Modular Modeling Approach for Analysis of Cellular Beams

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Abstract

This paper presents the modular modeling approach adapted for the analysis of a cellular beam. These beams can be very useful for reducing the overall floor thickness in order to improve the Architectural design of a building. The cellular beam is made of a steel I-section with a total of 32 holes throughout its length. The finite element model of the beam is divided into 32 partitions and each partition is analyzed on a different processor. This partitioning method is based on a domain decomposition technique using dual partition super-elements. The analysis results are compiled and it is shown that with the help of the partitioning method, the nonlinear analysis of cellular beams has become more efficient and more accurate.

Keywords: : Cellular Beams, Thin Floors, Modular Modeling, Domain Decomposition, Nonlinear Analysis

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INTRODUCTION

The cellular beam is a type of structural member that has gained increased popularity because of its ability to withstand gravitational loads over large spans whilst allowing the integration of services within the beam depth, resulting in floors of a much smaller overall depth compared to conventional steel beams. This feature of the cellular beams makes this type of elements very important from Architectural point of view as it can provide additional freedom to the designer for bringing out an aesthetically pleasant look of the building while maintaining its structural integrity.

The presence of holes in the web of the beam, however, causes local buckling in the web-post and/or compression regions around the openings. Work on the simplified and detailed analysis of this type of structure is currently ongoing at Imperial College London as part of an independent PhD research program (Zainal Abidin & Izzuddin, 2011; Zainal Abidin, 2012). This example demonstrates that the modeling of such seemingly complex structures is simplified with the use of modular modeling, and highlighting the computational efficiency of the proposed partitioned approach when utilized with parallel processing.

Overview of the Partitioning Method

A new method was been developed for structural domain decomposition presented elsewhere (Jokhio, 2012; Jokhio and Izzuddin, 2013), which facilitates scalable parallel processing over distributed memory networks, thus overcoming memory bottlenecks for large scale problems. The new method introduces the concept of dual partition super-elements, where a child partition is wrapped at the interface boundary by a super-element and is represented in the parent structure by a dual super-element. With the parent structure and child partition attached to separate processes, one super-element is used in the parent process and the dual super-element is used as a wrapper around the partitioned boundary in the child process. It is worth noting here that this method can accommodate multiple child partitions for a specific parent structure. Furthermore, the method is hierarchic, in the sense that a child partition can be further partitioned similar to a parent structure, and it can thus be mapped to hierarchic parallel processing architectures.

In addition to the benefits of traditional partitioning approaches, the current approach provides the facility for using mixed methods such as implicit-explicit integration schemes (Belytschko, Yen, & Mullen, 1979; Hughes, Levit, &Winget, 1983) as well as dimensional coupling (McCune, Armstrong, & Robinson, 2000; Shim, Monaghan, & Armstrong, 2002) between partitions. A further important benefit of the developed approach is that it allows the recovery of child partition forces and condensed tangent stiffness matrix at the interface boundary relatively easily via the dual super-element, which can be achieved in a frontal solution method by placing the child super-element at the end of the element ordering list. When all the other elements of the partition are assembled and the associated interior freedoms are eliminated, the remaining equilibrium equations contain the forces and condensed tangent stiffness matrix for the super-element only, which can be communicated to the dual super-element in the parent structure. The parent process treats the partition super-element similar to other finite elements, providing displacements that can be communicated to child processes of the dual super-elements and receiving interface forces and condensed tangent stiffness matrices in return. Thus, this approach is effectively identical in performance to the monolithic approach with high speed-ups due to parallel processing and ease of modeling.

A very significant advantage of the partitioning method is that it supports the modular modeling approach in that a model partition finite element model can be created that can be replicated with minor adjustments to represent other partitions. The applicability of the modular approach using the developed domain decomposition method is utilized here for the analysis of the cellular beam.

The Cellular Beam

The cellular beam under consideration spans over 30 meters and has a total of 32 holes in its web that are spaced 0.92 m apart center to center. The first hole is situated at a distance of 0.74 m from the left end of the beam as shown in Figure 1. The beam has 25.4 mm thick and 268 mm wide flanges, 15.6 mm thick web, a total depth of 1165.2 mm and hole diameters of 800 mm each as shown in Figure 2. The web posts between two consecutive holes have a minimum width of 120 mm.

The entire model is made of elements of type 'cvs9', which are shell elements that can be used to model 2D slabs as well as flanges and webs of frame elements. Further details about this element can be found elsewhere (Izzuddin, 2007).

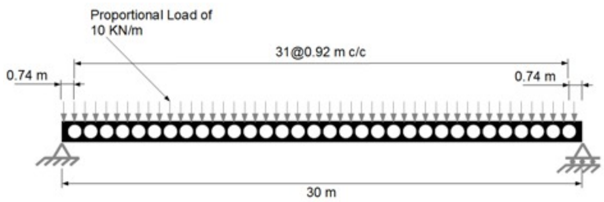
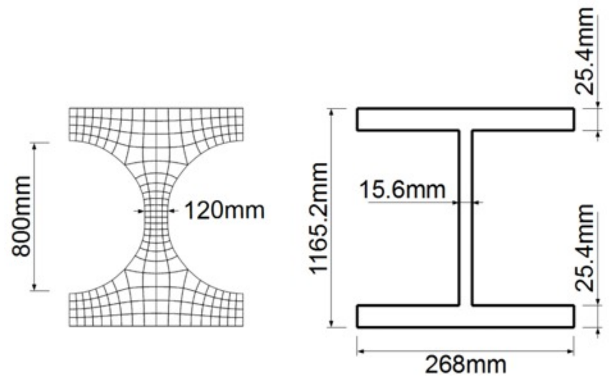


Figure 1: Side elevation of the cellular beam



(a) side view of a unit cell **(b)** cross-section

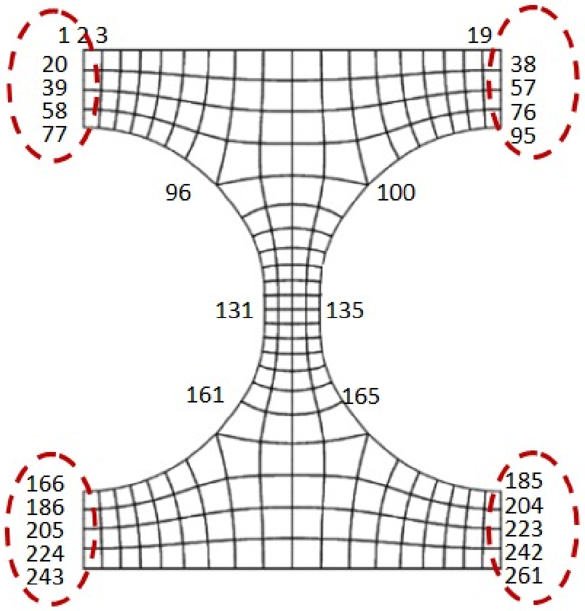
Figure 2: Side elevation of a unit cell and cross-section of the cellular beam

Modular modeling

For making the finite element model of the cellular beam, the modular modeling approach is adapted. According to this approach, a partition module is created, which can later be replicated to represent other partitions. In this particular case,the domain partitioning is quite simple due to the fact that it consists of 31 identical unit cells in addition to the 2 end units. Therefore, it is advantageous to make each unit cell a child partition, resulting in the need to create only

3 data files in addition to the parent structure which consists of nodes for the 32 cross-sections, equally spaced at 0.92 m. The mesh used for a typical unit cell can be seen in Figure 2 (a). Each unit cell child partition is exactly similar and is created by making copies of a single data file. The corresponding data file can therefore be identified as the module from which all these partitions are generated.

For making one partition module, a total of 200 'cvs9' elements are used as stated earlier. This type of elements require four nodes for connectivity resulting in a total of 261 nodes as shown below in Figure 3.



If the nodes for the finite element model are numbered as shown in Figure 3, then the nodes shown within the dotted circles represent the partitioned boundary for that particular module. In the present example, therefore, there are 4 sets of nodes that form the partitioned boundary i.e. nodes sets 1, 20, 39, 58, 77; 19, 39, 57, 76, 95; 166, 186, 205, 224, 243; and 185, 204, 223, 242, 261. All these 4 sets of nodes at the partitioned boundary i.e. a total of 20 nodes are connected to a single partition super-element that wraps around the boundary of this partition. At the start of each loading step, the coordinator process sends such an instruction along with the load factor for that loading step to this partition, and after performing the analysis the partition, through the partition super element communicates the tangent stiffness matrix and the resistance forces vector for these 20 nodes at the partitioned boundary to the coordinator. The coordinator process, after getting this information from all the partitions, finds out the out of balance and checks the equilibrium.

For the example under consideration, therefore, the tangent stiffness matrix at the parent structure level is assembled as:

$$K = \sum_{i=1}^{33} K_{\Omega_i}^{c}$$

Where K is the tangent stiffness matrix, $K_{\Omega_i}^{c}$ is the condensed tangent stiffness matrix received from the child partitions $\sum_{i=1}^{33}$ and does not represent the conventional summation, instead, it represents the assembly of contributions from the child partitions as per their element connectivity. It is worth noting here that the child partitions are represented in the parent structure through the partition super-elements.

It should be noted that super-elements are treated in an identical way to conventional finite elements with regard to assembly of element contributions, and therefore the order of element assembly may be varied from what is assumed in the above discussion without loss of generality. Indeed, in a frontal solution method, the order of assembly at the parent level, considering super-elements and conventional elements individually, would typically be determined as one that minimizes the front width.

Once the tangent stiffness matrix has been assembled, the change in resistance due to the iterative corrections of displacements, including the contribution from the partition super-elements, is approximated as:

$$\delta R = K \delta d$$

where δd and δR refer here to finite iterative increments of displacement and resistance, respectively.

Similar to the tangent stiffness matrix, the resistance forces vector R is assembled as contributions from conventional finite elements and super-elements, where in the latter case the condensed resistance forces at the partitioned boundary $R_{\Omega_m}^{c}$ are considered, as discussed in the next section. The out-of-balance at the parent level is obtained as:

$$G = R - P$$

It is worth noting here that the load vector in the above equation does not contain the loads that are applied to the internal nodes of the partitions, as these are dealt with inside the child partitions.

If there is any out-of-balance remaining, the correction to the displacements at the parent level is calculated as:

$$\delta d = K^{-1}(-G)$$

The correction to the displacements for the nodes connected to the partition super-elements are provided to the dual partition super-elements:

$$\delta d^{c}_{\Omega_i} = \delta d_{\{m:n\}}$$

so as to determine the displacement corrections for the internal nodes. It should be noted here that the range *m:n* is representative only and it may be varied, without the loss of generality, depending upon the element connectivity. Once the partition displacements are updated with the iterative corrections, the condensed resistance forces and tangent stiffness from the dual super-elements are provided to the parent structure for the next iteration. The process is repeated until convergence to an acceptably small out-of-balance at both the parent and child levels.

Following this, every partitioned module is connected to the parent structure in the manner as shown in Figure 4 below:

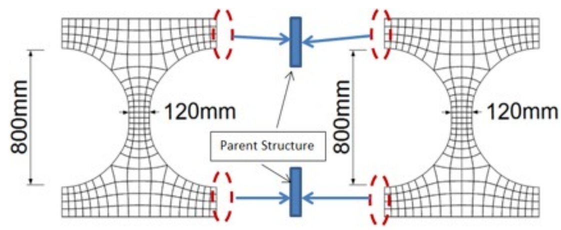


Figure 4: Connectivity of partition modules with the parent structure

Computation Efficiency

The parent structure of this example consists of 32 such sets of nodes as shown in Figure 4, which results in a total of 1280 nodes for the parent structure. The monolithic finite element model for such a structure would have consisted of about 6000 additional nodes. The computational time required for such a model in terms of No. of Multiplication Operations required for bringing the tangent stiffness matrix to the row-echelon form can be given as:

$$\Psi = \frac{(n-1)n(n+2)}{3}$$

Where, Ψ represents the No. of Multiplication Operations and is the size of the matrix that is to be brought to the row-echelon form. The No. of Multiplication Operations, for the monolithic structure, therefore would have been approximately 0.13 Trillion. It is worth noting here that this is just for bringing the matrix to the row-echelon form in the forward elimination step of one iteration. The solution of the system of nonlinear equations for such an structure requires several iterations, the number of which generally remains around 10. Because of the complexity of the problem, this structure could not be completely analysed using the load control method. The displacement control technique will, therefore, have to be employed, which renders the analysis of this structure in a monolithic setting impractical.

In a parallel setting, since all the partitions are being analysed on different processors in parallel, the computational time in terms of No. of Multiplication Operations for one iteration should be that for one of the partitions in addition to the parent structure. The No. of Multiplication Operations for one partition as per Equation 6 is 5.9 Million and that for the parent structure is approximately 0.7 billion. The total computational requirement, therefore, remains about 0.7 billion as the effect of the partition is very small. There is expected to be some communication overhead as well because of the message passing between parallel processors. This communication overhead depends upon the type of network as well as physical location of the processing cores. Ignoring the communication network, the computational requirement of the partitioned model using the modular modelling approach is negligible compared to the monolithic model. Thus, the parallelisation approach has made the analysis of such structures parallel as shown by the example presented here. This is shown in the following section.

RESULTS AND DISCUSSION

The cellular beam is subjected to a proportional uniformly distributed load, with a nominal value of 10kN/m, which is specified internally at the partition level. Since this is a problem dominated by local buckling of the web-post, random imperfections are introduced in the two end units via very small out-of-plane loads. Importantly, as the post-buckling response is associated with snap-back behavior, the arc-length displacement control method is used beyond the limit point after an initial phase of load control.

Deflected shapes of the beam are shown in Figure 4, and for illustrative purposes only a close up of partitions 1 and 2 with contours of the normal stress in the longitudinal direction is provided in Figure 4. It can be seen that, as expected, that the web-posts buckle near the support due to significant shear forces combined with compression resulting from applying the UDL on top of the beam.

Figure 5: Deflected shape of the cellular beam (Displacement scale = 5)

The load-deflection response of the beam is provided in Figure 5, where it is clear that the arc-length method is successful with the proposed partitioned approach in tracing the snap-back post-buckling response. Importantly, the whole analysis is undertaken on 34 processors in 30 minutes of wall-time when it would have taken over 16 hours with a monolithic model besides the excessive memory requirements that such a model demands.

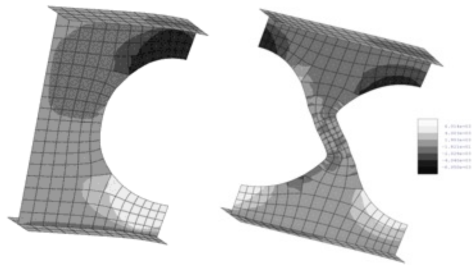


Figure 6: Contours for normal traction in longitudinal direction for partitions 1 and 2 (Units: N/m as thickness is included in stress resultant)

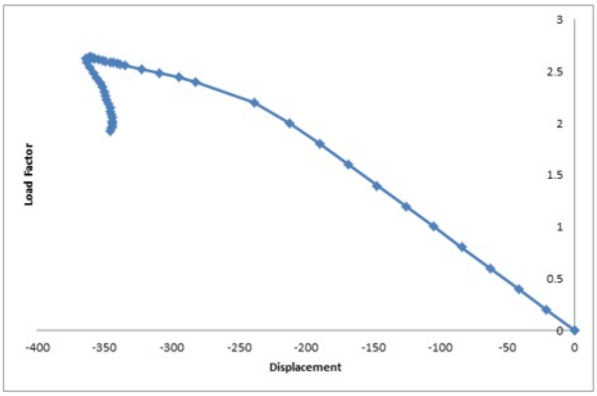


Figure 7: Displacement at mid-span in the cellular beam

CONCLUSION

The modular modelling approach was adopted for the nonlinear analysis of a cellular beam. The analysis results have shown that accurate structural response of such beams can be easily and efficiently obtained with the help of the developed partitioning approach. The use of this type of beams for making the supporting structures for large spanned floors will help reducing the overall floor thickness significantly by incorporating the services within the beam depth. This will add to the aesthetical look of the building and may provide more freedom to the Architect

REFERENCES

- Jokhio GA, Izzuddin BA, (2013). Parallel Nonlinear Analysis using Dual Partition Super Elements. *Advances in Engineering Software.* 60-61: 81–88.

- Izzuddin BA. (2007). An Optimisation Approach towards Lock-Free Finite Elements, *Proceedings of the 11th International Conference on Civil, Structural and Environmental*

Engineering Computing, Paper No. 105, B.H.V. Topping (Editor), Civil-Comp Press, Stirlingshire, Scotland.

- Jokhio GA. (2012). *Mixed Dimensional Hierarchic Partitioned Analysis on Nonlinear Structural Systems*. PhD Thesis. Imperial College London, UK.
- Belytschko T, Yen HJ and Mullen R. (1979). Mixed methods for time integration. *Computer Methods in Applied Mechanics and Engineering.* 17/18: 259-275.
- Hughes TJR, Levit I and Winget J. (1983). An element-by-element solution algorithm for problems of structural and solid mechanics. *Computer Methods in Applied Mechanics and Engineering.* 36: 241-254.
- McCune RW, Armstrong CG and Robinson DJ. (2000). Mixed-dimension coupling in finite element models. *International Journal for Numerical Methods in Engineering.* 49: 725-750.
- Shim KW, Monaghan DJ and Armstrong CG. (2002). Mixed dimension coupling in finite element stress analysis. *Engineering with Computers.* 18: 241-252.