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Common Fixed Point Theorems for Converse Commuting and OWC Maps in Fuzzy Metric Spaces

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Abstract. The intent of this paper is to prove some common fixed point theorems for converse commuting and occasionally weakly compatible (owc) maps using implicit relations in fuzzy metric spaces. Our results extend, generalize, unify and fuzzify several results existing in the literature.

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1. Introduction

It was a turning point in the development of mathematics when Zadeh [26] introduced the concept of fuzzy set. This laid the foundation of fuzzy mathematics. Consequently, the last three decades were very productive for fuzzy mathematics.

The concepts of weak commuting, compatibility, non-compatibility and weak compatibility were frequently used to prove fixed point theorems for single and set valued maps satisfying certain conditions in different spaces.

The idea of converse commuting maps was first introduced by Lii [13] in 2002 which discuss the relation from the reverse and proved fixed point theorems for single valued maps in metric spaces. Since then many authors see [14], [19] have introduced the new concepts of converse commuting multi valued mappings and proved some fixed point theorems using these concepts.

As fuzzy mathematics is the hottest area of research now-a-days and new concepts are emerging very rapidly in this field. Among these concepts one is that of occasionally weakly compatible maps introduced by Thagafi and Shazad [3].This concept is more general among all the commutativity concepts and has opened a new venue for many mathematicians. See [1], [2], [4], [5], [6], [7], [8] etc.

The main purpose of this paper is to extend the concepts of converse commuting and occasionally weakly compatible (owc) maps to fuzzy metric spaces and prove some common fixed point theorems for single and set valued maps using implicit relations under strict contractive condition.

Our improvements in this paper are four-fold as;

(1) Relaxed the continuity of maps completely.
(2) Completeness of the space removed.
(3) Minimal type contractive condition used.
(4) The condition $\lim_{n\to\infty} M(x, y, t) = 1$ is not used.

We first give some preliminaries and definitions.

2. PRELIMINARIES

Definition 1. [9] A binary operation $* : [0, 1] \times [0, 1] \to [0, 1]$ is a continuous t-norm if $*$ is satisfying the following conditions:

(1) $*$ is commutative and associative;
(2) $*$ is continuous;
(3) $a * 1 = a$ for all $a \in [0, 1]$;
(4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$, $a, b, c, d \in [0, 1]$.

Definition 2. [9] A triplet $(X, M, *)$ is said to be a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following;

(FM-1) $M(x, y, t) > 0$;
(FM-2) $M(x, y, t) = 1$ if and only if $x = y$;
(FM-3) $M(x, y, t) = M(y, x, t)$;
(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(FM-5) $M(x, y, \cdot) : (0, \infty) \to (0, 1]$ is continuous.

Note that $M(x, y, t)$ can be thought of as the degree of nearness between x and y with respect to t.

Example 3. [20] *(Induced fuzzy metric space) Let (X, d) be a metric space, denote $a * b = ab$ for all $a, b \in [0, 1]$ and let M_d be fuzzy set on $X^2 \times (0, \infty)$ defined as follows:*

$$M_d(x, y, t) = \frac{t}{t + d(x, y)}$$

Then $(X, M_d, *)$ is a fuzzy metric space. We call this fuzzy metric induced by a metric d.

Throughout this paper X will represent the fuzzy metric space $(X, M, *)$.

Definition 4. [23] Let $\mathcal{CB}(X)$ denote the set of all non-empty closed and bounded subsets of X. Then for every $A, B \in \mathcal{CB}(X)$ and for every $t > 0$, denote

$$H(A, B, t) = \sup\{M(a, b, t), a \in A, b \in B\}$$

and

$$\delta_M(A, B, t) = \inf\{M(a, b, t), a \in A, b \in B\}.$$

If A consists of a single point a, we write $\delta_M(A, B, t) = \delta_M(a, B, t)$. If B also consists of a single point b, we write $\delta_M(A, B, t) = \delta_M(A, b, t)$.

It follows immediately from definition that

$$\delta_M(A, B, t) = \delta_M(B, A, t) \geq 0;$$
$$\delta_M(A, B, t) = 1 \Leftrightarrow A = B = \{a\} \text{ for all } A, B \in \mathcal{CB}(X).$$

Definition 5. [12] A point $x \in X$ is called a coincidence point (resp. fixed point) of $A : X \to X, B : X \to \mathcal{CB}(X)$ if $Ax \in Bx$ (resp. $x = Ax \in Bx$)

Definition 6. [12] Maps $A : X \to X$ and $B : X \to \mathcal{CB}(X)$ are said to be weakly compatible if they commute at coincidence points. i.e., if $ABx = BAx$, whenever $Ax \in Bx$.

Definition 7. [12] Maps $A : X \to X$ and $B : X \to \mathcal{CB}(X)$ are said to be occasionally weakly compatible (owc) if there exists some point $x \in X$ such that $Ax \in Bx$ and $ABx \subseteq BAx$.

Clearly weakly compatible maps are occasionally weakly compatible (owc). However, the converse is not true in general as shown in [12].

S. Sedghi et al. [22] proved the following result:

Theorem 8. . *Let F, G be mappings of a complete fuzzy metric space $(X, M, *)$ with $t * t = t$ for all $t \in [0, 1]$ into $\mathcal{CB}(X)$ and also f, g be mappings of X into itself satisfying*

(1) *$Fx \subseteq g(X)$, $Gx \subseteq f(X)$ for every $x \in X$;*
(2) *The pairs (F, f) and (G, g) are weakly compatible;*
(3) *There exists a constant $k \in (0, 1)$ such that*

$$\phi\begin{pmatrix} \delta_M(Fx, Gy, kt), M(fx, gy, t), H(fx, Fx, t), H(gy, Gy, t) \\ H(fx, Gy, \alpha t) * H(gy, Fx, (2-\alpha)t) \end{pmatrix} \geq 0 \tag{2. 1}$$

for every x, y in X, for every $t > 0$ and $\alpha \in (0, 2)$. Suppose that one of g and f is a closed subset of X then there exists a unique $p \in X$ such that $\{p\} = \{fp\} = \{gp\} = Fp = Gp$, and where $\phi \in \Phi = \{\phi : [0, 1]^5 \to [-1, 1]\}$ is a continuous function satisfying the following conditions:

(ϕ_1) $T(t_1, t_2, t_3, t_4, t_5)$ is increasing in t_1 and decreasing in $t_2, \ldots, t_5$.

(ϕ_2) $T(u, v, v, v, v) \geq 0$ implies that $u > v$ for all $v \in [0, 1]$ and $u \in [0, 1]$.

The aim of this paper is to establish common fixed point theorems by dropping the hypothesis of completeness of the space and deleting the two conditions $Fx \subseteq g(X)$, $Gx \subseteq f(X)$ of above theorem and using more general concept of occasionally weakly compatible maps.

3. Results and Discussion

We now, prove the following result.

Theorem 9. *Let $(X, M, *)$ be a fuzzy metric space with $t * t = t$ for all $t \in [0, 1]$ and let $A, B : X \to X$ and $S, T : X \to \mathcal{CB}(X)$ be single and set valued mappings respectively such that the pairs (A, S) and (B, T) are occasionally weakly compatible satisfying*

$$\phi\begin{pmatrix} \delta_M(Sx, Ty, t), M(Ax, By, t), H(Ax, Sx, t), \\ H(By, Ty, t), H(Ax, Ty, t) * H(By, Sx, t) \end{pmatrix} \geq 0 \tag{3. 1}$$

for every x, y in X, for every $t > 0$ and where $\phi : [0, 1]^5 \to [-1, 1]$ is a continuous function satisfying

(ϕ_1) ϕ is decreasing in t_2 and t_5 for all $t > 0$.

(ϕ_2) $\phi(u, u, v, v, u) \geq 0 \Rightarrow u \geq v$, for all $u, v \geq 0$.

Then A, B, S and T have a unique common fixed point in X.

Proof. Since the pairs (A, S) and (B, T) are occasionally weakly compatible (owc) maps, therefore, there exist two elements u, v in X such that $Au \in Su$, $ASu \subseteq SAu$ and $Bv \in Tv$, $BTv \subseteq TBv$.

First we prove that $Au = Bv$. As $Au \in Su$, $Bv \in Tv$, so,

$M(Au, Bv, t) \geq \delta_M(Su, Tv, t)$, $M(Au, Tv, t) \geq \delta_M(Su, Tv, t)$ and $M(Bv, Su, t) \geq \delta_M(Su, Tv, t)$.

If $Au \neq Bv$, then $\delta_M(Su, Tv, t) < 1$. Using (3. 1) for $x = u$, $y = v$

$$\phi \left(\begin{array}{c} \delta_M(Su, Tv, t), M(Au, Bv, t), H(Au, Su, t), \\ H(Bv, Tv, t), H(Au, Tv, t) * H(Bv, Su, t) \end{array} \right) \geq 0.$$

As $H(Au, Su, t) \geq M(Au, Su, t) \geq \delta_M(Au, Su, t)$, $a * b \geq c * d$, whenever $a \geq c$, $b \geq d$, so we have

$$\phi\left(\delta_M(Su, Tv, t), \delta_M(Su, Tv, t), 1, 1, \delta_M(Su, Tv, t)\right) \geq 0.$$

Also ϕ satisfies (ϕ_2), so $\delta_M(Su, Tv, t) = 1$, which gives $Au = Bv$. Now, we claim that $A^2u = Au$. Suppose not, then $\delta_M(Su, Tv, t) < 1$.

Again using (3. 1) for $x = Au$, $y = v$, we get

$$\phi \left(\begin{array}{c} \delta_M(SAu, Tv, t), M(AAu, Bv, t), H(AAu, Su, t), \\ H(Bv, Tv, t), H(AAu, Tv, t) * H(Bv, SAu, t) \end{array} \right) \geq 0,$$

i.e.

$$\phi\left(\delta_M(SAu, Tv, t), M(AAu, Bv, t), 1, 1, M(AAu, Tv, t) * M(Bv, SAu, t)\right) \geq 0.$$

Also, $Au \in Su$ and $ASu \in SAu$, so $AAu \in ASu \subseteq SAu$ and $Bv \in Tv$ and $BTv \subseteq TBv$, thus

$M(AAu, Bv, t) \geq \delta_M(SAu, Tv, t)$ and $M(Bv, SAu, t) \geq \delta_M(SAu, Tv, t)$ hence

$$\phi\left(\delta_M(SAu, Tv, t), \delta_M(SAu, Tv, t), 1, 1, \delta_M(SAu, Tv, t)\right) \geq 0.$$

But ϕ satisfies (ϕ_2), so $\delta_M(SAu, Tv, t) = 1$, a contradiction and hence $A^2u = Au = Bv$.

Similarly, we can show that $B^2v = Bv$.

Putting $Au = Bv = z$, then $Az = z = Bz$, $z \in Sz$ and $z \in Tz$.

Therefore, z is a fixed point of A, B, S and T.

For uniqueness , let $z \neq z'$ be another common fixed point of A, B, S and T, then by using (3. 1), we have,

$$\phi \left(\begin{array}{c} \delta_M(Sz, Tz', t), M(Az, Bz', t), H(Az, Sz, t), \\ H(Bz', Tz', t), H(Az, Tz', t) * H(Bz', Sz, t) \end{array} \right) \geq 0,$$

i.e.,

$$\phi\left(\ \delta_M(Sz, Tz', t), \delta_M(Az, Bz', t), 1, 1, \delta_M(Az, Tz', t) * \delta_M(Tz', Sz, t) \ \right) \geq 0.$$

By (ϕ_2), we get $\delta_M(Sz, Tz', t) = 1$ and hence $z = z'$ i.e., $z = z'$ is unique common fixed point of A, B, S and T. □

Now, we furnish an example to our theorem .

Example 10. [21] *Let $(X, M, *)$ be a fuzzy metric space in which $X = R^+$, $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ such that $M(x, y, t) = \frac{t}{t+|x-y|}$ for all $t > 0$. Now, we define the mappings $A, B : X \to X$ and $S, T : X \to \mathcal{CB}(X)$ as follows:*

$$A(X) = \begin{cases} 2x - 1, & \text{if } x \leq 5; \\ 2x, & \text{if } x > 5. \end{cases} \quad B(X) = \begin{cases} 3 - 2x, & \text{if } x \leq 1; \\ x + 1, & \text{if } x > 1. \end{cases}$$

$$S(X) = \begin{cases} \{1\}, & \textit{if } x < 2; \\ \textit{[2x,2x+5]}, & \textit{if } x \geq 2. \end{cases} \quad T(X) = \begin{cases} \{1\}, & \textit{if } x = 1; \\ \textit{[x,x+2]}, & \textit{otherwise.} \end{cases}$$

Define $\phi : [0,1]^5 \to [0,1]$ *as* $\phi(t_1, t_2, t_3, t_4, t_5) = \min\{t_1, t_2, t_3, t_4, t_5\}$. *Here the pairs* (A, S) *and* (B, T) *are owc and the contractive condition is also satisfied and '1' is unique common fixed point of* A, B, S *and* T.

Remark 11. Theorem 9 is a generalization of corresponding theorems of [15] and [25], since we extended the setting of single valued maps to the one of single and set valued maps by removing the condition of the completeness of the space and continuity of maps and concept of weakly compatibility has been replaced by more general concept of occasionally weakly compatible maps with a strict contractive condition.

Also, we deleted some assumptions of the function ϕ which are superfluous for our result but necessary for [15] and [25].

If we set $A = B$ in Theorem 9, then we get the following corollary.

Corollary 12. *Let* $(X, M, *)$ *be a fuzzy metric space with* $t * t = t$ *for all* $t \in [0, 1]$ *and let* $A : X \to X$ *and* $S, T : X \to \mathcal{CB}(X)$ *be single and set valued mappings respectively such that the pairs* (A, S) *and* (A, T) *are occasionally weakly compatible satisfying*

$$\phi \begin{pmatrix} \delta_M(Sx, Ty, t), M(Ax, Ay, t), H(Ax, Sx, t), \\ H(Ay, Ty, t), H(Ax, Ty, t) * H(Ay, Sx, t) \end{pmatrix} \geq 0 \tag{3. 2}$$

for every x, y *in* X*, for every* $t > 0$ *and* ϕ *satisfies* (ϕ_1) *and* (ϕ_2).

Then A, S *and* T *have a unique common fixed point in* X.

If we set $A = B$ and $S = T$ in Theorem 9, we get the following corollary.

Corollary 13. *Let* $(X, M, *)$ *be a fuzzy metric space with* $t * t = t$ *for all* $t \in [0, 1]$ *and let* $A : X \to X$ *and* $S : X \to \mathcal{CB}(X)$ *be single and set valued mappings respectively such that the pair* (A, S) *are occasionally weakly compatible satisfying*

$$\phi \begin{pmatrix} \delta_M(Sx, Sy, t), M(Ax, Ay, t), H(Ax, Sx, t), \\ H(Ay, Sy, t), H(Ax, Sy, t) * H(Ay, Sx, t) \end{pmatrix} \geq 0 \tag{3. 3}$$

for every x, y *in* X*, for every* $t > 0$ *and* ϕ *satisfies* (ϕ_1) *and* (ϕ_2).

Then A *and* S *have a unique common fixed point in* X.

Definition 14. [21] A point $x \in X$ is called a commuting point of $A, B : X \to X$, if $ABx = BAx$.

Definition 15. [13] Maps $A, B : X \to X$ are said to be converse commuting if $ABx = BAx$ implies $Ax = Bx$.

Remark 16. The concept of converse commuting mappings introduced by Lii [13] is reverse process of weakly compatible mappings.

Let in this note $C(A, S)$ denotes the set of converse commuting points of A and S.

Now, we prove the following result.

Theorem 17. *let* $A, B, S, T : X \to X$ *be self maps such that the pairs* (A, S) *and* (B, T) *are converse commuting maps satisfying*

$$\phi \begin{pmatrix} M(Sx, Ty, t), M(Ax, By, t), M(Ax, Sx, t), \\ M(By, Ty, t), M(Ax, Ty, t), M(By, Sx, t) \end{pmatrix} \geq 0, \tag{3. 4}$$

for every x, y *in* X*, for every* $t > 0$ *and* $\phi : [0,1]^6 \to [0,1]$ *is a continuous function satisfying*

(ϕ_1) $\phi(u,u,v,v,u,u) \geq 0$ or $\phi(u,u,u,v,u,u) \geq 0 \Rightarrow u \geq v$, for all $u,v \geq 0$.

If A and S, B and T have a commuting point, then A,B,S and T have a unique common fixed point in X.

Proof. Let $u \in C(A,S)$ and $v \in C(B,T)$, therefore, $ASu = SAu \Rightarrow Au = Su$, hence $M(Au,Su,t) = 1$, also $Au = Su \Rightarrow AAu = SAu = ASu$, and hence $M(AAu,SAu,t) = 1$. Similarly, $BTv = TBv \Rightarrow Bv = Tv$ therefore $M(Bv,Tv,t) = 1$ and so $M(BBv,TBv,t) = 1$.

First we prove that $Au = Bv$. If $Au \neq Bv$, then $M(Au,Bv,t) < 1$. Using (3. 4) for $x = u$, $y = v$ we get

$$\phi\left(\begin{array}{c} M(Su,Tv,t), M(Au,Bv,t), M(Au,Su,t), \\ M(Bv,Tv,t), M(Au,Tv,t), M(Bv,Su,t) \end{array}\right) \geq 0,$$

i.e.,

$$\phi\left(M(Su,Tv,t), M(Su,Tv,t), 1, 1, M(Su,Tv,t), M(Su,Tv,t)\right) \geq 0.$$

Also ϕ satisfies (ϕ_1) so $M(Su,Tv,t) = 1$, which gives $Au = Bv$. Now, we claim that $A^2u = Au$. Suppose not, then $M(AAu,Au,t) < 1$.

Again using (3. 4) for $x = Au$, $y = v$, we get

$$\phi\left(\begin{array}{c} M(SAu,Tv,t), M(AAu,Bv,t), M(AAu,SAu,t), \\ M(Bv,Tv,t), M(AAu,Tv,t), M(Bv,SAu,t) \end{array}\right) \geq 0,$$

i.e.,

$$\phi\left(\begin{array}{c} M(SAu,Tv,t), M(AAu,Bv,t), M(AAu,SAu,t), \\ 1, M(AAu,Tv,t), M(Bv,SAu,t) \end{array}\right) \geq 0.$$

As, $Au = Su$ and $AAu = ASu = SAu$ and ϕ satisfies (ϕ_1), so

$$\phi\left(M(AAu,Au,t), M(AAu,Au,t), 1, M(AAu,Au,t), M(AAu,Au,t)\right) \geq 0.$$

But ϕ satisfies (ϕ_1), so $M(AAu,Au,t) = 1$, a contradiction and hence $A^2u = Au = Bv$. Similarly, we can show that $B^2v = Bv$.

On the other hand, $AAu = ASu = SAu$, and $Bv = BBv = BTv = TBv$. Hence $Au = z$, is a common fixed point of A,B,S and T and uniqueness of fixed point follows easily from (3. 4). □

Example 18. [21] *Let $(X,M,*)$ be a fuzzy metric space in which $X = R^+$, $a*b = ab$ for all $a,b \in [0,1]$ such that $M(x,y,t) = \frac{t}{t+|x-y|}$ for all $t > 0$. Now, we define the mappings $A,B : X \to X$ and $S,T : X \to \mathcal{CB}(X)$ as follows:*

$$A(X) = \begin{cases} 2x-1, & \text{if } x < 2; \\ 2x, & \text{if } x \geq 2. \end{cases} \quad B(X) = \begin{cases} 2x-1, & \text{if } x \leq 2; \\ x+2, & \text{if } x > 2. \end{cases}$$

$$S(X) = \begin{cases} x^2, & \text{if } x < 2; \\ x^2-1, & \text{if } x \geq 2. \end{cases} \quad T(X) = \begin{cases} 3-2x^2, & \text{if } x \leq 2; \\ x^2-1, & \text{otherwise.} \end{cases}$$

Define $\phi : [0,1]^6 \to [0,1]$ as $\phi(t_1,t_2,t_3,t_4,t_5,t_6) = \min\{t_1,t_2,t_3,t_4,t_5,t_6\}$. Here the pairs (A,S) and (B,T) are converse commuting satisfying the contractive condition and '1' is unique common fixed point of A,B,S and T.

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