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# Damage Localization in Plate-type Structure using 2D-Bayesian Technique

Vishal Sindhu[1], Ummul Baneen[*], Syed Abbas Zilqurnain Naqvi, Ayisha Nayyar

*Department of Mechatronics & Control Engineering, University of Engineering and Technology, Lahore*

* **Corresponding Author:** Email: u_baneen@uet.edu.pk

## Abstract

*Failure in civil, mechanical, and aerospace structures are commonly due to heavy loads, environmental hazards and not to mention their lifespan. Structural health monitoring (SHM) has become vital to maintain the design and strength of these structures. For detection of damage, prior data of the structure is crucial but in the absence of the prior data, for example in case of antiquated structures, this becomes quite challenging. Gapped smoothing method (GSM) has been used successfully to detect damage with the response data only. However, noise in the measurement data is detrimental to its effectiveness. Moreover, single damage has been identified with most of the techniques particularly in plate-type structures, but those methods are not as successful in case of multiple damage. In this paper, a 2D-Bayesian technique is presented to suppress noise along with 2D-GSM to detect multiple damage in plate-type structures without using any prior data. The technique uses the fusion approach to combine the information from the available mode shape data by reducing the noise and highlighting the damage locations. Different damage scenarios in plate-type structures are simulated in ANSYS. To evaluate the effectiveness of the proposed method, the simulated data is contaminated with noise and results are compared with the published work.*

**Key Words**: 2D-Bayesian, Damage Detection, Gapped Smoothing Method, Multiple Damage Detection, Plate-Type structures

## 1. Introduction

Without SHM the structures are prone to damage which could result in the loss of human life and financial losses as well. When a structure is damaged, it is essential to ensure structure's safety and necessary measures should be taken to restore its functionality by adopting some reliable retrofitting scheme. Numerous techniques have been proposed and implemented for damage identification in the last decade [1-4]. Although researchers are continuously making efforts to introduce latest techniques, but many problems still needed to be resolved which are given below,

a) Structures with few modes,

b) Structures with many members,

c) Structures whose baseline data models are unavailable (i.e., old structures),

d) In an environment of uncertainty associated with modeling, measurement, and processing errors [5].

Generally, damage localization techniques consume too much resources and time [6]. Many non-destructive techniques are available. Few of them involve usage of strain gauges, eddy currents, penetration of liquids and so on. Engineers are continuously investigating about new methods for condition monitoring and assessment of structures as its components can be inspected frequently without any hindrance and damage can be detected in its early stages. Modal and structural properties can be used to obtain position, severity of damage by usage of different unfailing techniques i.e. frequency response functions, resonance frequency and mode shapes [6,7]. Natural frequencies and mode shapes of a structure are affected by changes in their mass or stiffness, it changes the dynamic properties of material which leads to detection of damage. Researchers have used second derivative of mode shapes, that is, curvature mode shapes to detect damage in structures [8,9], the derivative of the frequency response function (FRF) curvatures difference that allows for an enhanced view of the region where the problem exists and may be used in a first localization step [10,11]. Resonant frequencies are most often used for detecting severity of damage, but it cannot localize it properly [7]. If we compare curvature mode shape methods and mode shape-based method the first one algorithm is effective for localization of damage while later one on contrary of it have to rely on any other technique or method [12,13] Baseline data is commonly missing of very old structures and almost every damage detection technique require some data to make comparison between parameters of healthy and damaged structures [14,15]. For localization of damage baseline data of undamaged structure is not required in case of one dimensional Gapped Smoothing Method (1-D GSM) and two dimensional Gapped Smoothing Method (2-D GSM) because they mainly can detect inconsistency of stiffness in structure. Due to development of irregularity in the indices, damage can be easily identified if the structure can easily be analyzed before and after damage. This irregularity in indices leads to changes in stiffness [10,16]. Cracks cause reduction in stiffness so they can easily be detected by the curvature mode shape-based methods. Therefore, variation in features of vibration leads to identification of damage in structure [6]. By using Gapped Smoothing Method (GSM) only curvature model information of the damaged structure is required while first few curvature mode shapes of the plate are needed in case of 2D mode shape curvature technique. In unhealthy structures, local damage induces irregularities in mode shapes where damage exists. Gapped Smoothing Method (GSM) is advantageous over other methods due to its smoothing function and makes it baseline free approach [17]. In order to reduce the noise a set of likelihood functions are synthesized by usage of damage indices which are provided by GSM, these functions are further processed underneath Bayesian approach. [18]. After successfully implementing damage detection technique based on mode shape on one dimensional structure, researchers are currently trying to implement those techniques on two dimensional structures. Two dimensional curvature mode shape is currently very popular among SHM researchers, it is briefly explicated in one dimensional structures while in two dimensional structures few issues are still unresolved which includes ambiguity about the mechanism of characterizing damage, vulnerability to noise, and sensitivity to sight damage [17,19-24].

This research is an extension of a previous published work from 1D structures to 2D structures with multiple damage. A Beta distribution, Beta probability density function (PDF) is introduced along with 2D-Bayesian technique to exclude the insensitive regions of plate in the data that might affect the results. To evaluate the effectiveness of 2D-GSM with Bayesian data fusion approach for multiple damage, the results are compared with the published literature work.

## 2. Gapped Smoothing Method

The GSM assumes that the curvature mode Healthy structures have smooth shape of model depiction such as mode shape and curvature mode shape. This smooth shape is basically represented by a smooth curve for 2D plate like structures. When the structure is damaged it will become irregular especially at damaged location. Gapped Smoothing Method (GSM) takes advantage of this irregularity. It assumes that curvature mode shape $K_i^f$ of a healthy structure has a smooth surface. This smoothness is approximated by using a polynomial with two variables for plate type structure by GSM as given in Eq.1 [8].

$$K_i^f(x,y) = \sum_{q=0}^{m}\sum_{r=0}^{n} \rho_{qr}\, x^q y^r \qquad (1)$$

By surface-fitting coefficient $\rho_{qr}$ will be acquired with polynomial order, $m$ and $n$. Eq.1 produces the fitted curvature, $K_i^f(x,y)$ for $i^{th}$ mode which represents curvature of the supposedly healthy plate. The curvature $K_i^d(x,y)$ will be acquired from the measured displacement $u_i$ of the $i^{th}$ mode by using central difference estimation, as in Eq.2.

$$K_i^d(x,y) = \frac{\{u_i(x+h,y) - 2u_i(x,y) + u_i(x-h,y)\}}{h_x^2} + \frac{\{u_i(x,y+h) - 2u_i(x,y) + u_i(x,y-h)\}}{h_y^2} \qquad (2)$$

Horizontal and vertical intervals between data points are denoted by $h_x$ and $h_y$ respectively. To measure damage index $\delta_i(x,y)$ for the $i^{th}$ mode at each measurement point, square of the alteration of the measured $K_i^d(x,y)$ and fitted value of curvature $K_i^f(x,y)$ is taken, given as

$$\delta_i(x,y) = \left\{K_i^d(x,y) - K_i^f(x,y)\right\}^2 \qquad (3)$$

Presence of any irregularity in curvature affects the damage index, this irregularity could be due to noise which is unpreventable and has the affinity to vitiate the method. Noise cannot be excluded totally without losing any useful data, but it can be reduced to take maximum advantage of this technique.

## 3. 2D-Bayesian Approach and Beta PDF

Inconsistency in damage indices is due to noise in the data, linear and nonlinear alteration because of un-modelled sensor dynamics and other source of ambiguity [24,25]. To lessen the influence of those unclear sources in approximating location of damage Bayesian fusion procedure has been employed. To define the presence of damage, probability density function (PDF) is produced, product of likelihood is executed for fusion of numerous sources of information as:

$$\rho(x, y) \propto \rho^0(x, y) * \prod_{i=1}^{n} Li\,(x, y) \qquad (4)$$

The calculated individual damage index functions results in the individual likelihoods which are represented by the factor $L_i(x, y)$. When several likelihoods are presented together the process not only increases the possibility of the detection of damage but also decreases the prospect of the inference of false negatives. The likelihoods are produced according to Eq.5.

$$L_i(x, y) = \int_{(u,v)\epsilon\Omega} k(u, v). M_i(x-u, y - v). du. dv \qquad (5)$$

Here the individual damage index function is $M_i(x, y)$ and kernel function is $k(u, v)$. The factor $p^0(x, y)$ in Eq.4 is a prior information about the damage position. Typically, a fully inadequate $p^0(x, y)$ can be used, that is, a uniformly distributed $p^0(x, y)$ with a constant probability of damage over the plate surface.

## 3.1 Beta PDF

Beta PDF is applied along with 2D-Bayesian technique to assign a low weightage to the insensitive portion of plate. Left side of the plate was fixed and there were many false peaks around that region which were affecting the damage indices. Beta PDF is employed to suppress these peaks from the calculation of damage indices.

## 4. Noise PDF

Imperfections in excitation mechanism, sensors, displacement measurement apparatus and external environment induces disturbance in the plate frequency response, which contribute noise in the FRF signal. Gaussian random variable is followed to model the noise under the assumption that noise distribution of component source adds up to the Gaussian random variable under the central limit theorem. According to the definition of Signal-to-noise ratio, magnitude of the noise is computed as the segment of the RMS of the FRF signal such that the metric remains consistent. Inverse Fast Fourier Transform of FRF is taken to add noise in the time-domain depiction of signal.

$$\hat{y}_i(t) = y_i(t) + \left(\frac{e}{100}\sigma(y_i(t))R(t)\right) \qquad (6)$$

In Eq.6 $\hat{y}_i(t)$ is the noisy signal which is acquired if we let $y_i(t)$ as the noiseless time-domain illustration of the FRF signal. The term $R(t)$ is the arbitrary value sampled (at time $t$) from zero mean and unit variance normal arbitrary variable. The expression infers $\hat{y}_i(t)$ as noisy FRF acquired by superimposing on the noiseless signal $(y_i(t))$, the noise level $e$% of the signal RMS value. Hence the extent of noise can be controlled by single variable $e$. Where $e$ denotes the noise amount express as fraction of the RMS value $\sigma(y_i(t))$ of the signal $y_i(t)$.After noise is added to the time-domain version of the FRF the subsequent signal is converted to frequency domain by taking the fast Fourier transform.

## 5. Numerical Analysis

## 5.1 Preliminary Analysis

Initially an analysis was conducted to comprehend which bending modes can be useful in detecting damage in noise free environment. Various scenarios were considered by changing the severity of damage, boundary conditions, and damage type. Damage was modelled in plate by reducing the stiffness at the damage location/s. The details of numerical model and simulated analysis are mentioned in section 5.2. From this initial analysis, results were generated as given in Table 1 for each scenario. These results provided a reference, based on which further analysis was performed in the presence of noise.

From the analysis in Table 1, we can observe that:

- Higher bending modes are required to detect damage with low severity.
- Boundary conditions have huge effect on damage peaks, peaks showing damage in free-free plate are sharper as compare to cantilever plate.
- Lower bending modes can be used to detect single damage, however for accurate detection of multiple damage, higher modes are required.
- Different types of damage even with same severity can be detected at different frequencies.

As it is challenging to detect multiple damage with low severity in cantilever plate. So, further analysis was carried out on multiple damage with 30% severity in a cantilever steel plate.

**Table 1:** Preliminary Analysis

| Damage Type | Severity of Damage | Boundary Conditions | Number of Damage | Modes which can detect damage |
|---|---|---|---|---|
| Square Type | 70% | Free-Free | Single | 7th or above |
| Square Type | 50% | Free-Free | Single | 9th or above |
| Square Type | 50% | Cantilever | Single | 5th or above |
| Square Type | 70% | Cantilever | Multiple | 7th or above |
| Square Type | 50% | Cantilever | Multiple | 9th or above |
| Square Type | 40% | Cantilever | Multiple | 9th with some false peaks |
| Square Type | 30% | Cantilever | Multiple | 10th or above |
| Diamond Type | 40% | Cantilever | Multiple | 9th or above |
| Diamond Type | 30% | Cantilever | Multiple | 10th or above |
| Delamination Type | 30% | Cantilever | Single | 5th or above |
| Delamination Type | 40% | Cantilever | Multiple | 5th or above |
| Delamination Type | 30% | Cantilever | Multiple | 6th or above |

**Table 2:** Specifications of the Plate Scenarios

| **Scenario** | **1. Delamination Type (mm)** | | **2. Diamond type (mm)** | |
|---|---|---|---|---|
| **Damage Location (X, Y, Z)** | **D1** (44, 36-56, 0-4) | **D2** (140-160, 24, 0-4) | **D1** (42-46, 40-44, 2-4) | **D2** (112-116, 20-24, 2-4) |

## 5.2 Depiction of Numerical Analysis

To demonstrate the effectiveness of the proposed method, analysis was carried out on plates with length, width, and thickness as 204 mm, 66 mm and 4 mm respectively, having different types of damage. For simplicity in this paper, two types of damage; delamination and diamond-type, are considered as shown in Fig.1 and Fig.2. Plates were modelled and analyzed in ANSYS. The location of each type of damage is detailed in Table 2.

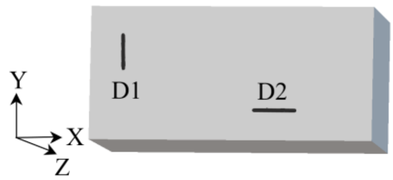


**Fig.1:** Plate with Delamination-type damage (*Not-to-scale*)

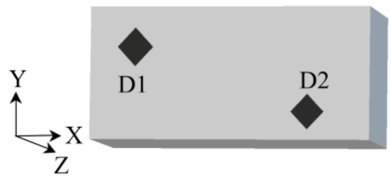


**Fig.2:** Plate with Diamond-type damage (*Not-to-scale*)

## 6. Damage Scenarios in Plate

### 6.1 Scenario 1: Delamination-type of Damage

Two, delamination-type of damages D1 and D2 are created in a steel plate. D1 is vertical to, while D2 is horizontal along the length of the plate. Both delaminations are through width crack of length 20mm. After modeling the damage, boundary conditions and force, are applied. It is observed that magnitude of force and boundary conditions have major impact on damage detection process. Strategies to improve the detection process might involve increasing the force or changing the boundary conditions, particularly when measurement noise is causing a detrimental effect on results. However, there is a limit to which the magnitude of the force can be increased otherwise the force itself might cause damage in the structure.

### 6.1.2 Effect of variation in applied force

Two scenarios were considered for comparison in this section. In the first case, a force of 1N was applied and damage indices were generated from the response. Then, in the second case a force of 100N was applied at the same location of the plate and damage indices were generated from this response. In both cases, the damage indices from modes 7 and 8 were considered because higher modes provide better detection of damage as already mentioned in Table 1. By changing the force from 1N to 100N, it can be seen from Fig.3 and Fig.4, that damage peaks are clearer and sharper. The red dashed boxes indicate the locations of damage in the figures. The force of 100N was considered after reviewing material properties of the plate. In this paper, however, a force of 1N is considered and the proposed method was employed on the responses of this excitation force. For simplicity, the labels and position values are not shown in figures from Fig.3 onwards. The horizontal axis of the figures represents the plate length while vertical axis refers to plate width in all figures.

### 6.1.2 Change in Boundary Conditions

From preliminary analysis in Table 1, it was perceived that boundary conditions affect the damage detection process. Hence, to investigate the proposed technique, the plate was analyzed with different boundary conditions. The damage indices from modes 6, 7, and 8 were obtained for free-free and cantilever plates as shown in Fig.5 and Fig.6.

From these figures, it is quite noticeable that damage peaks in free-free plate are much sharper as compared to cantilever. It means that when the constraints on the plates are reduced, there will be substantial response with the same force as compared to the plate with more constraints. Higher response will provide more useful information in the form of damage localization with clear peaks. In this paper, a cantilever plate is used instead of free-free plate because of the challenge it offers.

### 6.1.3 Damage Detection with GSM

Different higher bending modes were used for detection of multiple damage using GSM. Looking at Table 1, multiple delamination with 30% severity can be detected from $6^{th}$ mode onwards. Whereas, in mode 5 only single damage can be detected. In the simulated tests of plates, the successful results were achieved up to a noise level of $e = 5.0 \times 10^{-2}$ [17,21]. So starting with the noise level $e = 3.5 \times 10^{-2}$ in the time domain response, damage indices were generated using GSM based on $5^{th}$- $8^{th}$ modes. As can be seen in Fig.7 , except mode 5 where only D2 is detected, the rest of the three modes clearly show the locations of both delaminations. There are few very small false peaks in higher modes.

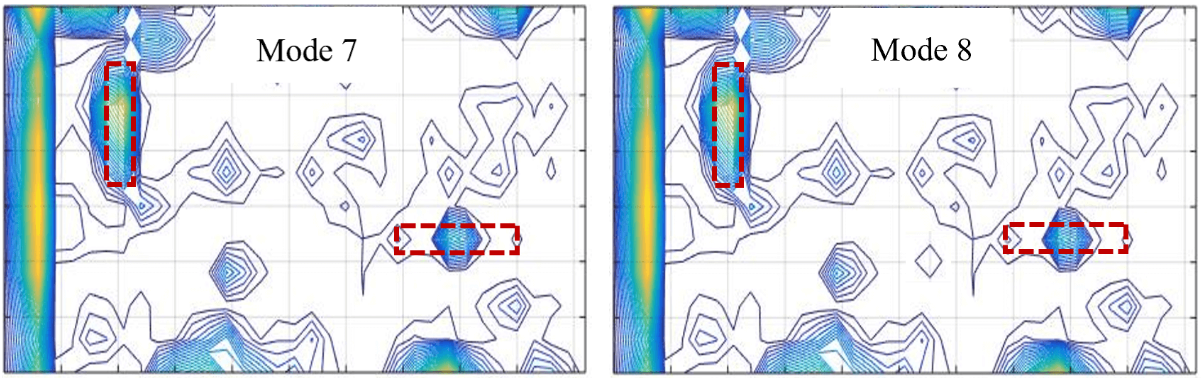


**Fig.3:** Damage index of $7^{th}$ and $8^{th}$ modes with 1 N force

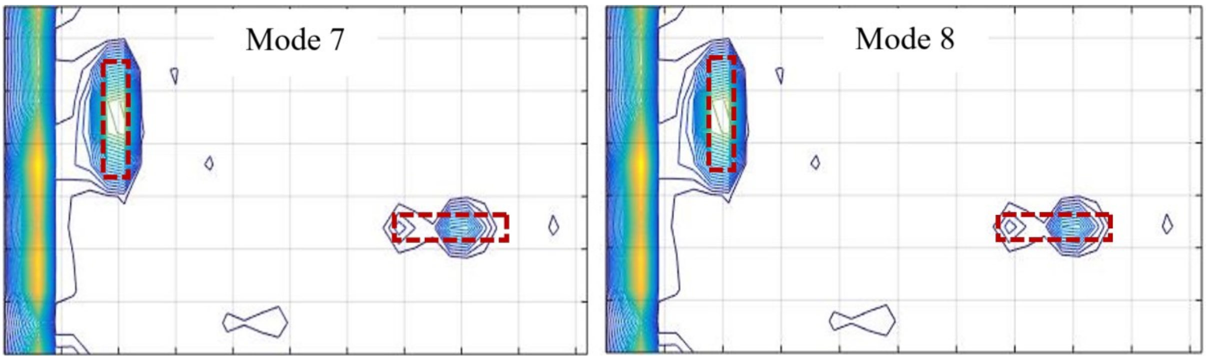


**Fig.4:** Damage index of $7^{th}$ and $8^{th}$ modes with 100 N force

The noise level was further increased to $e = 1 \times 10^{-1}$ and again the damage indices were acquired using GSM. Once more, mode 5 showed only D2 while modes 6, 7 and 8 displayed peaks of both D1 and D2 along with some false peaks ,as shown in Fig.8. The peak of D1 is dominant in these higher modes as compared to D2. The peak of D2 can be distinguished, however there are large false peaks at the edges of the plate which become dominant in higher modes. The peaks at the extreme left of the plate can be disregarded as this is the fixed end of the plate. The area around fixed end gives a small response and hence the peaks in this region can be ignored. The difference of this insensitive region between the fixed-end plate and free-free plate can be clearly seen in Fig.5 and Fig.6.

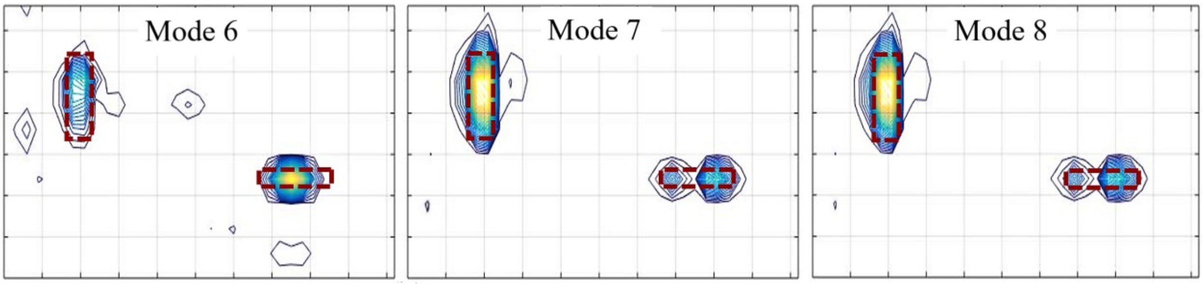


**Fig.5:** Damage index of 6th, 7th, and 8th modes for free-free plate

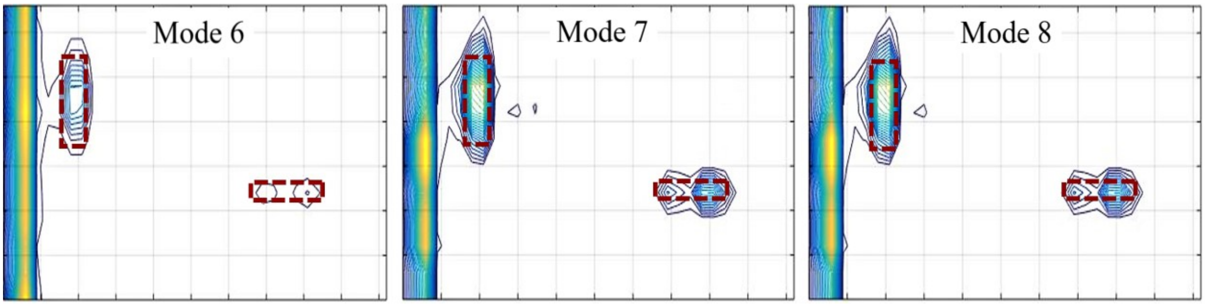


**Fig.6:** Damage index of 6th, 7th, and 8th modes for cantilever plate

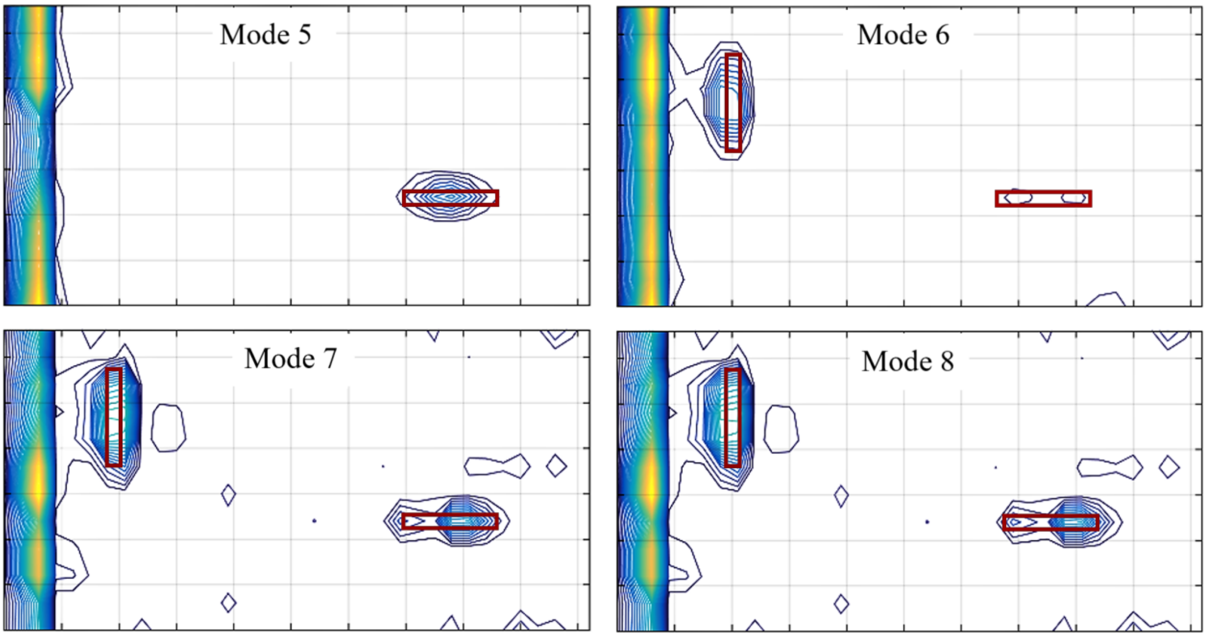


**Fig.7:** Damage indices of 5th – 8th modes with $e = 3.5 \times 10^{-2}$

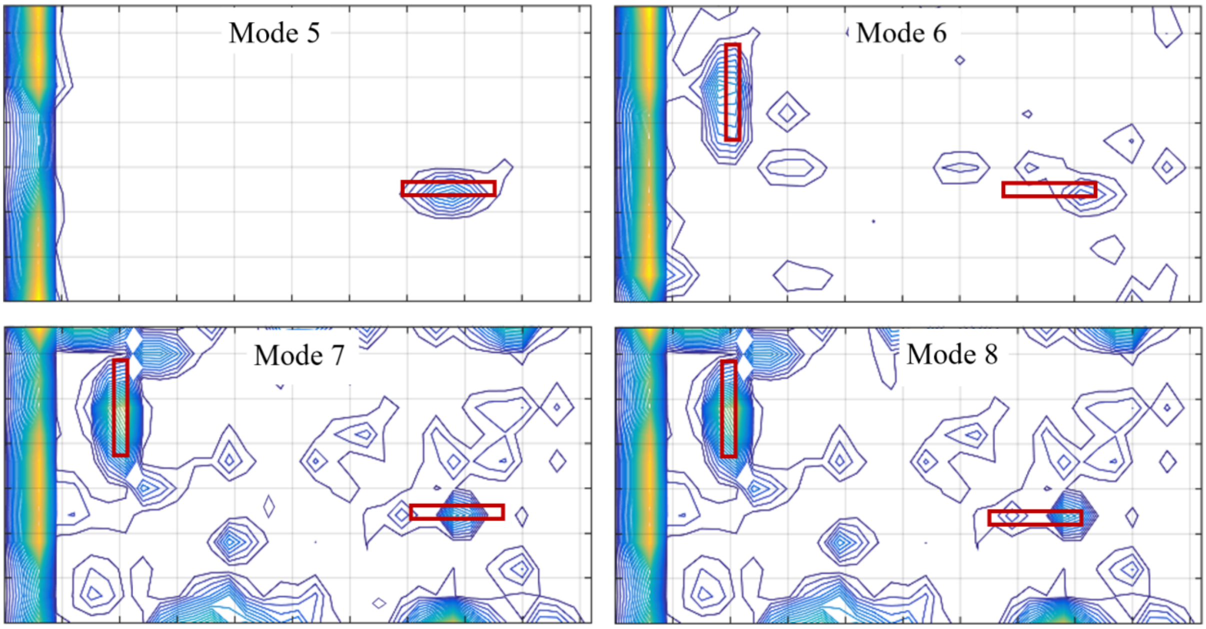


**Fig.8:** Damage indices of 5th – 8th modes with $e = 1 \times 10^{-1}$

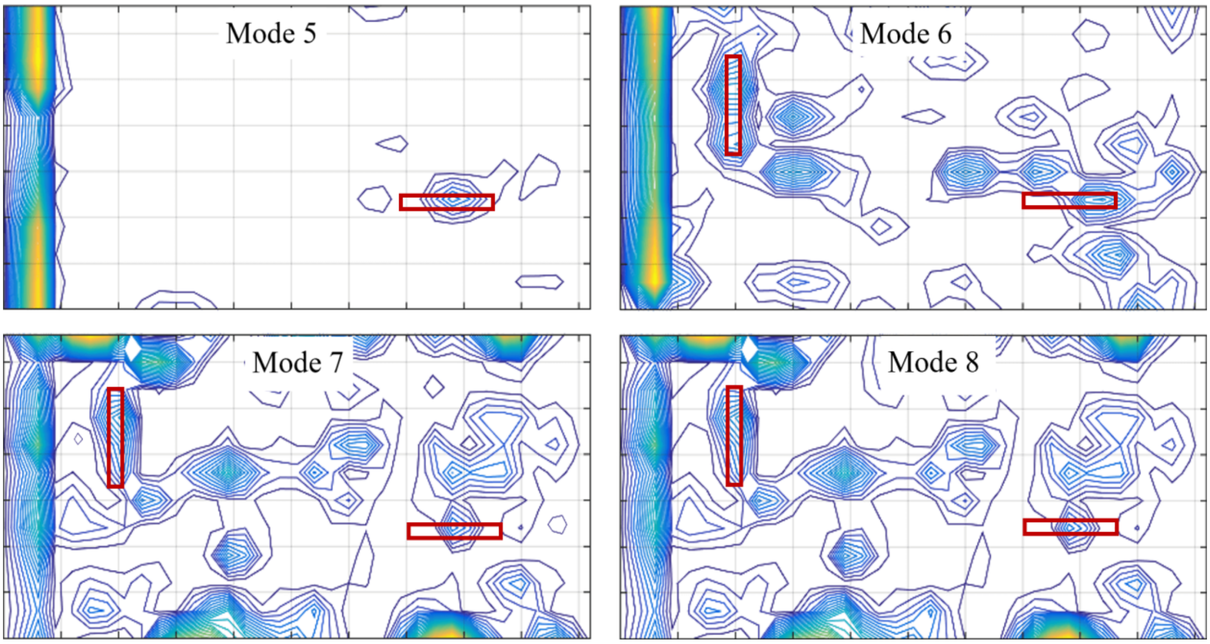


**Fig.9:** Damage indices of 5th – 8th modes with $e = 2 \times 10^{-1}$

Again, the noise level was further increased to $e = 2.0 \times 10^{-1}$ and damage indices were obtained from GSM. From Fig.9, it can be clearly seen that false peaks become dominant and it is difficult to detect damage at this noise level.

#### 6.1.4 Effect of 2D-Bayesian Technique with Beta PDF on Noisy Data

As discussed, it becomes difficult to detect damage when the measurement noise increases. Now to suppress this noise, 2D-Bayesian technique was applied to generate the estimated damage indices using the higher modes. After employing 2D-Bayesian along with Beta PDF, the results were significantly improved. It can be seen from Fig.10, that uniting the information of higher modes through 2D-Bayesian technique helped in highlighting the useful peaks that were consistent in all the modes. Moreover, beta PDF helped in removing the false peaks at the insensitive locations of the plate, closer to the support (left side of plate).

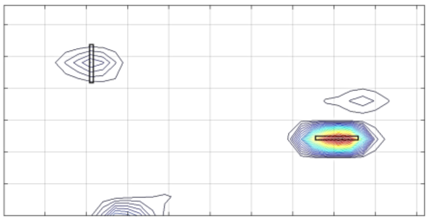

**Fig.10:** Estimated damage indices using 2D-Bayesian technique with $e = 2 \times 10^{-1}$

## 6.2 Scenario 2: Diamond-Shaped Damage

For the second scenario, two diamond-shaped damages D1 and D2 with 30% severity were created in plate. The severity was introduced by reducing the stiffness of the damaged elements in plate. The damage locations with plate parameters are already mentioned in Table 2. From the preliminary analysis in Table 1, it was observed that diamond-shaped damages with 30% and 40% severity can only be detected by using higher modes such as from mode 9 onwards. So, for this analysis, mode 9 and mode 10 were used to generate damage indices using GSM initially. The results can be seen in Fig.11, which shows the locations of both damage without any false peaks. The damage locations in Fig.11 onwards are highlighted by black boxes.

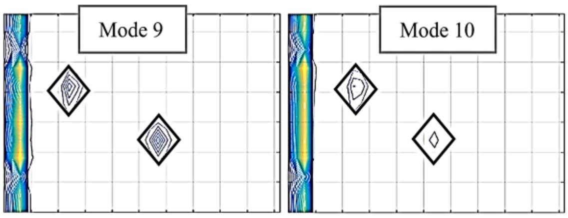


**Fig.11:** Damage indices for noise-free case

### 6.2.1 Addition of Noise:

In practical life, the measurement noise is unavoidable. Many devised methods that can accurately detect damage on noise-free data, did not work as effectively in the presence of noise. The effectiveness of the 2D-Bayesian technique with Beta PDF is already checked for delamination case, in section 6.1.4. Like scenario 1, noise was added in the time domain response of the damaged plate considering scenario 2. The results in the form of damage indices were first generated by using GSM only. With a noise level of $e = 4 \times 10^{-1}$, only mode 9 shows the locations of both damages while mode 10 has many false peaks in damage indices as can be seen in Fig.12.

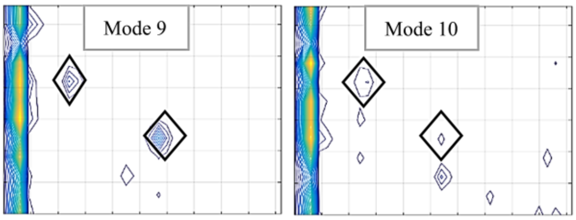


**Fig.12:** Damage indices with $e = 4 \times 10^{-1}$

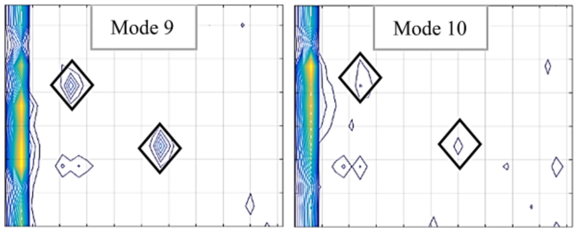


**Fig.13:** Damage indices with $e = 5 \times 10^{-1}$

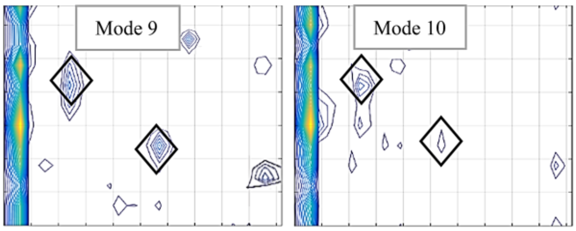


**Fig.14:** Damage indices with $e = 6 \times 10^{-1}$

When the noise level is further increased gradually from $e = 5 \times 10^{-1}$ to $e = 6 \times 10^{-1}$, it can be observed in Fig.13 and Fig.14, that the false peaks in the damage indices of mode 10 also increased. Hence mode 10 cannot be used to detect this type of multiple damage in the presence of noise. Now if mode 9 is examined, the peaks at the locations of damage are consistent for both noise levels, however, there are false peaks also. These false peaks increased with the increase in noise. Hence mode 9 alone cannot be used to detect diamond-shaped multiple damage with good accuracy using GSM.

These damage indices were then employed in 2D-Bayesian technique along with Beta PDF. Bayesian technique helped highlighting the consistency of the peaks at the damage locations using both mode 9 and mode 10. The estimated damage indices were generated for the highest level of noise considered here, which is $e = 6 \times 10^{-1}$. It can be seen in Fig.15, that both damage locations are clearly identified while almost all false peaks are removed due to their randomness in each mode. There is a very small false peak on the right side of the plate, however the peaks at the damage location are dominant.

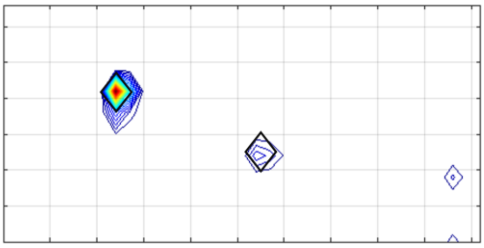

**Fig.15:** Estimated damage indices using 2D-Bayesian technique with $e = 6 \times 10^{-1}$

## 7. Conclusion

This paper presented a 2D-Bayesian technique for plate-type structures. Initially a preliminary analysis is carried out to examine the modes that are effective for damage detection. This analysis was carried out on a plate by varying the damage type, damage severity, and boundary conditions. Once the effective modes were found, those modes were used in further analysis. It was observed that in plates, higher modes were usually effective in detecting single and multiple damage with low severity. Moreover, the sensitivity of damage detection process changes with the change in boundary conditions.

To investigate the 2D-Bayesian technique in plates, multiple delamination-type and diamond-type damages were considered. The technique was used in conjunction with Beta PDF filter. This filter was used to assign low weightage to the portions of the plate that were insensitive in damage detection process. This is the portion that is around the support in cantilever case and it frequently shows false peaks in damage indices. The results in this paper indicated that traditional 2D-GSM was not that effective when the noise levels were increased. However, when 2D-Bayesian technique was employed along with the Beta PDF the results drastically improved localizing the multiple damage in both cases. In comparison to the published work detailed in Table 3, the results were much better particularly in the presence of noise. In this paper, the localization of multiple damages was achieved even with noise level of $e = 2 \times 10^{-1}$ for delamination-type and $e = 6 \times 10^{-1}$ for diamond-type damage. This is in comparison to the published work where single damage with same severity was detected with noise level up to $e = 2.5 \times 10^{-4}$ [17].

Multiple damage with 27% severity was also successfully detected considering the noise level up to $e = 1 \times 10^{-1}$, however the detection was possible only when many higher modes such as modes 10 – 15 were involved [20]. Moreover, the measurement points were dense all on the surface of the plate which is usually not feasible for real structures. In this paper the measurement points were sparse and only two higher modes were used. Even then, the method was successful in detecting multiple damage by using the data from the damaged structure only and at higher noise level.

**Table 3:** Comparison of this research work with already published work

| Parameters | Fan & Qiao [21] | Baneen & Guivant [17] | Cao et al. [20] | This Research Work |
|---|---|---|---|---|
| **Damage Type** | Delamination, Diamond and Square | Delamination, Diamond and Square | Square | Delamination and Diamond |
| **Boundary Conditions** | Cantilever | Free-Free | Cantilever | Cantilever+ Free-Free |
| **Noise Level $e$** | $1 \times 10^{-5} - 5 \times 10^{-5}$ | $1 \times 10^{-4} - 2.5 \times 10^{-4}$ | $1 \times 10^{-1}$ | Up to $2 \times 10^{-1}$ and $6 \times 10^{-1}$ |
| **Number of Damage** | Single | Single | Multiple | Multiple |
| **Techniques** | GSM | GSM + Bayesian | PCA + LoG | GSM + Bayesian with Beta PDF |
| **Damage Severity** | 30% stiffness reduction | 30% stiffness reduction | 10% thickness reduction ≈ 27.1% stiffness reduction with 10-15 modes | 30% stiffness reduction |

Further experimental work is being carried out to validate the effectiveness of the technique. Moreover, if the curvature mode shapes can be directly measured instead of using a second difference of mode shape data, the errors related to approximations can also be avoided.

## 8. References


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