A Modified Reference-free Modal Strain Energy Method for Damage Identification in 2D Structures

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Abstract

Health monitoring of structures is gaining the interest of many engineers because of its role in early detection of structural damage. The commonly used damage detection techniques based on vibrations, require data from both pristine and damaged form of the structure to discriminate the damage. In most cases the data from pristine structure are not available. To make the process of damage detection baseline-free, the curvature mode shapes of the damaged structure are fitted with a smoothing polynomial curve. These smoothed curvature mode shapes can be further used as reference data. Among many damage detection methods, modal strain energy method has proven to be more effective as it considers the strain energy of the structure at each mode. However, noise in the acquired response affects the success rate of the said method. To deal with this problem, a modification function is employed to amend the basic form of modal strain energy method. This modified modal strain energy method helped in giving damage indices which are less clouded with noisy peaks. However, it was still not able to detect and localize the damage with good accuracy. In this paper, along with this modified modal strain energy method which is made reference-free, a 2D Bayesian approach is presented. The approach uses the damage indices generated from the modified modal strain energy method and delivers an estimated damage index indicating the damage locations. The approach was tested for different scenarios of damage in terms of their severities, locality and noise levels and the results are compared with the published literature.

Key Words: Modal strain energy method, reference-free data, multiple damage detection, 2D structures

1. Introduction

Today a major challenge with modern civil, aerospace engineering and transport metallic and composite structures is to secure their normal operation by damage detection, health monitoring and condition assessment. Damage detection and quantification has been a challenging task of structural mechanics for many decades. The failure of such structure leads to catastrophic consequences in term of time, money and valuable human lives. For these reasons damage in the structures needs to be identified at an earliest possible stage and for that purpose engineers have tried to devise different methodologies. When a structure is damaged, dynamic characteristics associated with it, are changed indicating a loss in stiffness [1-3].

Vibration based methods have been broadly applied for non-destructive inspection (NDI) in health monitoring of structures as they are capable of providing local and global damage features [4,5]. Based on different criteria vibration-based methods can be categorized as linear or non-linear vibration responses, and methods that use reference data or reference-free data [1,5-7]. In past three decades many damage detection techniques have been investigated considering different types of material of the structure and varying types of damage such as cracks, slots, delaminations etc. From most of the studies it is observed that although mode shapes of a structure are easier to extract but these are less sensitive to localized damage and more prone to noise. So, curvature mode shapes are used as an alternative to mode shapes [8,9]. When a crack is introduced, bending/flexural stiffness is reduced increasing the modal curvatures in the vicinity of the damaged area thus allowing modal curvatures to detect damage. When curvature mode shapes are obtained by taking the second derivative of mode shapes, noise in the mode shapes also get amplified which masks the damage or give false alarms [10,11]. So, obtaining accurate mode shapes is a challenging task. A finite element model updating based method has also been used to detect multiple damages in steel plate and shell giving high quality results in localizing and quantifying the damaged elements [12].

In most of the vibration-based techniques reference data are a fundamental requirement to localize the damage. The reference data are not always available as in case of most of the existing structures. Many reference-free techniques have been developed to overcome this flaw. A frequency domain Fourier analysis without the data of pristine structure was employed on an aluminum plate [13]. A baseline-free modal curvature method based on discrete Fourier transform was used for damage detection in glass-fiber-reinforced polymer beam [14]. A modal curvature based on discrete Fourier transform with continuous wavelet transform was also used to detect damage without baseline data [15]. Wavelet Transform has been used to detect damage in aluminum plate without data from healthy structure [16]. Some of the reference-free damage detection methods that have been used are gapped smoothing method [11], regression analysis with polynomial approximation [1], Chebyshev filter and cubic polynomial regression [16].

Another common technique in damage detection is modal strain energy method (MSEM) that uses the variation in modal strain energy to define a damage index [17-19]. MSEM have been used to detect single damage in composite laminated plates and multiple damage in aluminum plates. MSEM is based on modal curvatures of the structure because according to Euler-Bernoulli beam theory, modal curvatures are linked with modal strain energy in case of beams and plates. However, as these modal curvatures are obtained by double differentiation of displacement mode shapes, hence these come with amplified noise [20,21]. Due to the noise in the measured data and the errors of approximations, MSEM generates damage indices with many false peaks. To overcome this flaw an improved modal strain energy-based method was developed for single and multiple damage in steel beam [11,22,23]. In 1D beam-type structures, the method was able to detect single damage with 15% severity while multiple damage with 10% severity. However, the method relied on the data from an intact structure. Hence, if the reference data are not available, the method is not expecting work. This method still needs to be tested on 2D structures.

From the above discussion, an improved modal strain energy method worked well on beam structures provided that the reference data are available. In this paper, a modified modal strain energy method is presented that is reference-free and that is expected to work well even with noise in the measurements. A 2D Bayesian approach is used in this paper to provide estimated damage indices with improved damage localization in 2D structures This research is an extension of previously published work [11].

2. Methodology

2.1 Modal Strain Energy Method

The main idea of the method of modal strain energy is to specify an index to identify the damage location by employing the change in modal strain energy in structure caused by the damage occurrence. In case of 2D plate structure of length l and width w, the location of each point is indicated by (x_i, y_j) along the length and width of the plate, respectively. The schematic of the plate is shown in Fig.1.

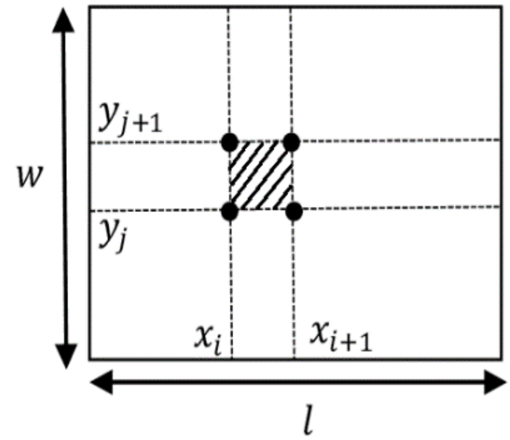


Fig. 1: Schematic of a 2D plate structure

The total strain energy U_k for a k^{th} normal mode, assuming a free vibration problem, can be obtained as

$$U_k = \frac{EI}{2}\int_0^w \int_0^l \left[c_{k,x}^2 + c_{k,y}^2 + 2\upsilon(c_{k,x})(c_{k,y}) + 2(1-\upsilon)\, c_{k,xy}^2\right] dxdy \quad (1)$$

Where $c_{k,x} = \frac{\partial^2 \phi_k}{\partial x^2}$; $c_{k,y} = \frac{\partial^2 \phi_k}{\partial y^2}$; and $c_{k,xy} = \frac{\partial^2 \phi_k}{\partial x \partial y}$; EI refers to the bending stiffness coefficient of the plate; ν, the Poisson's ratio; and ϕ_k the k^{th} displacement mode shape. The strain energy related to a single sub-region (i,j) of the plate as marked in Fig.1, in k^{th} mode can be calculated as

$$U_{k,ij} = \frac{(EI)_{ij}}{2}\int_{y_j}^{y_{j+1}} \int_{x_i}^{x_{i+1}} \left[c_{k,x}^2 + c_{k,y}^2 + 2\upsilon(c_{k,x})(c_{k,y}) + 2(1-\upsilon)\, c_{k,xy}^2\right] dxdy \quad (2)$$

In case of damaged plate, U_k^* represents the total strain energy of the plate while $U_{k,ij}^*$ refers to the strain energy of plate's sub-region for the k^{th} mode shape ϕ_k^*. The asterisk $(*)$ is used to represent the damaged plate. The curvature mode shapes of the plate with respect to both x and y axes can be calculated by using central difference approximation [16]. The fractional energies in case of undamaged and damaged plate can be obtained by Eqs.3 and 4, respectively,

$$F_{k,ij} = \frac{U_{k,ij}}{U_k} \tag{3}$$

$$F^*_{k,ij} = \frac{U^*_{k,ij}}{U^*_k} \tag{4}$$

By assuming EI to be constant along the length of the beam, the damage index in sub-region (i,j) for N number of displacement modes can be defined as

$$\beta_{ij} = \frac{1 + \sum_{k=1}^{N} F^*_{k,ij}}{1 + \sum_{k=1}^{N} F_{k,ij}} \tag{5}$$

In an improved modal strain energy method, the damage index β_{ij} was operated with a modification function M. For each k^{th} mode, M_k can be calculated by dividing each curvature mode shape with the maximum value of the curvature. Using this function, low weightage is assigned to the points that are at or near nodal points along the beam length. This assisted in removing many false peaks that were generated due to the nodal points. The damage index for the k^{th} mode and element having sub-region (i,j) can be calculated as [22].

$$(\beta_k)_{ij} = [\beta_{ij} - 1] \times |M_k| \tag{6}$$

2.2 Bayesian Approach for 2D Structures

As mentioned in section 2.1, the results from the modal strain energy method can be improved by using the modification factor, which reduces the false alarms around nodal points. However, the noise in the measurement data could also be due to sensor dynamics, linear or nonlinear distortions or some other sources causing uncertainty in the data. The noise that cannot be treated or reduced, generates false peaks in the damage indices and masks the true peak of damage. The false peaks in most cases are random and inconsistent for each mode. Whereas, the true peaks in damage indices, even if these are of very small magnitude, are consistent in each mode. Bayesian approach, which is employed here, exploits the consistency of the true peaks that are present at damage location. The damage index at i^{th} mode acts as an individual likelihood function $L_i(x,y)$. When multiple damage indices are extracted from number of modes n, the probability density function is generated by taking the product of likelihoods as:

$$\rho(x,y) \propto \rho^0(x,y) * \prod_{i=1}^{n} L_i\,(x,y) \tag{7}$$

The factor $\rho^0(x,y)$ in Eq.7 is *a priori* belief regarding the location of damage. A fully uninformative $p^0(x,y)$ can be used, but if the information about the possible damage locations are available then this factor can be set. The individual damage index functions $M_i(x,y)$ are first convolved with a kernel function $k(u,v)$. Associated to each i^{th} mode available, the kernel function can be also adjusted to generate the likelihood functions as

$$L_i(x,y) = \int_{(u,v)\in \square} k_i(u,v) \cdot M_i(x-u, y-v) \cdot du \cdot dv \tag{8}$$

The pruning process expressed in Eq.9, is then employed in which N available likelihoods are readjusted across each measurement point in descending order of magnitude. The K higher values of the available likelihood functions are finally used to generate estimated damage indices.

$$p(x_m, y_n) = p^0(x_m, y_n) . \prod_{k=1}^{K} L_{i(k)}\,(x_m, y_n)$$
$$L_{i(k)}(x_m, y_n) \geq L_j(x_m, y_n)\,, \forall j/j \neq i(r), 1 \leq r \leq K \tag{9}$$

3. Numerical Simulation

An Aluminum plate with dimensions and material properties given in table 1, was modelled in ANSYS. The plate is cantilever and the clamped end is located on the left side. Firstly, three multiple-damage scenarios were considered in the plate as shown in Fig.2. In scenario 1, the two square damages were modelled along same horizontal axis and centered at $(x_1, y_1) = (0.1, 0.115)\,m$ and $(x_2, y_2) = (0.21, 0.115)\,m$ with an equal area of $0.02 \times 0.02\,m^2$. For scenario 2, the two square damages were modelled along same vertical axis and centered at $(x_1, y_1) = (0.155, 0.075)\,m$ and $(x_2, y_2) = (0.155, 0.115)\,m$ with an equal area of $0.02 \times 0.02\,m^2$. Lastly, for scenario 3 a multiple diamond-shaped damage were modelled at locations centered at $(x_1, y_1) = (0.1, 0.115)\,m$ and $(x_2, y_2) = (0.21, 0.115)\,m$ with an equal area of $0.02 \times 0.02\,m^2$. The damage was modelled by reducing the stiffness coefficient E of the damaged elements by 30%. To obtain the displacement mode shapes of the plate, modal analysis of the plate is carried out by using the ANSYS.

Table 1: Properties of Plate Structure

Physical Characteristics of the Plate	
Plate Dimensions	$0.35 \times 0.23 \times 0.003\ m^3$
Boundary Condition	Cantilever
Element type	3-D 8-Node Solid185
Matrix Grid	70×46 elements
Element Size	$0.005 \times 0.005 \times 0.003\ m^3$
Material Characteristics	
Young's Modulus, *E*	69 GPa
Poisson's Ratio, *v*	0.35
Material	Aluminum
Density	$2700\ kg/m^3$

It has commonly been assumed that stiffness K, of the plate is proportional to the thickness t of the plate as, $K \propto t^3$ [24]. The stiffness is also proportional to the modulus of elasticity E as, $K \propto E$. Hence, the modulus of elasticity of the damaged area can be related to the thickness of the plate as in Eq.10.

$$E = (constant).t^3 \quad (10)$$

By using the relationship in Eq.10, change in stiffness reduction can be related to change in thickness reduction. So, a stiffness reduction of 10%, 20% and 30% is considered in this study which is equivalent to approximately 3.5%, 7.2% and 11.2% thickness reduction for comparison purposes. To check the effectiveness of the proposed methodology, Gaussian white noise is added in the numerical mode shapes according to Eq.11 [25].

$$m_i' = m_i(x,y) + \gamma.r.m_{i,rms} \quad (11)$$

where m_i is the i^{th} displacement mode shape with x and y representing the location and m_i' is the noisy mode shape; r is the random variable with normal distribution having a zero mean and a variance of 1; γ represents the noise level; and $m_{i,rms}$ is the root mean square value for the i^{th} mode shape.

(a)

(b)

(c)

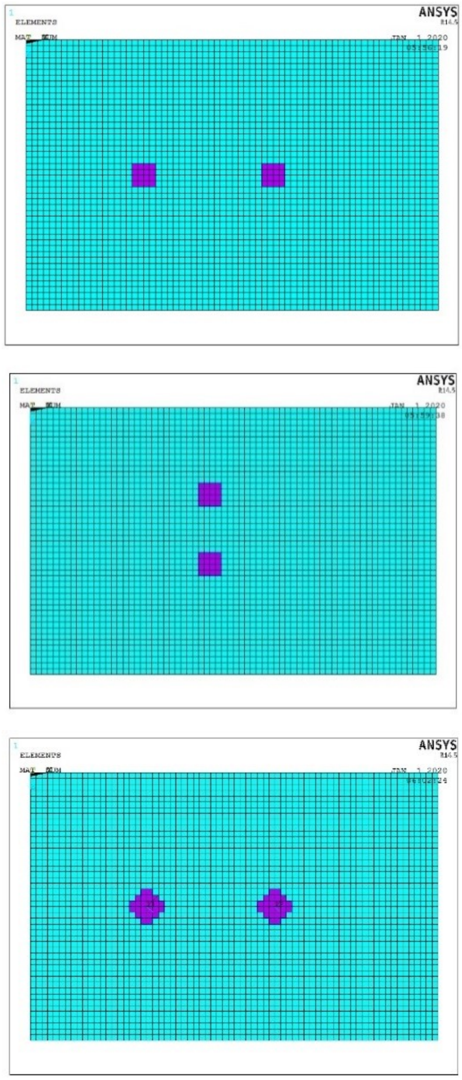


Fig. 2: Finite element model of the plate with damage zones: (a) Scenerio 1, (b) Scenerio 2, (c) Scenario 3

4. Results and Discussions

Curvature mode shapes were extracted from displacement mode shapes by using the central difference formula, which were then employed in the calculation of damage indices from the modified modal strain energy method. Bending mode shapes from 6th to 9th mode were used in the calculations because these are more sensitive to damage in plates as compare to the lower modes. Initially, the noise-free bending modes were used and results for scenario 1 and 2 were obtained for damage severities of 1.7%, 3.5%, 7.2% and 11.2% reduction in thickness. From Fig.3, it can be clearly seen that the proposed method is able to successfully detect the multiple damage in scenario 1, even for 1.7% thickness reduction. While, for scenario 2, both damages can be clearly detected up to 3.5% thickness reduction as can be seen in Fig.4.

Noise with $\gamma = 0.1\% \ (1 \times 10^{-3})$ was then added in the displacement mode shapes and damage indices for both scenario 1 and 2 were acquired as shown in Figs.5 and 6. It can be seen that as the severity is reduced, it becomes more difficult to identify the damage. Therefore, Bayesian approach was then applied and estimated damage indices were obtained as illustrated in Figs.7 and 8. To obtain estimated damage indices from noisy data, higher modes from mode 10 to 15 were used because of their good performance in plate-type structures. It is clear from the results that for both scenarios, the two damages were successfully detected for 11.2% and 7.2% severity but in case of 3.5% severity only one damage was detected with some noisy peaks.

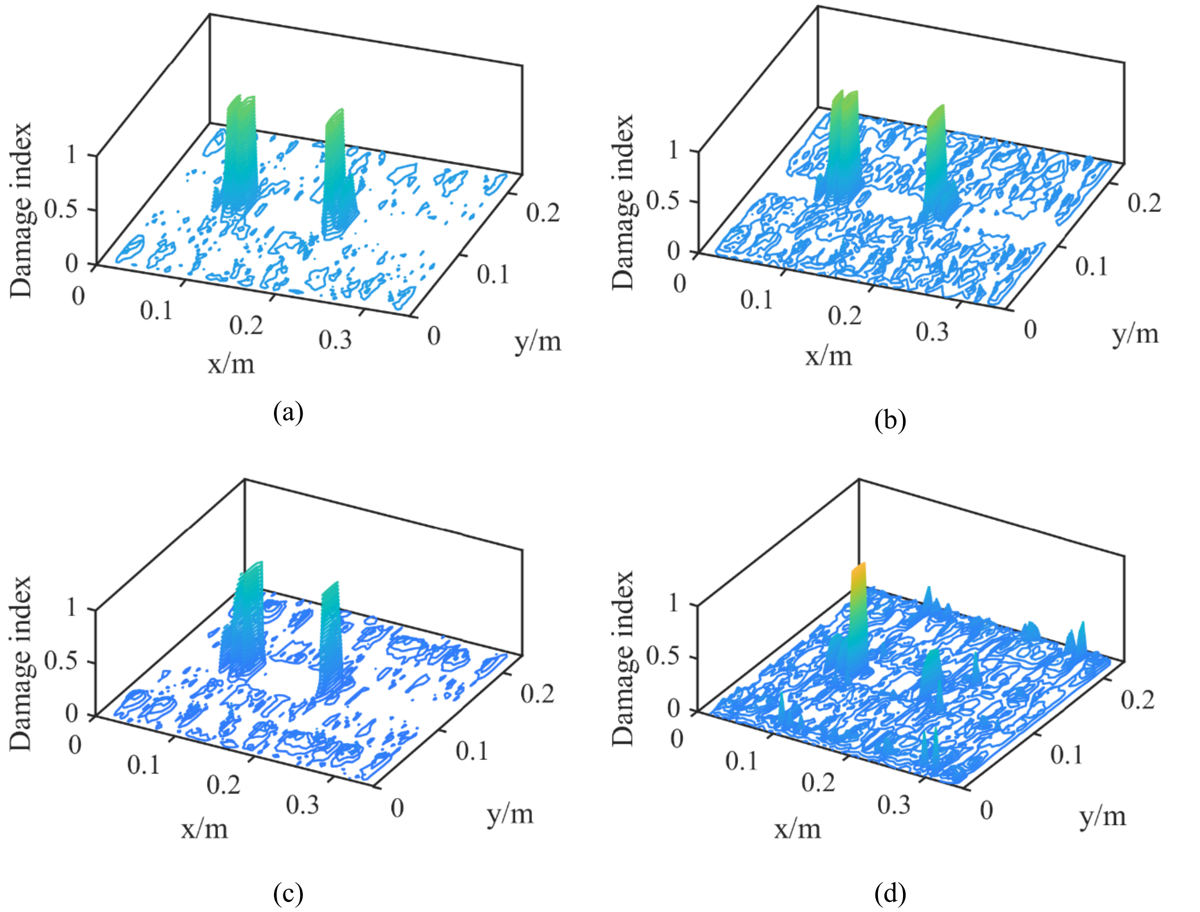
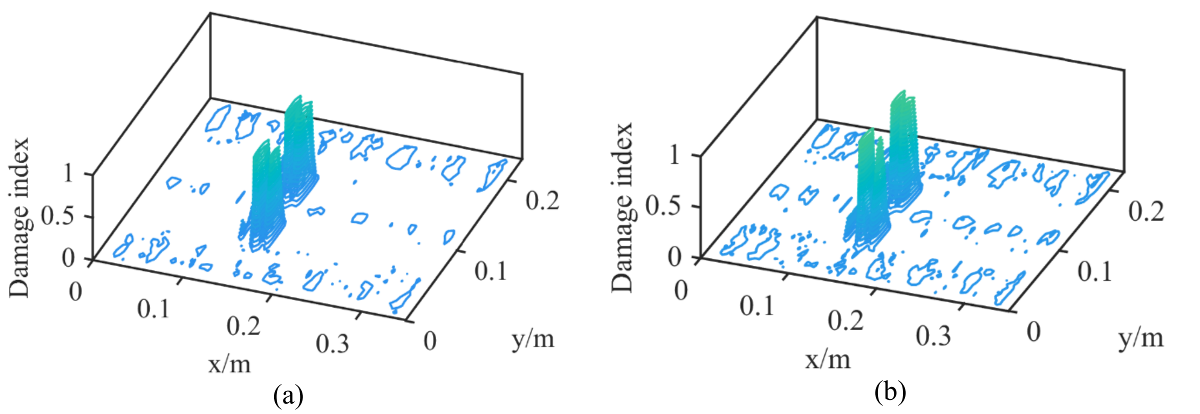


Fig. 3: Damage index for different severities of damage scenario 1: (a) 11.2% severity, (b) 7.2% severity, (c) 3.5% severity and (d)1.7% severity



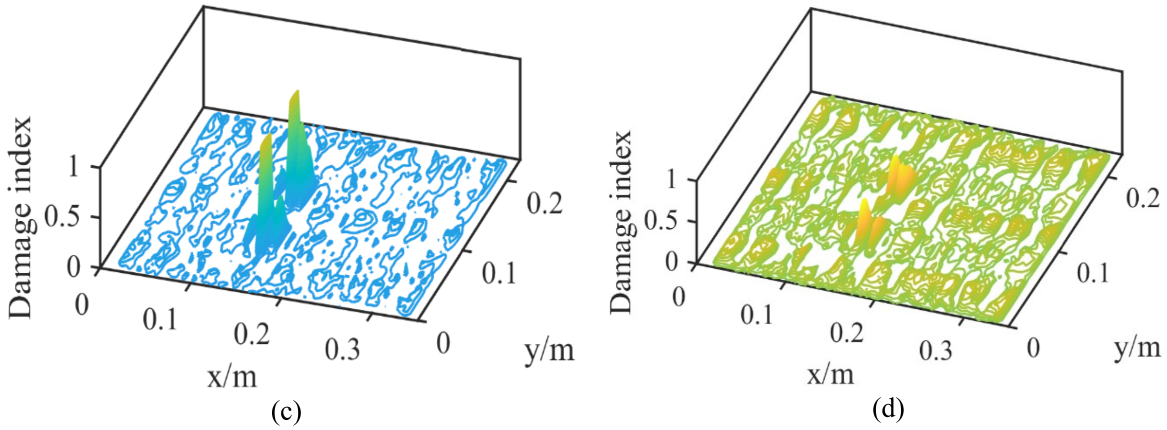


Fig. 4: Damage index for different severities of damage scenario 2: (a) 11.2% severity, (b) 7.2% severity, (c) 3.5% severity and (d)1.7% severity

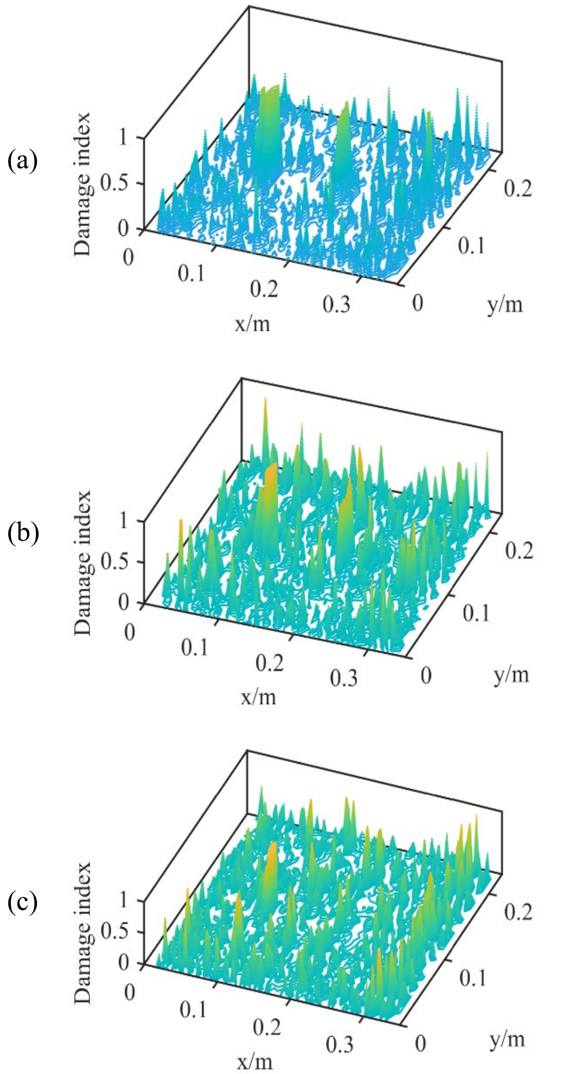


Fig. 5: Damage indices with 0.1% noise for damage scenario 1; (a) 11.2% severity, (b) 7.2% severity, (c) 3.5% severity

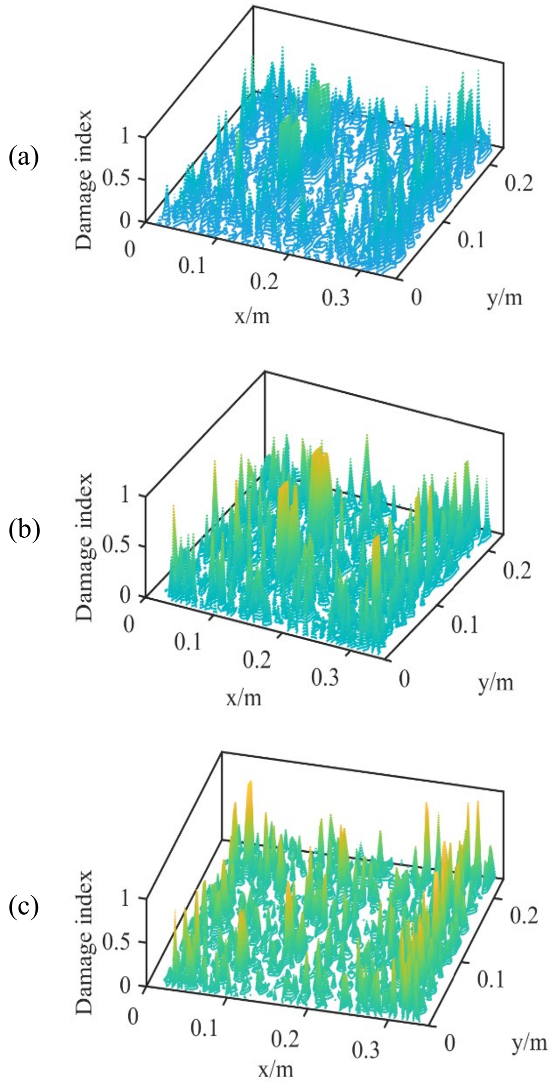


Fig. 6: Damage indices with 0.1% noise for damage scenario 2; (a) 11.2% severity, (b) 7.2% severity, (c) 3.5% severity

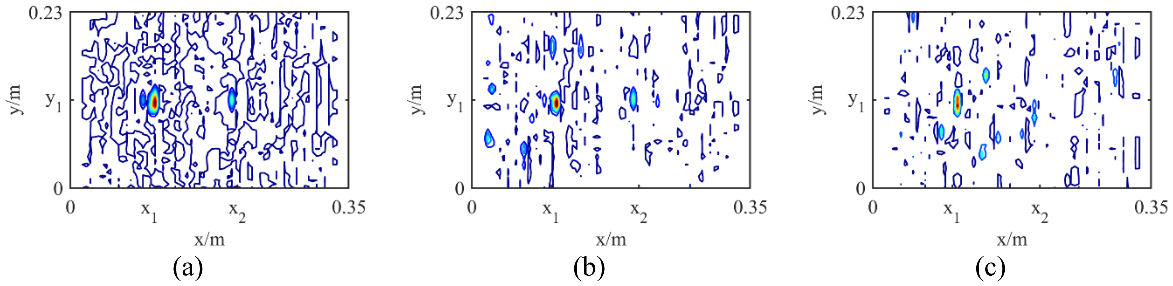


Fig. 7: Estimated damage indices after using Bayesian approach for scenario 1: (a) 11.2% severity, (b) 7.2% severity, (c) 3.5% severity

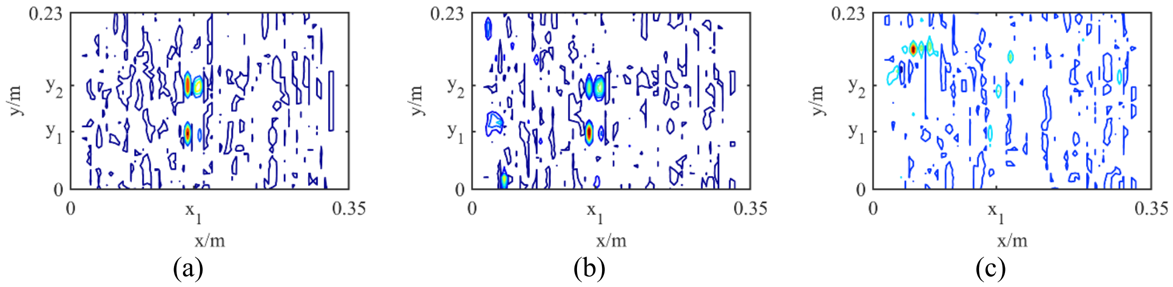


Fig. 8: Estimated damage indices after using Bayesian approach for scenario 2: (a) 11.2% severity, (b) 7.2% severity, (c) 3.5% severity

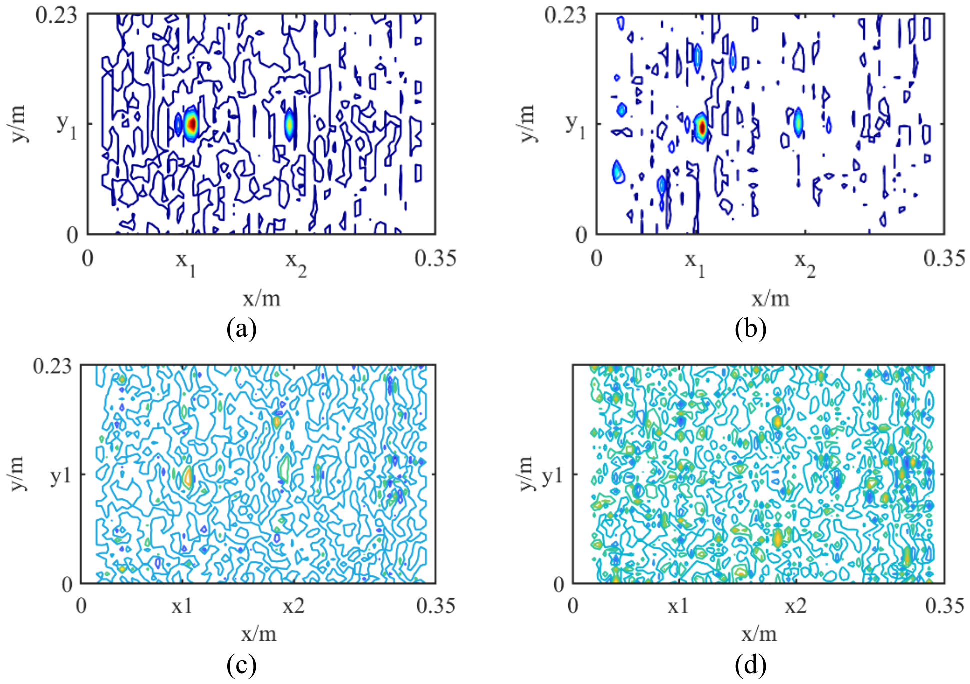


Fig. 9: Estimated damage indices for Scenario 1 with noise levels of: (a) 5×10^{-4}, (b) 1×10^{-3}, (c) 5×10^{-3} and (d) 1×10^{-2}

Now for 7.2% severity of damage, scenario 1 and 3 were considered and investigated for varying levels of noise. The noise was increased from a low noise level of $\gamma = 5 \times 10^{-4}$ to a high noise level of $\gamma = 1 \times 10^{-2}$. The estimated damage indices were generated for each case, as shown in Fig.9 and 10. For both scenarios 1 and 3, the two damages were detected successfully up to the noise level of 1×10^{-3}while single damage can be seen with some noisy peaks for 5×10^{-3} noise level. For a further high noise level, both the damages were lost in noise.

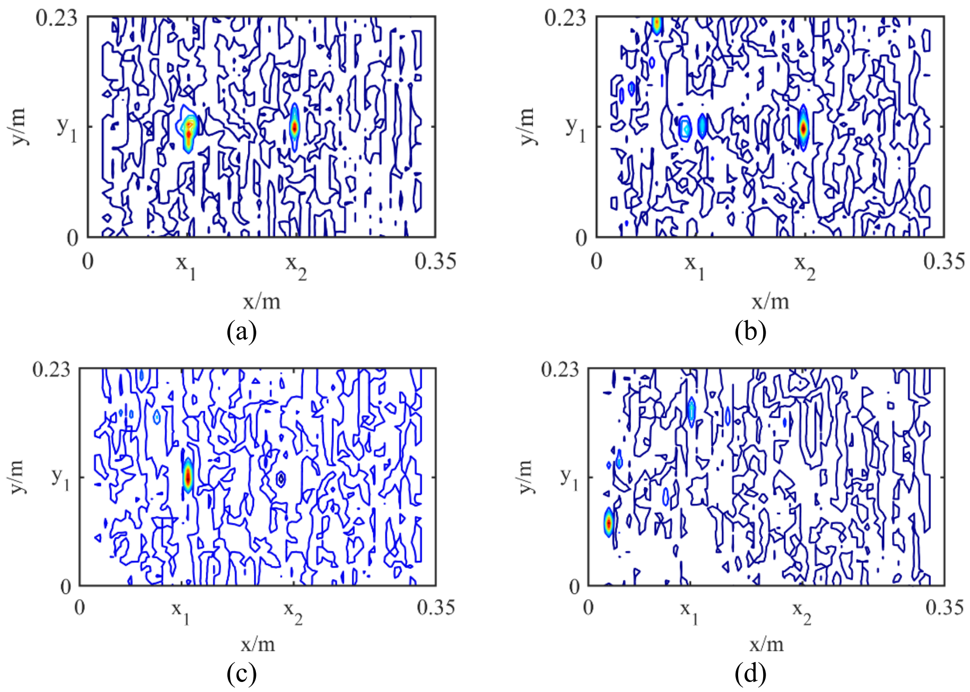


Fig. 10: Estimated damage indices for Scenario 3 with noise levels of: (a) 5×10^{-4}, (b) 1×10^{-3}, (c) 5×10^{-3} and (d) 1×10^{-2}

Table 2: Comparison with published literature

	Damage severity in terms of thickness reduction	**Damage type in plate**	**Good results up to noise level**
Fan et al. [25]	11.2% -single damage is localized	Single square, diamond shaped damage	2.5×10^{-4}
Cao et al. [26]	10% - both damages are localized	Multiple square damage	1×10^{-3}
This Paper	7.2% - both damages are localized 3.5% - single damage can be detected	Multiple square and diamond shaped damage	1×10^{-3}- both damages 5×10^{-3}- single damage

5. Conclusion

A modified reference-free modal strain energy method along with Bayesian approach is presented in this study to detect multiple damage in plate-type structures. Initially, the results were generated by using modified reference-free modal strain energy method on noise-free modes. These results identified the modes which were effective in damage detection process. Then varying levels of noise were added in those modes and damage indices for each case were obtained. It was found that reference-free modal strain energy method alone cannot work successfully in detecting multiple damage. Thus, Bayesian approach was then employed, and the estimated damage indices indicated much better results even in the presence of high noise levels such as 1×10^{-3}. These results were finally compared with the same published literature with single and multiple damage [25,26]. In comparison to the single damage detection by Fan et al., the results were far better in terms of detection of multiple damage at high noise level. When compared with the recent published work of Cao et al., the method presented in this paper can even detect multiple damage of low severity such as 7.2% at same noise level and can also detect single damage for very low severity of 3.5% thickness reduction. From the comparison, it can be concluded that the modified reference-free modal strain energy method along with Bayesian approach has the potential to detect multiple damage with good accuracy. The method does not require any reference data and just rely on the response from the damaged structure. The results can be further improved if instead of using central difference approximation, direct strains can be calculated that will help in avoiding the approximation errors. Furthermore, data obtained at more measurement points can also help to improve the detection capability of the method.

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