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An Empirical Assessment of Alternative Methods of Variance-Covariance Matrix

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Abstract

The current study aims at the estimation of a group of variance-covariance methods using the data set of the non-financial sector of the Pakistan stock exchange. The study compares nine covariance estimators using two assessment criteria of root mean square error and standard deviation of minimum variance portfolios to gauge on accuracy and effectiveness of estimators. The findings of the study based on RMSE and risk behaviour of MVPs suggest that portfolio managers receive no additional benefit for using more sophisticated measures against equally weighted variance-covariance estimators in the construction of portfolios.

Keywords: *Variance-Covariance Estimators, Portfolio Construction, Mean-Variance Optimization.*

Introduction

Markowitz (1952) first time proposed the idea of mean-variance portfolio optimization since then many applied the concept and remain in the favor of standard variant of the mean-variance approach (Chan, Karceski, & Lakonishok, 1999; Jagannathan & Ma, 2003). His revolutionary idea of investment allocation for risky assets gave birth to modern finance, which later labeled as modern portfolio theory (MPT). Backed by certain assumptions, the concept of Markowitz to develop portfolios for risky assets gained widespread recognition in the literature.

Over time, financial engineers developed different techniques for portfolios management but the fundamental work of Markowitz (1952) is still considered by many in active portfolio management (Tu & Zhou, 2011). The concept of Markowitz for portfolio selection using mean-variance criteria is a nonstop topic-of-debate in finance (Fletcher, 2009). DeMiguel, Garlappi, and Uppal (2007) and Elton and Gruber (1973) laud the phenomenal idea of the mean-variance for portfolio management and stress on the use of variance-covariance estimator for successful implementation of the mean-variance framework. Meanwhile, the method was criticized at a number of avenues. Disatnik and Benninga (2007) argue that the mean-variance method yields dubious and unhealthy estimates. Michaud (1989) calls the optimization by the mean-variance framework as an “estimation error maximizer” and titled the method an “enigma”. Ledoit and Wolf (2003) term estimation of the covariance matrix, a troubling part of the technique. Chopra and Ziemba (1993) call the portfolio model of Markowitz as input centered or sensitive to inputs. Best and Grauer (1991) question the robustness of the mean-variance optimization portfolio strategy. Chow, Cioffi, and Bingham (1995) criticize the framework on the investors’ utility function. According to Chow et al.

(1995), investors reject mean-variance optimization and claim that investors' utility is not just centered on expected return and risk function. Literature shows that researchers could not end up on a single comprehensive yet error-free method to portfolio optimization (Ly, 2019; Zakamulin, 2017).

Apart from the exceptional work of Markowitz (1952), the work on portfolio optimization was further enhanced by many other researchers. Konno and Yamazaki (1991) applied the mean absolute deviation model for portfolio optimization. Samuelson (1958) presented the idea of higher-order moments and their significance in portfolio construction which later enabled the development of the mean-variance skewness method. The work of Konno and Yamazaki (1991) was further extended by Feinstein and Thapa (1993). Levy and Samuelson (1992) applied linear programming, It was further extended by Tamiz and Jones (1996) in the form of goal programming for successful portfolio construction. In addition to these theories and models, there are other prominent strategies, portfolio rules, non-theory based diversification methods, and behavioral perspective of portfolio construction.

The literature on the topic of portfolio optimization is divided into two streams first, theoretical approach, and second implementation approach. The theoretical approach discusses assumptions whereas the implementation approach focuses on two inputs required for the successful implementation of the framework. So, an investor requires the computation of two fundamental inputs i.e. future return vector and variance-covariance matrix. This research primarily focuses on the second input, the estimation of a group of variance-covariance matrix, and their evaluation to recommend a suitable estimator to the investor of PSX.

The section below reviews literature related to a group of variance-covariance estimators actively considered by portfolio managers and researchers.

Literature Review

Ample literature can be found on the subject of portfolio optimization however, this section reviews the literature relating to variance-covariance estimators which gained prominent position and are generally recognized by researchers for the construction of portfolios.

The method of sample covariance matrix estimates pairwise covariance of the sample asset group and is based on historical covariance. The pairwise estimation of covariance is error-prone, specifically at that time when underlying asset-groups are greater than sample asset groups (Hwang, Xu, & In, 2018; Michaud, 1989; Pafka & Kondor, 2004). Sharpe (1963) improves the method of sample covariance matrix by suggesting a comparatively more vigorous method of covariance using a common factor (the market factor). Other researchers also tried to enhance the efficiency of the sample covariance estimation method. King (1966) considers other factors besides a single-common-factor. Blume (1971) and Vasicek (1973) improves the performance of the covariance estimator with an idea of mean-reverting tendency and by adjusting variation of betas respectively.

Husnain, Hassan, and Lamarque (2016) argue that the standard technique for the estimation of the covariance matrix is error-prone, either due to specification or estimation error. In the literature, non-theory-based or statistical measures are also used for the identification of factors relating to sample covariance i.e principal component analysis (PCA). Elton and Gruber (1973) recommend the usage of average correlation-based variance-covariance estimators for the placement of assets. Due to the instability of numerical estimators and estimation errors, statistical or non-theory-based diversification outperforms sophisticated theory-based optimization strategies (DeMiguel et al., 2007). The decision theory of statistics guides toward an optimal point between specification and estimation error. Researchers consider this fundamental norm of statistics for optimization between specification and estimation error. Stein (1956) argues that the optimal point can be derived by taking the weighted mean of both estimators.

Chan et al. (1999), Bengtsson and Holst (2002), and Ledoit and Wolf (2004) provide empirical evidence on the effectiveness of shrinkage-based estimation and portfolio of estimators for the estimation of the covariance matrix. Ledoit and Wolf (2004) propose the Bayesian shrinkage approach for use in the process of portfolio optimization in comparison to conventional sample covariance technique and single index covariance technique. This method reduces both estimation and specification errors in sample covariance. The technique provides a shrunk matrix, of the conventional sample matrix. In which all off-diagonal components (covariances) get shrunk without any change in diagonal components. Ledoit and Wolf (2004) remain successful in shrinking conventional sample covariance matrix to constant correlation covariance matrix. The technique of shrinkage covariance matrix was later challenged by Jagannathan and Ma (2003).

Jagannathan and Ma (2003) criticize the usage of more complex Ledoit and Wolf (2004) covariance estimator and propose the usage of the equally weighted mean of the sample covariance matrix for estimation or any other suitable covariance estimator in the process of portfolio optimization. Disatnik and Benninga (2007) argue that investors get no extra advantage for using complex shrinkage covariance estimators in comparison to equally weighted portfolios of covariance matrix estimators. However, equally-weighted covariance estimators and shrinkage based covariance estimator are relatively superior to sample covariance estimator. Theoretically, shrinkage based covariance estimators are more complex compared to the equally weighted portfolio of estimators and the conventional simple covariance estimator.

The literature documents a shred of evidence, as there exists no consensus on the usage of variance-covariance estimators in equity markets of Asian countries. Considering the fact, the current study compares nine variance-covariance estimators in the non-financial sector of the Pakistan stock exchange (PSX). To comment upon the performance of covariance estimators, literature reports usage two assessment criteria. The first is, the root mean square error (RMSE) and the second is the minimum variance portfolio (MVP) risk behavior measure, to gauge how effective an estimator is for the selection of an MVP.

The remainder of the study is arranged as follows: Section 3 explains the data description and methodology of the study followed by evaluation criteria. Section 4 presents the results and discussion followed by the conclusion at the end in section 5.

Data Description and Methodology

The sample data consists of equity prices of listed, Non-financial sector companies of Pakistan stock exchange (PSX). The data is collected from the official data portal of PSX. This study uses monthly data ranging from 01st of January 2001 to 27th of December 2019, for a relatively larger period of 18 years. The sample data is divided into two subsets, first from 01st of January 2001 to 29th of December 2009 to estimate covariance matrices, second from 01st of January 2010 to 27th of December 2019 for testing on the ex-post assessment of covariance matrices.

In the study, equally-weighted indices are developed, consisting of 22 non-financial sectors of PSX. The returns are calculated as per the assumption of continuous compounding.

The return $(R_{s,t})$ for each asset group is calculated as follows: $(R_{s,t}) = ln(P_t/P_{t-1})$. Here, ln represents the natural log of function, P_t represents the current price while P_{t-1} represents the price of the previous period for the asset group.

Variance Covariance Matrix Estimation

It is a square matrix composed of variances and covariances for all asset groups involved. The diagonal elements contain variances of each asset group while off-diagonal elements contain covariances for all potential pairs of asset groups. In other words, variance means squared mean deviation, whereas the covariance show movement of two asset groups.

The mathematical expression of the variance-covariance matrix is as follows:

$$\sum = \begin{bmatrix} \sum m_1^2/n & \sum m_1 m_2/n & \dots & \sum m_1 m_i/n \\ \sum m_2 m_1/n & \sum m_2^2/n & \vdots & \sum m_2 m_i/n \\ \dots & \dots & \dots & \dots \\ \sum m_i m_1/n & \sum m_i m_2/n & & \sum m_i^2/n \end{bmatrix}$$

Where $\sum$ represents variance-covariance matrix of $(i\times i)$,n shows data points for every asset group, m_1 show a mean deviation, m_1^2/n represents covariance between asset groups *i and j*.

Sample Variance-covariance Matrix

For a vector ($e \in P^n$), where, sample variance is denoted by σ^2 and the average of sample denoted by $\hat{y}$, then:

$$\hat{e} = \frac{1}{n} * (e_1 + e_2 + e_3 + \cdots + e_n) \text{ and}$$

$$\sigma 2 = \frac{1}{n} * ((e_1 - \hat{e})^2 + (e_2 - \hat{e})^2 + \cdots + (e_n - \hat{e})^2$$

Let, $Y = [y_1 + y_2 + y_3 + \cdots + y_n] \in P^{m*n}$, here each column y_i denotes a value in P^n. Now, for variance, the values are taken from the data projection following $\rho \in P^m$, s.t.:

$$e = (\rho^T x_1 + \rho^T x_2 + \rho^T x_3 + \cdots \rho^T x_n) = \rho^T Y \in P^n$$

While the parallel sample mean and variance are:

$$\sigma^2(\rho) = \frac{1}{n}\sum_{k=1}^{n}(\rho^T y_k - \rho^T \hat{y})^2, \hat{e} = \rho^T \hat{y}$$

In the previous equation, the sample mean for $\hat{y} = \frac{1}{n}(x_1 + x_2 + x_3 + x_4 + \cdots + x_n) \in P^m$ while the variance in the quadratic form with direction ρ is as follows:

$$\sigma^2(\rho) = \frac{1}{m}\sum_{k=1}^{m}[\rho^T(y_k - \hat{y})]^2 = \rho^T \Sigma \rho$$

In the previous equation, the sample variance-covariance is shown by Σ, which can be presented as follows:

$$\Sigma_{sample} = \frac{1}{n} * \Sigma_{k=1}^{n}(y_k - \hat{y})(y_k - \hat{y})^T \text{ ………………(Eq-01)}$$

In equation-1, the covariance matrix holds the properties of semi-definite, positive and symmetrical. By using this covariance matrix, the variance can be found in any direction.

Constant Correlation Covariance Matrix (Overall Mean)

Chan et al. (1999) argue on the usefulness of the covariance matrix compared to other alternatives, claiming that the covariance matrix measure is more appropriate. Elton and Gruber (1973) estimate the covariance matrix through the constant correlation method. Under the assumption that the return variance of each asset group is a sample return while the covariance of an asset class is linked with the same correlation coefficient. For this reason, here average coefficient of correlations is taken for all asset groups. We know $\sigma_{eu} = \varphi_{e,m}\sigma_e\sigma_u$, then:

$$\sigma_{eu}\begin{cases}\sigma_{eu} = \sigma_e^2 \text{ if } e = u \\ \sigma_{eu} = \varphi_{e,u}\,\sigma_e\sigma_u \text{if } e \neq u\end{cases} \quad \text{.............................(Eq-02)}$$

The Single Index Covariance Matrix

The single index method, presented by Sharpe (1963) based on the assumption that the return of an asset group formed as liner function of the market portfolio. It shows there exists a linear relationship between the return of asset and market portfolios. This linear relationship is significant and positive. The equation can be written as:

$$R_{st} = \alpha_s + \beta_s x_s + \varepsilon_t$$

In the previous equation, x_s presents a market portfolio that does not correlate with the error term. And, $E(\varepsilon_{st} * \varepsilon_{zt}) = 0$. Here, Variance $(\sigma^2 *(\varepsilon_{st}) = \vartheta_{sz})$ inside the asset group remains unchanged. So, the covariance matrix for (σ_{sz})can be written as:

$$\sigma_{sz} = \beta\sigma^2 * \beta^2 + \partial$$

In the previous equation, β represents the slope of the vector, σ^2 represents the market variance while ∂ represents the error term of the variance of the matrix. Under the single index model, the covariance matrix can be expressed as follow:

$$\Sigma_{(\text{single index})} = \lambda\sigma^2\hat{\lambda} + \omega \quad \text{...........................(Eq-03)}$$

In the previous equation, λ represents estimates for the slope of vector, σ^2 represents sample variance of the market while ω represents estimates for the matrix of the variance of the error term.

The Sample Covariance by PCA

The principal component analysis (PCA) can be used to investigate the causal intentions of co-movements of asset groups. Without any economic or theoretical justification, the PCA transforms vector space of *S* asset groups into *S* factors. This transformation by PCA is done using the singular value decomposition of the sample covariance. In the model, each factor from *S* symbolizes a linear combination of original *S* asset groups. The sample covariance can be expressed in the following way:

$$S_j^e = \sum_{j=1}^{s} \rho_{ij} L_j$$

$$\sum = \rho\, T_L \acute{\rho}$$

Where, ρ represents an eigenvectors matrix, order *1*N* while *T* denotes an eigenvalue of matrix, order *N*N*. As PCA aims at cutting dimensions, from *S* factors. Here, selection can be done only for *Z* factors, When:

$$\frac{\sum_{b=0}^{z}\sigma_{L,b}^{2}}{\sum_{b=0}^{s}\sigma_{L,b}^{2}} \cong 1$$

So, the sample covariance matrix based on PCA can be expressed as follows:

$$\Sigma = \rho\widetilde{T_L}\tilde{\rho}' + T_\varepsilon \quad \text{……………………………} \text{(Eq-4)}$$

The equation-4 is utilized to estimate the PCA based covariance matrix. Here, ρ denotes to first Z-eigenvectors' matrix and T denotes the diagonal matrix in reference to Z-eigenvalues.

Portfolio of Estimators

The idea of optimal weighted intensity came under the criticism of Jagannathan and Ma (2003), they proposed a relatively easy concept, the estimation of equally-weighted covariance. This proposed model gained widespread acceptance. Corresponding to work done by Disatnik and Benninga (2007), Jagannathan and Ma (2003), and Liu and Lin (2010), this study estimates five equally weighted (EW) portfolios.

i. The portfolio of sample and diagonal matrix:

The following equation represents the sample covariance matrix and the diagonal covariance matrix in terms of the equally-weighted average. In the matrix, all off-diagonal components contain 0 values whereas diagonal components contain variance of asset groups.

$$\Sigma_{(portfolio1)} = \frac{1}{2}\Sigma_{(SampleMatrix)} + \frac{1}{2}\Sigma_{(DiagonalMatrix)}$$

ii. The portfolio of sample and single index matrix

The following equation represents the sample covariance matrix and the single index covariance matrix in terms of the equally-weighted average.

$$\Sigma_{(portfolio2)} = \frac{1}{2}\Sigma_{(SampleMatrix)} + \frac{1}{2}\Sigma_{(SingleIndexMatrix)}$$

iii. The portfolio of sample and constant correlation covariance matrix

The following equation represents the sample covariance matrix and the constant correlation covariance matrix (the overall mean of constant correlation) in terms of the equally-weighted average.

$$\Sigma_{(portfolio3)} = \frac{1}{2}\Sigma_{(SampleMatrix)} + \frac{1}{2}\Sigma_{(OverllMean)}$$

iv. The portfolio of sample matrix, single index matrix and constant correlation matrix

The following equation represents the sample covariance matrix, the single index matrix and the constant correlation covariance matrix (the overall mean of constant correlation) in terms of equally-weighted average.

$$\Sigma_{(portfolio4)} = \frac{1}{3}\Sigma_{(SampleMatrix)} + \frac{1}{3}\Sigma_{(SingleIndexMatrix)} + \frac{1}{3}\Sigma_{(OverllMean)}$$

v. The portfolio of sample, overall mean and single index matrix

The following equation represents the sample covariance matrix, the single index matrix, the constant correlation covariance matrix (the overall mean of constant correlation) and the diagonal matrix in terms of equally-weighted average.

$$\Sigma_{(\text{portfolio5})} = \frac{1}{4}\Sigma_{(\text{SampleMatrix})} + \frac{1}{4}\Sigma_{(\text{SingleIndexMatrix})} + \frac{1}{4}\Sigma_{(\text{OverllMean})} + \frac{1}{4}\Sigma_{(\text{DiagonalMatrix})}$$

List of estimation methods

Literature supports the use of alternative methods of the variance-covariance matrix for the construction of portfolios. Table 1 lists various methods selected for the estimation of variance-covariance and their symbols. The table also lists the diagonal method of covariance estimation which is denoted by the D symbol. The diagonal method servers as a fundamental component for the estimation of variance-covariance methods from E5, E6, E7, E8 and E9. The method of sample variance-covariance matrix utilizes historic values for covariances, Elton and Gruber (1973) suggest the usage of measure based on historical design however, this method is formed of the poor structure compared to other methods of variance-covariance estimators.

Table 1. List of estimation methods with symbols

Sr.	Covariance Estimators	Symbols
1	Diagonal method	D
2	Sample variance-covariance matrix	E1
3	Constant correlation covariance matrix	E2
4	Single index covariance matrix	E3
5	Sample covariance by PCA	E4
6	Portfolio of sample and diagonal matrix	E5
7	Portfolio of sample and single index matrix	E6
8	Portfolio of sample and constant correlation covariance matrix	E7
9	Portfolio of sample matrix single index matrix & constant correlation matrix	E8
10	Portfolio of sample, overall mean and single index matrix	E9

Note: D symbol represents diagonal method, symbols from E1 to E9 represents corresponding covariance estimators.

Sharpe (1963) utilizes the systematic factor of risk for estimation of the variance-covariance matrix. The method received much criticism due to its property of single risk factor that also tends to specification errors however, this method proved a more sophisticated estimator in comparison to the sample variance-covariance method on the grounds of estimation error. Jagannathan and Ma (2003) suggest the use of an equally-weighted average approach for the estimation for two or more than two variance-covariance estimators.

Assessment Criteria

Consistent with prior research, the current study adopts two assessment methods for the evaluation of alternate variance-covariance estimators. As mentioned, this study uses two data sets for analysis. The first data set starting from 01st of January 2001 to 29th of December 2009 is considered for the estimation of alternate variance-covariance matrices and to check the pairwise accuracy of covariance estimators. And, the second data set starting from 01st of January 2010 to the 27th of December 2019 is taken to check the ex-post accuracy of variance-covariance estimators received in the second sub-sample.

Two assessment criteria; root mean square error (RMSE) and measure of risk behavior of minimum variance portfolios (MVPs) are selected for evaluation. The RMSE is calculated to analyze the pairwise accuracy of a covariance matrix.

Root mean square error (RMSE) is calculated as:

$$RMSE = \sqrt{\frac{S(S-1)}{2}\sum_{l=1}^{S} \times \sum_{m=1,\ l\neq l}^{S} (\hat{\sigma}_{lm} - \sigma_{lm})^2}$$

Where, $S(S-1)/2$ denotes to pairwise variance-covariance estimators for order *S*S* of the covariance matrix, σ_{lm} denotes to actual covariances and σ_{lm} denotes estimated covariances between *m* and *1*. Here, A relatively low value of RMSE over high is considered better for pairwise accuracy of the variance-covariance estimator.

Inline with the study of Chan et al. (1999) another assessment criterion being considered for evaluation of covariance estimators is the MVP method. MVP is estimated to know the effectiveness of covariance estimators in the selection of an MVP. Which is used for the comparison of resultant variance-covariance estimators. A minimum variance portfolio (MVP), is a unique portfolio that does not rely on returns of asset group but the covariance matrix.

The study uses the first sub-sample to estimate weights through a minimum variance portfolio for each variance-covariance estimator. These weights are later used to estimate and take note for out of sample returns (second subsample of data) of MVP. This calculated return series of portfolio guides for estimation of average values of the mean of MVP and risk characteristics or standard deviation (SD) of MVP.

The weight of *m* risky asset for minimum variance portfolio stated as:

$$Min._w\, w^{\varphi}\Sigma w,\ Such\ that.\ w^{\varphi}e = 1$$

The above expression is restructured, using Lagrangian multiplier (λ) as:

$$Min._w\, \partial = w^{\varphi}\Sigma w - 2\lambda \times (w^{\varphi}e - 1)$$

$$\frac{\tau\partial}{\tau w} => 2\Sigma w - 2\lambda e = 0 \text{ (As per the 1}^{st}\text{ order rule)}$$

By solving the above equation for w, we find: $w = \lambda(\Sigma^{-1}e)$. Suppose, v is a *P*1* vector column. Written as *v=1/*$\lambda \times w$ that can be written as *v=*$\Sigma^{-1}e$. As the total of all weights is equal to 1 so $v^{\varphi}e = \frac{1}{\lambda}w^{\varphi}e = \frac{1}{\lambda}$. Hence, the weight for any investment for minimum variance portfolios is as follow:

$$w_{mvp} = \frac{v}{v^{\varphi}e}$$

Results and Discussion

Table 2 shows the results of root mean square error (RMSE) for nine variance-covariance estimators. Consistent with the study of Liu and Lin (2010), the results of RMSE show pairwise estimates of covariance matrices and the out of sample corresponding values. An estimator is said to be better if its RMSE value is comparatively low from other competing estimators.

Table 2. Results of RMSE for covariance estimators

Covariance Estimators	Values
Sample variance-covariance matrix (E1)	0.0109
Constant correlation covariance matrix (E2)	0.0085
Single index covariance matrix (E3)	0.0084
Sample covariance by PCA (E4)	0.0001
Portfolio of sample and diagonal matrix (E5)	0.0054
Portfolio of sample and single index matrix (E6)	0.0087
Portfolio of sample and constant correlation covariance matrix (E7)	0.0078
Portfolio of sample matrix single index matrix & constant correlation matrix (E8)	0.0074
Portfolio of sample, overall mean and single index matrix (E9)	0.0055

Table 2 exhibits that E4 the PCA-based sample covariance estimator outperformed all other covariance measures. From (Table 2) it is clear that E1, the sample variance-covariance method performed worst. The results show that rules proposed by Jagannathan and Ma (2003) of portfolio of estimator for E5, E9, E8 and E7 performed relatively better in comparison to single index covariance matrix E3 and constant correlation covariance matrix E2 except for portfolio of sample and single index matrix E6. Results also show that the constant correlation covariance estimation approach to matrix E2 performed a little better compared to the portfolio of sample and single index matrix E6 and Sample variance-covariance matrix E1. It is evident there is no extra gain from using sticky measures for the estimation of variance-covariance methods as opposed to equally-weighted covariance estimators.

Table 3 shows the results of standard deviation (SD) for minimum variance portfolios (MVPs) for nine variance-covariance estimators based on sample data. Both RMSE and MVPs evaluation criteria show similar findings however MVPs report slight differences in the performance of estimators.

Table 3. Results of standard deviation of MVP for covariance estimators

Covariance Estimators	Values
Sample variance-covariance matrix (E1)	0.0179
Constant correlation covariance matrix (E2)	0.0170
Single index covariance matrix (E3)	0.0174
Sample covariance by PCA (E4)	0.1009
Portfolio of sample and diagonal matrix (E5)	0.0174
Portfolio of sample and single index matrix (E6)	0.0172
Portfolio of sample and constant correlation covariance matrix (E7)	0.0175
Portfolio of sample matrix single index matrix & constant correlation matrix (E8)	0.0172
Portfolio of sample, overall mean and single index matrix (E9)	0.0171

Consistent with the result of RMSE the sample variance-covariance matrix E1 again remains a poor estimator in terms of standard deviation. However, the constant correlation covariance matrix E3 outperformed on the scale of standard deviation. Results show that the sample covariance by PCA E4 methods performs worst concerning standard deviation compared the RMSE accuracy measure. Overall, the equally weighted covariance estimators (E9, E6 and E8) performed better compared to sample variance-covariance matrix E1 and the sample covariance by PCA E4. Table A-1 in annexure shows mean results for MVP for the comparison of related risk characteristics. It is clear from results that usage of more sophisticated methods give no extra benefit over equally weighted covariance estimators or portfolio of estimators.

Table 4 reports the Sharpe ratio for the comparison of sequential portfolios consisting of their MVP for different methods of covariance estimation. The Sharpe ratio shows risk-adjusted yield for multiple inputs to the minimum variance portfolio (MVP).

Table 4. Results of Sharpe ratio of MVPs for covariance estimators

Covariance Estimators	Values
Sample variance-covariance matrix (E1)	0.0535
Constant correlation covariance matrix (E2)	0.0582
Single index covariance matrix (E3)	0.0409
Sample covariance by PCA (E4)	-0.0193
Portfolio of sample and diagonal matrix (E5)	0.0399
Portfolio of sample and single index matrix (E6)	0.0555
Portfolio of sample and constant correlation covariance matrix (E7)	0.0475
Portfolio of sample matrix single index matrix & constant correlation matrix (E8)	0.0511
Portfolio of sample, overall mean and single index matrix (E9)	0.0453

From Table 4, looking at the Sharpe ratio, it is clear that the constant correlation covariance matrix E2 outperformed other covariance estimators while the portfolio of sample and single index matrix E6 and Sample variance-covariance matrix E1 remain on second and third best performers, respectively. However, the sample covariance by the PCA E4 estimator performed worst among all estimators. For overall results, equally-weighted covariance estimators performed relatively better. It further shows that non of the estimators consistently beaten others. It confirms that usage of more complicated covariance estimators does not guarantee any incremental gain over equally-weighted estimators.

Conclusion

This study is an attempt to compute and evaluate a group of variance-covariance estimators; one of the integral parts of portfolio construction. In study nine of covariance matrices are estimated using data of non-financial sector companies listed at Pakistan stock exchange (PSX) whereas these estimators are also evaluated for accuracy and effectiveness using two alternate criteria RMSE and MVP method, respectively. The results show that the PCA method of sample covariance estimator outperformed competing estimators but the sample variance-covariance measure performed worst on RMSE criteria. Both of the evaluation measure RMSE and MVP produce dissimilar results for covariance estimators. The sample covariance matrix estimator remains poor whereas the estimator of constant correlation base covariance matrix outperformed on standard deviation (SD) evaluation measure. The overall results reveal that equally weighted variance-covariance matrices proposed by Jagannathan and Ma (2003) perform relatively much better compared to other variance-covariance estimators. Consistent with prior studies conducted by Nguyen (2018) and Husnain et al. (2016), findings confirm that investors or portfolio managers cannot obtain additional benefit from relatively more sophisticated variance-covariance models over equally weighted covariance estimators in the non-financial sector of Pakistan stock exchange. Investment executives are advised to exercise due care while formulating an investment policy incorporating tricky methods of covariance in comparison to equally weighted portfolio estimators.

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Appendix

Detail of Mean Values for Minimum Variance Portfolios (MVPs)

In the study, variance-covariance matrices are analyzed based on minimum variance portfolios (MVPs). By using weights of MVP for various variance-covariance estimators, values of note and out-of-sample performance (returns) are calculated. This estimated return series directs to the calculation of average values of mean for MVPs, as shown in Table A-1.

Table A-1 Mean values for MVPs for covariance estimators

Covariance Estimators	**Values**
Sample variance-covariance matrix (E1)	0.0020
Constant correlation covariance matrix (E2)	0.0021
Single index covariance matrix (E3)	0.0015
Sample covariance by PCA (E4)	-0.0042
Portfolio of sample and diagonal matrix (E5)	0.0015
Portfolio of sample and single index matrix (E6)	0.0020
Portfolio of sample and constant correlation covariance matrix (E7)	0.0018
Portfolio of sample matrix single index matrix & constant correlation matrix (E8)	0.0018
Portfolio of sample, overall mean and single index matrix (E9)	0.0017