paper graphs under discussion are simple and without directed lines. In a graph G, $V(G)$ and $E(G)$ both are finite sets of vertices and edges respectively. All basic definitions related to graph theory are present in [6, 22]. There are many types of labeling such as graceful labeling, odd graceful labeling, prime magic labeling, anti-magic labeling. The labeling which is discussed in this paper is SEMT labeling. First time SEMT assigning rule was put forward in [8] and the following Conjecture was proposed by authors:

Conjecture 1.1. [8] *Every tree is SEMT.*

Labeling scheme means assigning the labels to the elements of graphs. If labels are assigned only to its vertices then it is called a *vertex labeling*, if graph elements to be assigned labels are only edges then it is called the *edge labeling* and if elements are both vertices and edges then this assigning rule is named as *total labeling* of graph G. Graph G is then called *vertex labeled*, *edge labeled*, total labeled graph respectively [4]. If $\tau : E(G) \to \{\overline{1, q}\}$ is an edge labeling, where q denotes the size of G. Then we can put forward notion of *vertex weight* [4]. Vertex weight of some vertex x of G is given by $\sum \tau(xy)$ such that $xy \in E(G)$. Similarly, having assigning rule $\psi : V(G) \cup E(G) \to \{\overline{1, p+q}\}$, where G is of order p [4]. The notions of *vertex and edge weights* do appear, where weight of vertex x is $\psi(x) + \sum\limits_{xy \in E(G)} \psi(xy)$ and weight of edge (say) $q = xy$ is given by sum of its label and sum of labels of its incident vertices.
A labeling ψ is called *VMT labeling* such that $\psi : V(G) \cup E(G) \to \{1, 2, 3, \dots p+q\}$ *s.t.* $\psi(x) + \psi(xy) = k$ *i.e.* the weight of every vertex in a graph is same [7]. If a graph G has a total magic labeling ψ assigning vertices the smallest possible labels then ψ is called *SVMT(super vertex magic total)* labeling. A graph G with p vertices and q edges admits an *EMT labeling* if there exist a one-one, onto map $\psi : V(G) \cup E(G) \to \{\overline{1, p+q}\}$ *s.t.* weight of every edge in a graph is same (say) k, then k is called the magic constant. If a graph G has a edge magic total labeling ψ giving smallest labels to vertices then ψ is called *SEMT(super edge magic total)* labeling. Super magic labeling of complete graphs is given in [21].

Conjecture 1.2. [20] *Every Tree is EMT.*

Theorem 1.3. [8] *k odd, is the necessary and sufficient condition for a cycle C_k to be SEMT.*

Formulation of all results in this work is based on Lemma 1 [11], which gives us necessary and sufficient condition for a graph to be SEMT. The SEMD of a graph G, *deficiency of super edge magic map* denoted by $\mu_s(G)$, is given as

$$\mu_s(G) = \begin{cases} \min(M(G)) & ; M(G) \neq \phi \\ \infty & ; M(G) = \phi \end{cases}$$

where $M(G) = \{n \geq 0 : G \cup nK_1 \textit{is a SEMT}\}$.
Super edge magic deficiencies of many graphs are provided in [10].

Definition 1.4. *A path is a tree denoted by P_n with n vertices and $n-1$ edges. Its set of vertices and edges are given by $\{x_i; i = \overline{1, n}\}$ and $\{x_i x_{i+1}; i = \overline{1, n-1}\}$* [23]. *In a* star *number of vertices is at least* 3 *and in general a star of order α is given by $K_{1,\alpha-1}$ with vertex and edge sets given as $\{y_i; i = \overline{1, \alpha}\}$, $\{y_1 y_i; i = \overline{2, \alpha}\}$ respectively. A* comb *is a tree which is obtained by adding the new end vertices $y_1, y_2, y_3 \dots y_{n-1}$ with the vertices $x_2, x_3, \dots x_n$ of a path. So the new edges of a comb obtained are $\{x_{i+1} y_i; 1 \leq i \leq n-1\}$ and it is denoted by Cb_n* [5].
Let $\{K_{1,m_\rho}; \rho = \overline{1, k}\}$ is a collection of disjoint stars with set of vertices $V(K_{1,m_j}) = \{c_j, a_{j\rho}; \rho = \overline{1, m_j}\}$ and $deg(c_j) = m_j; 1 \leq j \leq k$. A tree in which we add a vertex a and make it adjacent to $a_{\rho 1}; 1 \leq \rho \leq k$ is called a banana tree *and it is denoted by $BT(m_1, m_2, \dots m_k)$* [13].

Definition 1.5. *Graph G obtained from the path $P_{s+1} : x_{0,1}, x_{1,1}, \ldots, x_{s,1}$, $s \geq 2$ by adding t_i with $1 \leq i \leq s$ new paths $x_{i,2}, x_{i,3}, \ldots, x_{i,t_i}$ of lengths $t_i - 2$, where $t_i \geq 2$, $1 \leq i \leq s$ and new edges $x_{i,1}x_{i,2}$ for $1 \leq i \leq s$ and this is denoted by $Cb_s(t_1, t_2, \ldots, t_s)$ and termed as* generalized comb. *If $t_1 = t_2 = t_3 = \ldots t_s = t$ then generalized comb becomes a* balanced generalized comb *and we write it as $Cb_s\,\underbrace{(t, t, t, \ldots, t)}_{s-times}$. In this paper $Cb_s\,\underbrace{(t, t, t, \ldots, t)}_{s-times}$ will be denoted shortly as $Cb_s(t, t, t, \ldots, t)$. $Cb_s(2, 2, \ldots, 2)$ is referred as a* comb *and it is denoted by Cb_s* [16]. *A* two sided generalized comb , *derived from n paths $\{x_{i,1}, x_{i,2}, \ldots, x_{i,m}; 1 \leq i \leq n, n \geq 2\}$ of length m, where m is odd by adding one new vertex $x_{0,\frac{m+1}{2}}$ and n new edges $\{x_{i,\frac{m+1}{2}}x_{i+1,\frac{m+1}{2}}; 0 \leq i \leq n-1\}$. It is denoted by $Cb^2_{n,m}$* [15].

In [14] it is proved that two sided generalized comb $Cb^2_{n,m}$ is SEMT. k-TEPC labeling for some families of convex polytopes for k = 3 was studied in [3]. [18] provides super totient labeling for several classes of graphs such as friendship graphs, wheel graphs, complete graphs and complete bipartite graphs. In [9], R_α, M_α, χ_α, ABC, GA, ABC_4 and GA_5 indices of $L(S(CNC_k[n]))$ were calculated. Existence of a super edge magic total (SEMT) labeling of some particular subclasses of the disjoint union of subdivided stars is provided in [2]. Super edge-magic deficiencies of acyclic graphs for instance disjoint union of shrub graph with star, disjoint union of the shrub graph with two stars and disjoint union of the shrub graph with path was investigated in [17]. [24] provides dimension and depth of monomial edge ideals of line and cycle graphs. Z. Raza [19] determined the abstract structure of the critical group of the graph $\widehat{W_{2n}}$ for $n \geq 2$, defined by removing the alternate spokes of a wheel graph with $2n$ rim vertices. S. Ahmad [1] proved that subdivided ladder admits magic evaluation having type $(1, 1, 1)$. He also proved that such a subdivision admits consecutive magic evaluation having type (1,1,0). In [12] authors carried the analysis of GCS segment by determining its aesthetic value using the log curvature graph (LCG).

Theorem 1.6. [14] *For $n \geq 2$,$m \geq 3$, the graph $G \cong Cb^2_{n,m}$* is a super edge magic total.

For graph $G \cong Cb^2_{n,m}$ with $|V(G)| = mn + 1$ and $|E(G)| = mn$, where $\{x_{i,j}; i = \overline{1,n}, j = \overline{1,m}\} \cup \{x_{0,\frac{m+1}{2}}\}$ and $\{x_{i,j}x_{i,j+1}; i = \overline{1,n}, j = \overline{1,m}\} \cup \{x_{i,\frac{m+1}{2}}x_{i+1,\frac{m+1}{2}};$ $i = \overline{1, n-1}\}$ respectively. It is proved that G is SEMT for this labeling $\psi : V(G) \to \{1, 2, \ldots, mn + 1\}$ defined in [14] as follows:
For $\frac{m-1}{2}$ is odd

$$\psi(x_{0,\frac{m+1}{2}}) = \tfrac{m+5}{4},$$

and for $j \equiv 1(mod2)$

$$\psi(x_{1,j}) = \begin{cases} \frac{j+1}{2} & ; 1 \leq j \leq \frac{m-1}{2} \\ \frac{j+3}{2} & ; \frac{m+3}{2} \leq j \leq m \end{cases}$$

$$\psi(x_{i,j}) = \begin{cases} \frac{m(i-1)+j+3}{2} & ; i \text{ odd}, j \text{ odd} \\ \frac{mi-j+4}{2} & ; i \text{ even}, j \text{ even} \\ \lceil \frac{mn}{2} \rceil + \frac{m(i-1)+j+2}{2} & ; i \text{ odd}, j \text{ even} \\ \lceil \frac{mn}{2} \rceil + \frac{im-j+3}{2} & ; i \text{ even}, j \text{ odd} \end{cases}$$

The set of edge weights given by the labeling ψ consists of the following mn consecutive integers $\{\lceil \frac{mn}{2} \rceil + 3, \lceil \frac{mn}{2} \rceil + 4, \ldots, \lceil \frac{3mn}{2} \rceil + 2\}$.
For $\frac{m-1}{2}$ is even

$$\psi(x_{0,\frac{m+1}{2}}) = \frac{m+3}{4},$$

and $j \equiv 0(\textit{mod } 2)$

$$\psi(x_{1,j}) = \begin{cases} \frac{j}{2} & ; 2 \leq j \leq \frac{m-1}{2} \\ \frac{j+2}{2} & ; \frac{m+3}{2} \leq j \leq m-1 \end{cases}$$

$$\psi(x_{i,j}) = \begin{cases} \frac{mi-j+3}{2} & ; i \text{ even}, j \text{ odd} \\ \frac{m(i-1)+j+2}{2} & ; i \text{ odd}, j \text{ even} \\ \lfloor \frac{mn}{2} \rfloor + \frac{m(i-1)+j+3}{2} & ; i \text{ odd}, j \text{ odd} \\ \lfloor \frac{mn}{2} \rfloor + \frac{mi-j+4}{2} & ; i \text{ even}, j \text{ even} \end{cases}$$

The set of edge weights given by the labeling ψ consists of the following mn consecutive integers $\{\lfloor \frac{mn}{2} \rfloor + 3, \lfloor \frac{mn}{2} \rfloor + 4, \ldots, \lfloor \frac{3mn}{2} \rfloor + 2\}$.

2. SEMT ASSIGNMENT AND SEMD OF DISJOINT UNIONS OF TWO SIDED GENERALIZED COMBS, STARS AND COMBS

Theorem 2.1. *For $n \geq 2$, $m \equiv 1(\text{mod } 2)$ and $m \geq 3$, we have*
(a) $Cb^2_{n,m} \cup K_{1,\alpha}$ is super edge magic total.
(b) $\mu_s(Cb^2_{n,m} \cup K_{1,\alpha-1}) \leq 1$, where $\alpha \geq 2$ and is given by

$$\alpha = \begin{cases} 2 + m(\frac{n}{2} - 1) + \lfloor \frac{m-1}{4} \rfloor + 2\lfloor \frac{m-2}{4} \rfloor & ; n \equiv 0(mod\, 2) \\ 3 + m(\frac{n-3}{2}) + 2\lfloor \frac{m-2}{4} \rfloor + 3\lfloor \frac{m-1}{4} \rfloor & ; n \equiv 1(mod\, 2) \end{cases}$$

Proof. (a): Consider the graph $G \cong Cb^2_{n,m} \cup K_{1,\alpha}$, where $V(K_{1,\alpha}) = \{y_q : 1 \leq q \leq \alpha + 1\}$ and $E(K_{1,\alpha}) = \{y_1 y_q;\, 2 \leq q \leq \alpha + 1\}$. Let $v = |V(G)|$ and $e = |E(G)|$, so we have $v = mn + \alpha + 2$ and $e = mn + \alpha$.
Case 1. For $\frac{m-1}{2} \equiv 1(mod\, 2)$.
Consider the map τ from $V(Cb^2_{n,m})$ to $\{1, 2, \ldots, mn + 1\}$ defined as follows:
For $1 \leq i \leq n, 1 \leq j \leq m$

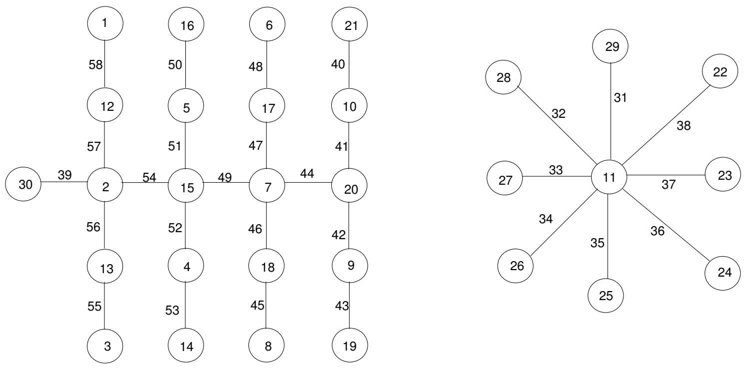

FIGURE 1. $Cb^2_{4,5} \cup K_{1,8}$

$$\tau(x_{i,j}) = \begin{cases} \frac{m(i-1)}{2} + \frac{j}{2} & ; i \text{ odd, } j \text{ even} \\ \left(\frac{mi}{2}\right) - \frac{j-1}{2} & ; i \text{ even, } j \text{ odd} \end{cases}$$

Now consider the labeling $\psi : V(G) \to \{1, 2, \ldots, v\}$ given by:
For $1 \leq q \leq \alpha + 1$

$$\psi(y_q) = \begin{cases} \lfloor \frac{mn}{2} \rfloor + 1 & ; q = 1 \\ mn + q & ; 2 \leq q \leq \alpha + 1 \end{cases}$$

Let $A = \lfloor \frac{mn}{2} \rfloor + 1$, $B = mn + \alpha + 1$

$$\tau(x_{i,j}) = \begin{cases} A + m(\frac{i-1}{2}) + \frac{j+1}{2} & ; i \text{ odd, } j \text{ odd} \\ A + \frac{mi}{2} - \frac{j}{2} + 1 & ; i \text{ even, } j \text{ even} \end{cases}$$

$$\tau(x_{\frac{m+1}{2},0}) = B + 1 = mn + \alpha + 2.$$

Edge weights for G is finite sequence of consecutive numbers starting with $\omega + 1$, ending at $\omega + e$, where $\omega = \lfloor \frac{mn}{2} \rfloor + 2$.
Case 2. For $\frac{m-1}{2} \equiv 0(\textit{mod } 2)$

$$\tau(x_{i,j}) = \begin{cases} m(\frac{i-1}{2}) + \frac{(j+1)}{2} & ; i \text{ odd, } j \text{ odd} \\ \frac{mi}{2} - \frac{j}{2} + 1 & ; i \text{ even, } j \text{ even} \end{cases}$$

$$\psi(y_q) = \begin{cases} \lceil \frac{mn}{2} \rceil + 1 & ; q = 1 \\ mn + q & ; 2 \leq q \leq \alpha + 1 \end{cases}$$

Let $A = \lceil \frac{mn}{2} \rceil + 1$, $B = mn + \alpha + 1$

$$\tau(x_{i,j}) = \begin{cases} A + \frac{mi}{2} - \frac{j-1}{2} & ; i \text{ even}, j \text{ odd} \\ A + m(\frac{i-1}{2}) + \frac{j}{2} & ; i \text{ odd}, j \text{ even} \end{cases}$$

$$\tau(x_{\frac{m+1}{2},0}) = B + 1 = mn + \alpha + 2.$$

Edge weights for G is arithmetic sequence with (common difference) $d = 1$, first term $a = \omega + 1$ and number of terms $n = e$, where $\omega = \lceil \frac{mn}{2} \rceil + 2$. Therefore by Lemma 1 [11], ψ gives desired labeling of G with magic sum $c = v + e + s$, with $s = \omega + 1$.

(b): Let $G_1 \cong Cb^2_{n,m} \cup K_{1,\alpha-1} \cup K_1$ with $V(G_1) = V(Cb^2_{n,m}) \cup V(K_{1,\alpha-1}) \cup \{z\}$ and $V(K_{1,\alpha-1}) = \{y_q; 1 \leq q \leq \alpha\}$ and $E(K_{1,\alpha-1}) = \{y_1 y_q; 2 \leq q \leq \alpha\}$. Let $\acute{v} = |V(G_1)| = mn + \alpha + 2$ and $\acute{e} = |E(G_1)| = mn + \alpha - 1$.

Consider the map $\acute{\psi}$ with domain $V(G_1)$ and co domain $\{1, 2, \dots, \acute{v}\}$, the mapping ψ at hand, in (a). Consider the assigning rule $\acute{\psi}$ given as:

$$\tau(x_{i,j}) = \acute{\psi}(x_{i,j}); 1 \leq i \leq n, 1 \leq j \leq m.$$

For $1 \leq q \leq \alpha$

$$\acute{\psi}(y_q) = \begin{cases} \lfloor \frac{mn}{2} \rfloor + 1 & ; q = 1, \frac{m-1}{2} \equiv 1 (mod\, 2) \\ \lceil \frac{mn}{2} \rceil + 1 & ; q = 1, \frac{m-1}{2} \equiv 0 (mod\, 2) \\ mn + q & ; 2 \leq q \leq \alpha \end{cases}$$

$$\acute{A} = \begin{cases} \lfloor \frac{mn}{2} \rfloor + 1 & ; \frac{m-1}{2} \equiv 1 (mod\, 2) \\ \lceil \frac{mn}{2} \rceil + 1 & ; \frac{m-1}{2} \equiv 0 (mod\, 2) \end{cases}$$

$$\acute{B} = mn + \alpha$$

$\acute{\psi}(z) = \acute{B} + 1 = mn + \alpha + 1$, $\acute{\psi}(x_{\frac{m+1}{2},0}) = \acute{B} + 2 = mn + \alpha + 2$.

Edge weights for G_1 is finite sequence of consecutive numbers starting with $\acute{\omega} + 1$ ending at $\acute{\omega} + e$, where

$$\acute{\omega} = \begin{cases} \lfloor \frac{mn}{2} \rfloor + 2 & ; \frac{m-1}{2} \equiv 1 (mod\, 2) \\ \lceil \frac{mn}{2} \rceil + 2 & ; \frac{m-1}{2} \equiv 0 (mod\, 2) \end{cases}$$

Therefore by Lemma 1 [11], $\acute{\psi}$ gives desired mapping of G_1. □

Theorem 2.2. *Consider $n \geq 2$, m odd, $m \geq 3$,*
(a) $Cb^2_{n,m} \cup Cb_\beta$ is super edge magic total.
(b) $\mu_s(Cb^2_{n,m} \cup Cb_{\beta-1}) \leq 1$ where $\beta \geq 2$ is given by

$$\beta = \begin{cases} 2 + m(\frac{n}{2} - 1) + \lfloor \frac{m-1}{4} \rfloor + 2\lfloor \frac{m-2}{4} \rfloor & ; n \equiv 0 (mod\, 2) \\ 3 + m(\frac{n-3}{2}) + 2\lfloor \frac{m-2}{4} \rfloor + 3\lfloor \frac{m-1}{4} \rfloor & ; n \equiv 1 (mod\, 2) \end{cases}$$

Proof. (a): Consider the graph $G \cong Cb^2_{n,m} \cup Cb_\beta$, where $V(Cb_\beta) = \{x_r; 0 \leq r \leq \beta\} \cup \{y_s; 1 \leq s \leq \beta\}$ and $E(Cb_\beta) = \{x_r x_{r+1}; 0 \leq r \leq \beta - 1\} \cup \{x_r y_s; 1 \leq r \leq \beta\}$. Let $v = |V(G)|$ and $e = |E(G)|$, so we get $v = mn + 2\beta + 2$ and $e = mn + 2\beta$. Consider mapping

η with domain $V(G)$ and range $\{1, 2, \ldots, v\}$, with the mapping τ at hand introduced in Theorem 2.1, consider mapping η given by:

$$\eta(x_{i,j}) = \tau(x_{i,j});\ 1 \le i \le n,\ 1 \le j \le m$$

For $0 \le r \le \beta$, $1 \le s \le \beta$,

$$\eta(x_0) = \begin{cases} \lfloor \frac{mn}{2} \rfloor + 1 & ; \frac{m-1}{2} \equiv 1(mod\,2) \\ \lceil \frac{mn}{2} \rceil + 1 & ; \frac{m-1}{2} \equiv 0(mod\,2) \end{cases}$$

$$\eta(x_r) = \begin{cases} \eta(x_0) + r & ; r \equiv 0(mod\,2) \\ mn + \beta + r + 1 & ; r \equiv 1(mod\,2) \end{cases}$$

and

$$\eta(y_s) = \begin{cases} mn + \beta + s + 1 & ; s \equiv 0(mod\,2) \\ \eta(x_0) + s & ; s \equiv 1(mod\,2) \end{cases}$$

Using the labeling τ defined in Theorem 2.1 with the following values of A and B

$$A = \begin{cases} \lfloor \frac{mn}{2} \rfloor + \beta + 1 & ; \frac{m-1}{2} \equiv 1(mod\,2) \\ \lceil \frac{mn}{2} \rceil + \beta + 1 & ; \frac{m-1}{2} \equiv 0(mod\,2) \end{cases}$$

$$B = mn + 2\beta + 1,\ \eta(x_{\frac{m+1}{2},0}) = B + 1 = mn + 2\beta + 2.$$

Edge weights for G is an Arithmetic sequence with $d = 1, a = \omega + 1$ and $n = e$, where

$$\omega = \begin{cases} \lfloor \frac{mn}{2} \rfloor + \beta + 2 & ; \frac{m-1}{2} \equiv 1(mod\,2) \\ \lceil \frac{mn}{2} \rceil + \beta + 2 & ; \frac{m-1}{2} \equiv 0(mod\,2) \end{cases}$$

Therefore by Lemma 1 [11], η gives desired labeling of G with magic sum $c = v + e + s$, with $s = \omega + 1$.

(b): Let $G_1 \cong Cb^2_{n,m} \cup Cb_{\beta-1} \cup K_1$, where $V(Cb_{\beta-1}) = \{x_r; 0 \le r \le \beta - 1\} \cup \{y_s; 1 \le s \le \beta - 1\}$, $V(K_1) = \{z\}$ and $E(Cb_{\beta-1}) = \{x_r x_{r+1}; 0 \le r \le \beta - 2\} \cup \{x_r y_s; 1 \le r \le \beta - 1\}$. Let $\acute{v} = |V(G_1)|$ and $\acute{e} = |E(G_1)|$, so we get $\acute{v} = mn + 2\beta + 1$ and
$\acute{e} = mn + 2(\beta - 1)$. Consider the labeling $\acute{\eta} : V(G_1) \to \{1, 2, \ldots, \acute{v}\}$, with labeling τ defined in Theorem 2.1, now we define the labeling $\acute{\eta}$ as follows:

$$\acute{\eta}(x_{i,j}) = \tau(x_{i,j});\ 1 \le i \le n,\ 1 \le j \le m.$$

For $0 \le r \le \beta - 1$, $1 \le s \le \beta - 1$

$$\acute{\eta}(x_0) = \eta(x_0),$$

$$\acute{\eta}(x_r) = \begin{cases} \acute{\eta}(x_0) + r = \eta(x_r) & ; r \equiv 0(mod\,2) \\ mn + \beta + r = \eta(x_r) - 1 & ; r \equiv 1(mod\,2) \end{cases}$$

$$\acute{\eta}(y_s) = \begin{cases} mn + \beta + s = \eta(y_s) - 1 & ; s \equiv 0(mod\,2) \\ \acute{\eta}(x_0) + s = \eta(y_s) & ; s \equiv 1(mod\,2) \end{cases}$$

Using labeling τ in Theorem 2.1 with the following values of A and B

$$A = \begin{cases} \lfloor \frac{mn}{2} \rfloor + \beta & ; \frac{m-1}{2} \equiv 1(mod\,2) \\ \lceil \frac{mn}{2} \rceil + \beta & ; \frac{m-1}{2} \equiv 0(mod\,2) \end{cases}$$

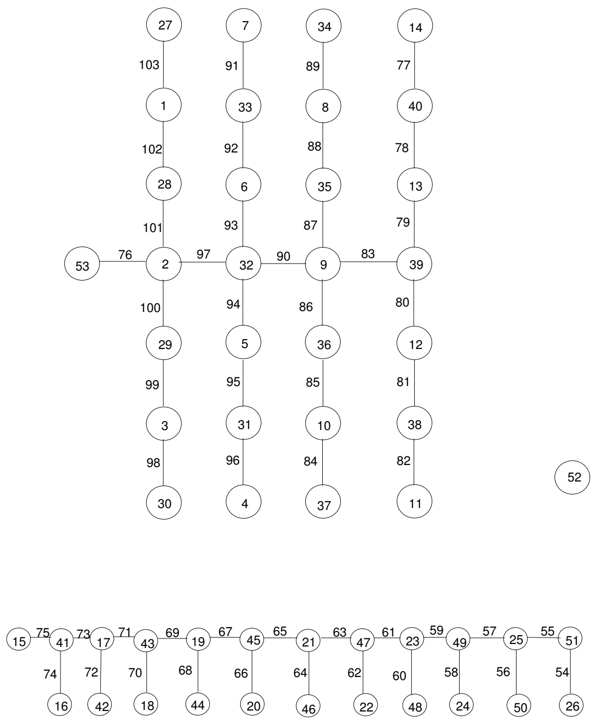

FIGURE 2. $Cb_{4,7}^2 \cup Cb_{11} \cup K_1$

$$B = mn + 2(\beta - 1) + 1,$$
$$\acute{\eta}(z) = B + 1 \text{ and } \acute{\eta}(x_{\frac{m+1}{2},0}) = B + 2.$$

The set of edge weights of graph G_1 is given as $\{\acute{\omega} + 1, \acute{\omega} + 2, ..., \acute{\omega} + \acute{e}\}$, where

$$\acute{\omega} = \begin{cases} \lfloor \frac{mn}{2} \rfloor + \beta + 1 & ; \frac{m-1}{2} \equiv 1(mod\, 2) \\ \\ \lceil \frac{mn}{2} \rceil + \beta + 1 & ; \frac{m-1}{2} \equiv 0(mod\, 2) \end{cases}$$

Therefore by Lemma 1 [11], $\acute{\eta}$ gives desired labeling of G_1. $\square$

3. SEMT ASSIGNMENT AND SEMD OF DISJOINT UNIONS OF TWO SIDED GENERALIZED COMBS, BANANA TREES AND BALANCED GENERALIZED COMBS

Theorem 3.1. *For $n \geq 2$, $m \equiv 1(mod\, 2)$, $m \geq 3$, $n_1 \geq 1$ and $n_2 \geq 2$*
(a) $Cb_{n,m}^2 \cup BT(n_1, n_2)$ *is super edge magic total.*
(b)(i) $\mu_s(Cb_{n,m}^2 \cup BT(n_1 - 1, n_2)) \leq 1$.
(ii)$\mu_s(Cb_{n,m}^2 \cup BT(n_1, n_2 - 1)) \leq 1$ *with*

$$n_1+n_2=\begin{cases} 2+m(\frac{n}{2}-1)+\lfloor\frac{m-1}{4}\rfloor+2\lfloor\frac{m-2}{4}\rfloor & ; n\equiv 0(mod\,2)\\ \\ 3+m(\frac{n-3}{2})+2\lfloor\frac{m-2}{4}\rfloor+3\lfloor\frac{m-1}{4}\rfloor & ; n\equiv 1(mod\,2)\end{cases}$$

Proof. (a): Consider the graph $G\cong Cb^2_{n,m}\cup BT(n_1,n_2)$,
where $V(BT(n_1,n_2))=\{a_{1\ell};1\leq l\leq n_1\}\cup\{a_{2t};1\leq t\leq n_2\}\cup\{c_1,c_2\}\cup\{a\}$ and $E(BT(n_1,n_2))=\{aa_{11}\}\cup\{aa_{21}\}\cup\{c_1a_{1\ell};1\leq l\leq n_1\}\cup\{c_2a_{2t};1\leq t\leq n_2\}$. Let $v=|V(G)|$ and $e=|E(G)|$, so we get $v=mn+(n_1+n_2)+4$ and $e=v-2=mn+(n_1+n_2)+2$.
With mapping τ at hand from Theorem 2.1, consider mapping ϑ from $V(G)$ to $\{1,2,\ldots,v\}$ given by

$$\vartheta(x_{i,j})=\tau(x_{i,j})\;;1\leq i\leq n\;,1\leq j\leq m,$$

$$\vartheta(a)=\begin{cases}\lfloor\frac{mn}{2}\rfloor+1 & ;\frac{m-1}{2}\equiv 1(mod\,2)\\ \\ \lceil\frac{mn}{2}\rceil+1 & ;\frac{m-1}{2}\equiv 0(mod\,2)\end{cases}$$

$$\vartheta(c_1)=\vartheta(a)+1,\;\vartheta(c_2)=\vartheta(a)+2$$

$$\vartheta(a_{1\ell})=\begin{cases} mn+4 & ;\ell=1\\ mn+n_1-\ell+5 & ;2\leq\ell\leq n_1\end{cases}$$

and

$$\vartheta(a_{2t})=\begin{cases} mn+n_1+4 & ;t=2\\ mn+n_1+5 & ;t=1\\ mn+(n_1+n_2)-t+6 & ;3\leq t\leq n_2\end{cases}$$

Considering map τ defined in Theorem 2.1 with the following values of A and B

$$A=\vartheta(a)+2,\;B=mn+(n_1+n_2)+3$$
$$\vartheta(x_{\frac{m+1}{2},0})=B+1.$$

Edge weights for G induced by ϑ is a sequence of consecutive integers starting with $\omega+1$ and ending at $\omega+(e-1)$, where

$$\omega=\begin{cases}\lfloor\frac{mn}{2}\rfloor+4 & ;\frac{m-1}{2}\equiv 1(mod\,2)\\ \\ \lceil\frac{mn}{2}\rceil+4 & ;\frac{m-1}{2}\equiv 0(mod\,2)\end{cases}$$

Therefore by Lemma 1 [11], ϑ gives desired SEMT mapping of G.
(b) Case (i): Consider the graph $G_1\cong Cb^2_{n,m}\cup BT(n_1-1,n_2)\cup K_1$ where $V(BT(n_1-1,n_2))=\{a_{1\ell};1\leq\ell\leq n_1-1\}\cup\{a_{2t};1\leq t\leq n_2\}\cup\{c_1,c_2\}\cup\{a\}\cup\{z\}$ and $E(BT(n_1-1,n_2))=\{aa_{11}\}\cup\{aa_{21}\}\cup\{c_1a_{1\ell};1\leq l\leq n_1-1\}\cup\{c_2a_{2t};1\leq t\leq n_2\}$.
Let $\acute{v}=|V(G_1)|$, $\acute{e}=|E(G_1)|$, so we get $\acute{v}=mn+(n_1+n_2)+4$ and $\acute{e}=v-3$
$=mn+(n_1+n_2)+1$.
Utilizing the mapping introduced in Theorem 2.1, consider mapping $\acute{\vartheta}$ from $V(G_1)$ to $\{1,2,\ldots,\acute{v}\}$ given by

$$\vartheta(x_{i,j})=\tau(x_{i,j});\,1\leq i\leq n,\,1\leq j\leq m,$$

$$\acute{\vartheta}(a_{1\ell})=\begin{cases} mn+4 & ;\ell=1\\ mn+(n_1-1)-\ell+5 & ;2\leq\ell\leq n_1-1\end{cases}$$

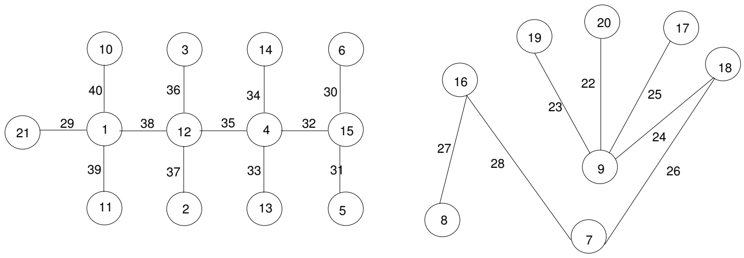

FIGURE 3. $Cb^2_{4,3} \cup BT(1,4)$

$$\acute{\vartheta}(a_{2t}) = \begin{cases} mn + (n_1 - 1) + 4 & ; t = 2 \\ mn + (n_1 - 1) + 5 & ; t = 1 \\ mn + (n_1 + n_2 - 1) - t + 6 & ; 3 \leq t \leq n_2 \end{cases}$$

Case (ii): Consider the graph $G_1 \cong Cb^2_{n,m} \cup BT(n_1, n_2 - 1) \cup K_1$, where $V(BT(n_1, n_2 - 1)) = \{a_{1\ell}; 1 \leq \ell \leq n_1\} \cup \{a_{2t}; 1 \leq t \leq n_2 - 1\} \cup \{c_1, c_2\} \cup \{a\} \cup \{z\}$ and $E(BT(n_1, n_2 - 1)) = \{aa_{11}\} \cup \{aa_{21}\} \cup \{c_1 a_{1\ell}; 1 \leq \ell \leq n_1\} \cup \{c_2 a_{2t}; 1 \leq t \leq n_2 - 1\}$. Let $\acute{v} = |V(G_1)|$, $\acute{e} = |E(G_1)|$, so we get $\acute{v} = mn + (n_1 + n_2) + 4$, $\acute{e} = v - 3 = mn + (n_1 + n_2) + 1$.

With τ introduced in Theorem 2.1, now we put forward the mapping $\acute{\vartheta}$ from $V(G_1)$ to $\{1, 2, \ldots, \acute{v}\}$ defined as

$$\acute{\vartheta}(x_{i,j}) = \tau(x_{i,j}); 1 \leq i \leq n, 1 \leq j \leq m,$$

$$\acute{\vartheta}(a_{1\ell}) = \vartheta(a_{1\ell}),$$

$$\acute{\vartheta}(a_{2t}) = \begin{cases} mn + n_1 + 4 & ; t = 2 \\ mn + n_1 + 5 & ; t = 1 \\ mn + (n_1 + n_2 - 1) - t + 6 & ; 3 \leq t \leq n_2 - 1 \end{cases}$$

$$\acute{\vartheta}(z) = \vartheta(x_{\frac{m+1}{2}}, 0) - 1, \acute{\vartheta}(x_{\frac{m+1}{2}, 0}) = \vartheta(x_{\frac{m+1}{2}}, 0).$$

Therefore by Lemma 1 [11], $\acute{\vartheta}$ gives desired labeling of G_1 giving same edge weight $\acute{c}$ for all edges, where $\acute{c} = \acute{v} + \acute{e} + s$ with $s = \acute{\omega} + 1$. □

Theorem 3.2. *For $n \geq 1$, $m \geq 3$ and $m \equiv 1(mod\, 2)$,*
$(a)(i)$ $Cb^2_{n,m} \cup Cb_d\, (\underbrace{\frac{\alpha}{d}, \frac{\alpha}{d}, \frac{\alpha}{d} \ldots \frac{\alpha}{d}}_{d-times})$ *is super edge magic total, where $1 \leq d \leq \alpha$ and $d \mid \alpha$.*
(ii) $Cb^2_{n,m} \cup Cb_{\acute{d}}\, (\underbrace{\frac{\alpha-1}{\acute{d}}, \frac{\alpha-1}{\acute{d}}, \frac{\alpha-1}{\acute{d}} \ldots \frac{\alpha-1}{\acute{d}}}_{\acute{d}-times})$ *is super edge magic total, where $1 \leq \acute{d} \leq \alpha - 1$ and $\acute{d} \mid \alpha - 1$.*
$(b)(i)$ $\mu_s(Cb^2_{n,m} \cup Cb_t\, (\underbrace{\frac{\alpha-2}{t}, \frac{\alpha-2}{t}, \frac{\alpha-2}{t} \ldots \frac{\alpha-2}{t}}_{t-times})) \leq 1$ *where $1 \leq t \leq \alpha - 1$ and*

$t \mid \alpha - 1$.
(ii) $\mu_s(Cb^2_{n,m} \cup Cb_{\acute{t}}\underbrace{(\frac{\alpha-3}{\acute{t}}, \frac{\alpha-3}{\acute{t}}, \frac{\alpha-3}{\acute{t}} \dots \frac{\alpha-3}{\acute{t}})}_{\acute{t}-times}) \leq 1$ *where* $1 \leq \acute{t} \leq \alpha - 3$ *and* $\acute{t} \mid \alpha - 3$.
Where

$$\alpha = \begin{cases} mn + m - 6\lfloor \frac{m}{4} \rfloor - 3 & ; n \equiv 0(mod\, 2), \frac{m-1}{2} \equiv 0(mod\, 2) \\ mn + m - 6\lfloor \frac{m}{4} \rfloor - 2 & ; n \equiv 1(mod\, 2), \frac{m-1}{2} \equiv 0(mod\, 2) \\ mn + m - 5\lfloor \frac{m+1}{4} \rfloor - \lfloor \frac{m-2}{4} \rfloor & ; n \equiv 0(mod\, 2), \frac{m-1}{2} \equiv 1(mod\, 2) \\ mn + m - 5\lfloor \frac{m+1}{4} \rfloor - \frac{m+1}{4} & ; n \equiv 1(mod\, 2), \frac{m-1}{2} \equiv 1(mod\, 2) \end{cases}$$

Proof. (a): Consider the graph $G \cong Cb^2_{n,m} \cup Cb_k(\ell, \ell, \ell, \dots, \ell)$; $k, \ell \geq 2$, where $V(Cb_k(\ell, \ell, \ell, \dots, \ell)) = \{y_{r,s}; 1 \leq r \leq k, 1 \leq s \leq \ell\} \cup \{y_{0,1}\}$ and $E(Cb_k(\ell, \ell, \ell, \dots, \ell)) = \{y_{r,s}y_{r,s+1}; 1 \leq r \leq k, 1 \leq s \leq \ell - 1\} \cup \{y_{r,1}y_{r+1,1}; 0 \leq r \leq k-1\}$, also we have $G_1 \cong Cb^2_{n,m} \cup Cb_k(\ell, \ell, \ell, \dots, \ell) \cup \{z\}$. Let $v = |V(G)|$, $e = |E(G)|$, $\acute{v} = |V(G_1)|$ and $\acute{e} = |E(G_1)|$. We get $v = mn + \ell k + 2$, $e = mn + \ell k$, $\acute{v} = mn + k\ell + 3$ and $\acute{e} = e$. Keep in mind labeling of τ already defined Theorem 2.1, now we define the labeling $\chi : V(Cb_k(\ell, \ell, \ell, \dots, \ell)) \to \{1, 2, \dots, \ell k + 1\}$.
For $\frac{m-1}{2} \equiv 1(mod\, 2)$,
For $n \equiv 1(mod\, 2)$,

$$\chi(y_{r,s}) = \begin{cases} \tau(x_{n,m-1}) + \ell(\frac{r-1}{2}) + \frac{s}{2} + 1 & ; r \equiv 1(mod\, 2), s \equiv 0(mod\, 2) \\ \tau(x_{n,m-1}) + \frac{\ell r}{2} - (\frac{s-3}{2}) & ; r \equiv 0(mod\, 2), s \equiv 1(mod\, 2) \\ \tau(x_{n,m}) + \ell(\frac{r-1}{2}) + (\frac{s-1}{2}) + 1 & ; r \equiv 1(mod\, 2), s \equiv 1(mod\, 2) \\ \tau(x_{n,m}) + \frac{\ell r}{2} - (\frac{s}{2}) + 1 & ; r \equiv 0(mod\, 2), s \equiv 0(mod\, 2) \\ \tau(x_{n,m-1}) + 1 & ; r = 0, s = 1 \end{cases}$$

For $n \equiv 0(mod\, 2)$

$$\chi(y_{r,s}) = \begin{cases} \tau(x_{n,1}) + \ell(\frac{r-1}{2}) + \frac{s}{2} + 1 & ; r \equiv 1(mod\, 2), s \equiv 0(mod\, 2) \\ \tau(x_{n,1}) + \frac{\ell r}{2} - (\frac{s-3}{2}) & ; r \equiv 0(mod\, 2), s \equiv 1(mod\, 2) \\ \tau(x_{n,2}) + \ell(\frac{r-1}{2}) + (\frac{s-1}{2}) + 1 & ; r \equiv 1(mod\, 2), s \equiv 1(mod\, 2) \\ \tau(x_{n,2}) + \frac{\ell r}{2} - (\frac{s}{2}) + 1 & ; r \equiv 0(mod\, 2), s \equiv 0(mod\, 2) \\ \tau(x_{n,1}) + 1 & ; r = 0, s = 1 \end{cases}$$

For $\frac{m-1}{2} \equiv 0(mod\, 2)$,
For $n \equiv 1(mod\, 2)$,

$$\chi(y_{r,s}) = \begin{cases} \tau(x_{n,m}) + \ell(\frac{r-1}{2}) + \frac{s}{2} + 1 & ; r \equiv 1(mod\, 2), s \equiv 0(mod\, 2) \\ \tau(x_{n,m}) + \frac{\ell r}{2} - (\frac{s-3}{2}) & ; r \equiv 0(mod\, 2), s \equiv 1(mod\, 2) \\ \tau(x_{n,m-1}) + \ell(\frac{r-1}{2}) + (\frac{s-1}{2}) + 1 & ; r \equiv 1(mod\, 2), s \equiv 1(mod\, 2) \\ \tau(x_{n,m-1}) + \frac{\ell r}{2} - (\frac{s}{2}) + 1 & ; r \equiv 0(mod\, 2), s \equiv 0(mod\, 2) \\ \tau(x_{n,m}) + 1 & ; r = 0, s = 1 \end{cases}$$

For $n \equiv 0(mod\,2)$

$$\chi(y_{r,s}) = \begin{cases} \tau(x_{n,2}) + \ell(\frac{r-1}{2}) + \frac{s}{2} + 1 & ; r \equiv 1(mod\,2), s \equiv 0(mod\,2) \\ \tau(x_{n,2}) + \frac{\ell r}{2} - (\frac{s-3}{2}) & ; r \equiv 0(mod\,2), s \equiv 1(mod\,2) \\ \tau(x_{n,1}) + \ell(\frac{r-1}{2}) + (\frac{s-1}{2}) + 1 & ; r \equiv 1(mod\,2), s \equiv 1(mod\,2) \\ \tau(x_{n,1}) + \frac{\ell r}{2} - (\frac{s}{2}) + 1 & ; r \equiv 0(mod\,2), s \equiv 0(mod\,2) \\ \tau(x_{n,2}) + 1 & ; r = 0, s = 1 \end{cases}$$

Utilizing the mapping τ introduced in Theorem 2.1 (a) with A and B as follows:
For $(a)(i)$, $(b)(i)$
For $k \equiv 1(mod\,2)$ put $A = \chi(y_{k,\ell})$ and $B = \chi(y_{k,\ell-1})$
For $k \equiv 0(mod\,2)$ put $A = \chi(y_{k,1})$ and

$$B = \begin{cases} \chi(y_{k,2}) & ; \ell \neq 1 \\ \chi(y_{k-1,\ell}) & ; \ell = 1 \end{cases}$$

For $(a)(ii)$, $(b)(ii)$ put

$$A = \begin{cases} \chi(y_{k,\ell-1}) & ; \ell \neq 1 \\ \chi(y_{k-1,\ell}) & ; \ell = 1 \end{cases}$$

and $B = \chi(y_{k,\ell})$.
For $(a)(i)$, $(a)(ii)$,
we have mapping φ from $V(G)$ to $\{1, 2, 3, \ldots, v\}$ given by:

$$\varphi(y_{r,s}) = \chi(y_{r,s}),\ \varphi(y_{0,1}) = \chi(y_{0,1}).$$

Case. 1. For $\frac{m-1}{2} \equiv 1(mod\,2)$.
With the mapping τ at hand of case.1 from Theorem 2.1, $\frac{m-1}{2} \equiv 1(mod\,2)$ with the above mentioned values of A, B.

$$\varphi(x_{i,j,}) = \tau(x_{i,j});\ 1 \leq i \leq n, 1 \leq j \leq m$$
$$\varphi(x_{0,\frac{m+1}{2}}) = B + 1$$

Case. 2. For $\frac{m-1}{2} \equiv 0(mod\,2)$.
We will take mapping τ of case.2 from Theorem 2.1, $\frac{m-1}{2} \equiv 0(mod\,2)$ with the above mentioned values of A, B.

$$\varphi(x_{i,j,}) = \tau(x_{i,j});\ 1 \leq i \leq n, 1 \leq j \leq m$$
$$\varphi(x_{0,\frac{m+1}{2}}) = B + 1$$

For $(b)(i)$, $(b)(ii)$.
We have assigning map $\acute{\varphi}$ from $V(G_1)$ to $\{1, 2, 3, \ldots, \acute{v}\}$ given by:

$$\acute{\varphi}(y_{r,s}) = \chi(y_{r,s});\ 1 \leq r \leq k, 1 \leq s \leq \ell$$
$$\acute{\varphi}(y_{0,1}) = \chi(y_{0,1})$$

Case. 1 For $\frac{m-1}{2} \equiv 1(mod\,2)$.
Considering the labeling τ of case.1 from Theorem 2.1, $\frac{m-1}{2} \equiv 1(mod\,2)$ with the above mentioned values of A and B.

$$\acute{\varphi}(x_{i,j,}) = \tau(x_{i,j});\ 1 \leq i \leq n, 1 \leq j \leq m$$
$$\acute{\varphi}(x_{0,\frac{m+1}{2}}) = B + 2$$

Case. 2 For $\frac{m-1}{2} \equiv 0(mod\,2)$.
Considering labeling τ of Case.2 from Theorem 2.1, $\frac{m-1}{2} \equiv 0(mod\,2)$ with the above mentioned values of A and B.

$$\acute{\varphi}(x_{i,j,}) = \tau(x_{i,j}); 1 \leq i \leq n, 1 \leq j \leq m$$
$$\acute{\varphi}(x_{0,\frac{m+1}{2}}) = B + 2 \text{ and } \acute{\varphi}(z) = B + 1.$$

Set of edge weights of graphs G and G_1 are the sets of consecutive integers *s.t*
For $(a)(i)$ $\{\omega + 1, \omega + 2, \ldots \omega + e\}$
For $(a)(ii)$, $(b)(i)$ $\{\omega, \omega + 1, \ldots \omega + e - 1\}$
For $(a)(ii)$ $\{\omega - 1, \omega, \ldots \omega + e - 2\}$, with

$$\omega = \begin{cases} mn + m - 5\lfloor \frac{m}{4} \rfloor & ; n \equiv 0(mod\,2), \frac{m-1}{2} \equiv 0(mod\,2) \\ mn + m - 4\lfloor \frac{m}{4} \rfloor - \lfloor \frac{m-2}{4} \rfloor & ; n \equiv 1(mod\,2), \frac{m-1}{2} \equiv 0(mod\,2) \\ mn + m + 2\lfloor \frac{m}{2} \rfloor - \lfloor \frac{m-2}{4} \rfloor & ; n \equiv 0(mod\,2), \frac{m-1}{2} \equiv 1(mod\,2) \\ mn + m - 2\lfloor \frac{m}{2} \rfloor - \frac{m+1}{4} & ; n \equiv 1(mod\,2), \frac{m-1}{2} \equiv 1(mod\,2) \end{cases}$$

Therefore by Lemma 1 [11], φ and $\acute{\varphi}$ give desired mappings of G and G_1 with all edges

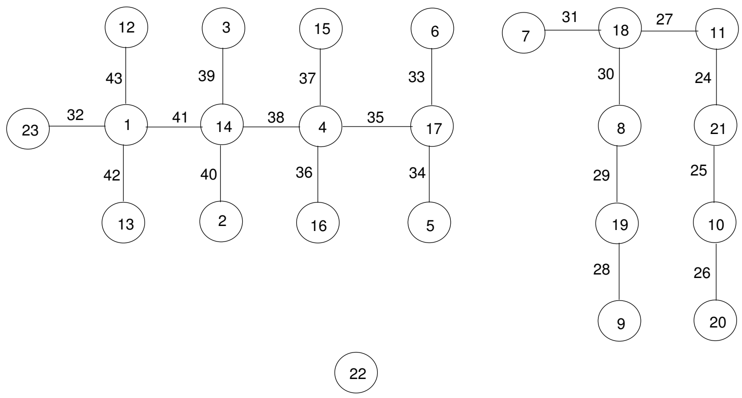

FIGURE 4. $Cb^2_{4,3} \cup Cb_2(4,4) \cup K_1$

having same weight c with $c = v + e + s$ and $\acute{c} = \acute{v} + \acute{e} + s$, with

$$s = \begin{cases} \omega + 1 & \text{for (a)(i)} \\ \omega & for\text{(a)(ii),(b)(i)} \\ \omega - 1 & for\text{(b)(ii)} \end{cases}$$

□

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