RATIONAL DESIGN OF RETAINING WALLS

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Abstract
Proposed a method and discussed the procedure for forming the structure of lightweight retaining wall. The method is based on the some new energy principles. Considered some examples for design of retaining walls. The construction of resulting structure is based on new technology.
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INTRODUCTION

Analysis and design of the structures with positive properties is one of the priorities of modern engineering science. The above determines development of methods and tools of design of efficient systems, which minimize the cost of the construction, weight, adverse impact on the environment, and etc. In this regard, the traditional (indirect) approach of the design can not provide integrated solutions positivity. Information technology is an alternative, which based on general systems theory, including topics such as mathematical modeling, synergetics, and informatics. Finite element method (FEM) became the basic of the modeling of deformable structures. Finite-element modeling provides not only effective solution of indirect problems of mechanics of deformable solid bodies, but also is an excellent application apparatus for formulating and solving optimization problems, i.e direct problems. A special class is problems of regulation of parameters of structures and their state, and in particular, in conjunction with the finding of their extreme values. Gorodtsly et al 2003 reported that the mangement (control) of the behavior of structures is the tool that can be used not only to significantly improve its technical and economic parameters, but also, most importantly, improve reliability of service. The principal feature of the formation of controlled structures is dual process involving algorithms for obtaining the necessary characteristics and appropriate technological sequences of their production. In this paper, using described approach, was found complex solution (including the application in practice) of the direct design of retaining walls. In this case, design and technological procedures are founded on new energetical principles (Ishlinskii 2008, Vasilkov 2008). As a result, the algorithms for search of rational method of design of structure were constructed, the geometry of which provides:
the given transformation of the diagram of horizontal active pressure on the wall;
quasi-energetical equi-strength of system.
In addition, proposed and implemented an effective method of construction of the discussed structure.

1 Geometry Generation of Ketaining Wall

1.1. Assumptions

The proposed formulation of the problem is founded on the hypotheses and assumptions of the corresponding Coulomb theories (Fig.1), namely (Klein 1996) the failure mode of biagregata consisting of retaining wall and held it soil, array is represented by the movement of the wall away from the soil, and simultaneously slipping of the some prism of the last along sliding surface - considered two sliding surfaces: the back side of the wall and a plane, which is the boundary of the stationary part of the soil; the slipping prism is absolutely rigid body, which allows to replace the existing volume and surface forces by their resultants of G; Q; R (G - self weight; Q - reaction of retaining wall; R - reaction of the fixed soil);

- The soil is a loose body, devoid of cohesion;
- Considered biagregat in the equilibrium limit state, corresponding to the initial stage of displacement of wall and sliding of the prism of soil. Therefore, it is assumed that reactive forces acting on the sliding prism by wall and the fixed part of the soil deviate from the vertical to the respective planes of φ_o and φ, equal to the angles of friction of soil on these planes;
- Considered the initial stage of the failure process, in this connection, the equilibrium conditions are written to its undeformed state;
- The problem is considered as planar.

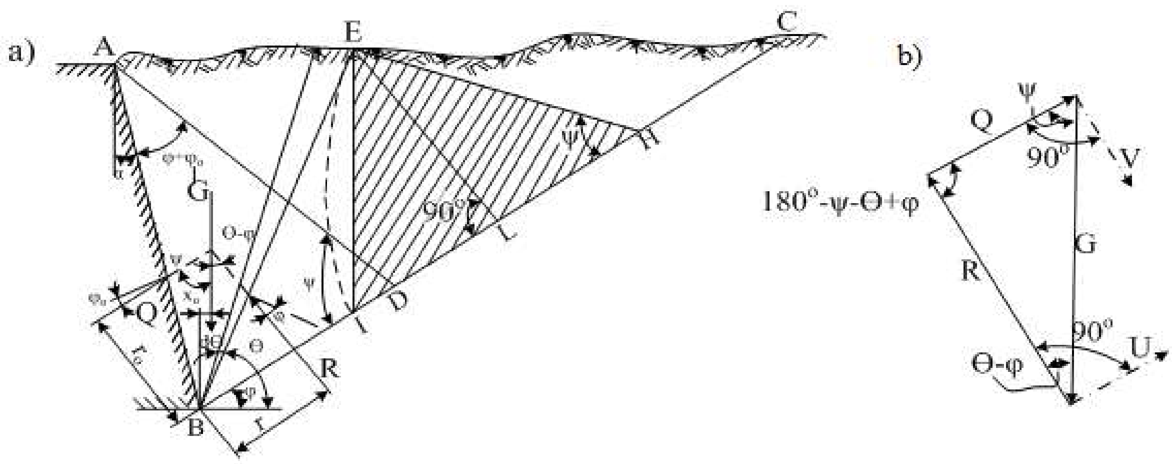


Figure1.1. The design scheme of the biaggregate.

The angles of force triangle (Figure 1.1) are:

$$\psi = 90^{o} - \alpha - \varphi_{o};$$

$$\theta - \varphi; \tag{1.1}$$

$$180^{o} - \psi - \theta + \varphi,$$

where α - the angle between the vertical plane and the back face of the wall;
θ - the unknown angle between the horizontal plane and slipping plane.
From the above follows that the reaction of retaining wall is equal in magnitude to the required active pressure and opposite in direction. On the basis of the assumed hypothesis, the expression for the resultant of active pressure (Klein 1996) is:

$$Q = \frac{\gamma \cdot h^2}{2}\lambda, \tag{1.2.}$$

where γ - specific weight of soil;
h – projection of height of wall on the vertical plane;
λ – the coefficient of active soil pressure is equal to

$$\lambda = \frac{\cos^2(\varphi - \alpha)}{\left[1 + \sqrt{\frac{\sin(\varphi + \varphi_o)\sin(\varphi - \bar{\beta})}{\cos(\alpha + \varphi_o)\cos(\alpha - \bar{\beta})}}\right]^2 \cos^2\alpha\cos(\alpha + \varphi_o)}, \tag{1.3}$$

where $\bar{\beta}$ – the angle of inclination of external surface of soil with respect to the horizontal (pitch angle).
Analysis of (1.3) allows considering the angle of inclination of the wall to the vertical α, as external regulating parameter (Ishlinskii 2008.) Further, given the logical direction of the problem, as well as to simplify its formulation (without loss of generality) we give:

The external surface of the soil is limited by horizontal plane, ie $\bar{\beta} = 0$;
The back side surface of the wall is considred perfectly smooth, then φ_o=0.
In this connection, (1.3) simplifies to:

$$\lambda = \left[tg\left(45 - \frac{\varphi + \alpha}{2}\right) + tg\alpha\right]^2 \cos\alpha \tag{1.4}$$

1.2. Mathematical Model.
Rationalization of the system, to some extent, can be achieved by reducing the soil pressure on retaining wall. The latter may be realized by making a certain shape for back surface of wall. Taking the concept of the independent formation of the priori distribution of the horizontal pressure (eg, uniform distributed load), per unit surface of the wall, we can write:

$$\sigma = \lambda \cdot \gamma \cdot (z_o + z_1), \tag{1.5}$$

Where: $z = z_o + z_1$ - current depth (Figure 1.2);
σ – intensity of normal pressure on the wall at depth z from the surface of the backfill; z_o – the depth, at which the horizontal pressure is taken as the initial (Figure 1.2).

$$l = \left[tg\left(45 - \frac{\varphi + \alpha}{2}\right) + tg\alpha \right]^2 \cos\alpha$$

The range of the angle α lies in the limit: $-\varphi < \alpha \le 0$, (1.6)

The limits set by the physical meaning of the problem.
Next, we introduce the notation:

$$\psi = 45^o - \frac{\varphi}{2},$$

$$l = \left[tg\left(\psi - \frac{\alpha}{2}\right) + tg\alpha \right]^2 \cos\alpha$$

Using the known trigonometric relationship: $\cos\alpha = \pm\sqrt{\frac{1}{1+tg^2\alpha}}$, Для дальнейших выкладок For further calculations need to choose the sign before the radical sign. So much so $\cos\alpha$ is an even function we take:

$$\cos\alpha = \sqrt{\frac{1}{1+tg^2\alpha}}.$$

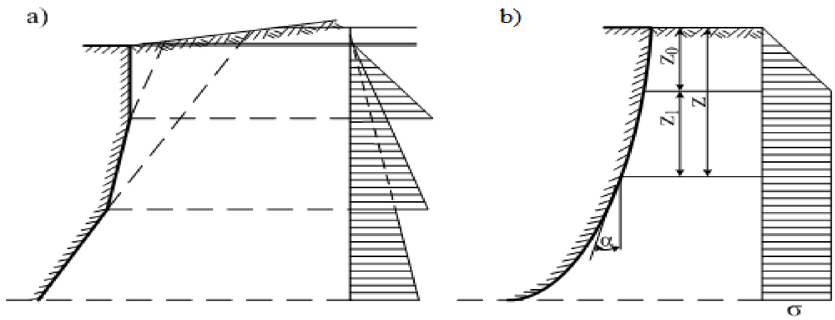


Figure 1.2. Search of geometry of wall.

Then:

$$l = \left[\frac{tg\psi - tg\left(\frac{\alpha}{2}\right)}{1 + tg\psi \cdot tg\left(\frac{\alpha}{2}\right)} + tg\alpha \right]^2 \sqrt{\frac{1}{1+tg^2\alpha}}$$

Use the trigonometric relation: $tg\frac{\alpha}{2} = \pm\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$, where is also necessary to choose the appropriate sign. From (1.6) we see that the angle α - is negative, then the function $tg\frac{\alpha}{2}$ is also negative. In this connection:

$$tg\frac{\alpha}{2} = -\sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}$$

$$l = \left[\frac{tg\psi + \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}{1 - tg\psi \times \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}} - tg\alpha \right]^2 \sqrt{\frac{1}{1+tg^2\alpha}} = \left[\frac{tg\psi + \sqrt{\frac{1-\sqrt{\frac{1}{1+tg^2\alpha}}}{1+\sqrt{\frac{1}{1+tg^2\alpha}}}}}{1 - tg\psi \times \sqrt{\frac{1-\sqrt{\frac{1}{1+tg^2\alpha}}}{1+\sqrt{\frac{1}{1+tg^2\alpha}}}}} - tg\alpha \right]^2 \sqrt{\frac{1}{1+tg^2\alpha}}$$

From the geometric meaning of the first derivative, it follows that

$$y' = \frac{dy}{dz} = tg\alpha\,, \tag{1.7}$$

where $y=y(z)$ – the function, which describe the geometry of the back surface of the retaining wall.

In view of (1.7) we have:

$$\lambda = \left[\frac{tg\psi + \sqrt{\dfrac{1-\sqrt{\dfrac{1}{1+y'^2}}}{1+\sqrt{\dfrac{1}{1+y'^2}}}}}{1 - g\psi \cdot \sqrt{\dfrac{1-\sqrt{\dfrac{1}{1+y'^2}}}{1+\sqrt{\dfrac{1}{1+y'^2}}}}} - tg\alpha \right]^2 \sqrt{\frac{1}{1+y'^2}}\,;$$

Substituting of variable.: $f = \sqrt{\frac{1}{1+y'^2}}$. We express out y':

$$f^2 = \frac{1}{1+y'^2} \quad ; \quad \frac{1}{f^2} = 1 + y'^2 \quad ; \quad y'^2 = \frac{1-f^2}{f^2} \quad ; \quad y' = tg\alpha = \pm\sqrt{\frac{1-f^2}{f^2}}$$

$$\lambda = \left[\frac{tg\psi + \sqrt{\dfrac{1-f}{1+f}}}{1 - g\psi \cdot \sqrt{\dfrac{1-f}{1+f}}} - tg\alpha \right]^2 f\,;$$

From (1.5): $\lambda = \frac{\sigma}{\gamma(z_o + z_1)}$, *where*: $z=z_o+z_1$ – current depth (Figure 1.2); finally: $\lambda = \frac{\sigma(z)}{z \cdot \gamma}$

Given that the values of γ, z_o, and φ are known, and the magnitude of the intensity of normal pressure can be represented as a known function of the depth $\sigma=\sigma(z)$, is permissible to write the following equation:

$$\left[\frac{tg\psi + \sqrt{\dfrac{1-f}{1+f}}}{1 - g\psi \cdot \sqrt{\dfrac{1-f}{1+f}}} - \sqrt{\frac{1-f^2}{f^2}} \right]^2 f = \frac{\sigma(z)}{z \cdot \gamma}\,; \tag{1.8}$$

We make one more substitution of variable, $F(z) = \sqrt{\frac{\sigma(z)}{z \cdot \gamma}} \;\Rightarrow\; F^2(z) = \frac{\sigma(z)}{z \cdot \gamma}$, then:

$$\left[\frac{tg\psi + \sqrt{\dfrac{1-f}{1+f}}}{1 - g\psi \cdot \sqrt{\dfrac{1-f}{1+f}}} - \sqrt{\frac{1-f^2}{f^2}} \right]^2 f = F^2(z).$$

Further, let that: $k^2 = \frac{1-f}{1+f}$

We express $f = f(k)$: $1 - f = (1+f)k^2 \quad ; \quad 1 - f = k^2 + f \cdot k^2 \quad ; \quad f \cdot k^2 + f = 1 - k^2 \quad ;$

$$f\left(k^2+1\right) = 1 - k^2 \quad ; \quad f = \frac{1-k^2}{1+k^2}\,,$$

$$\frac{1-f^2}{f^2}=\frac{1}{f^2}-1=\frac{1}{\left(\frac{1-k^2}{1+k^2}\right)^2}-1=\frac{\left(1+k^2\right)^2}{\left(1-k^2\right)^2}-1=\frac{1+2k^2+k^4}{1-2k^2+k^4}-1=$$

$$=\frac{1+2k^2+k^4-1+2k^2-k^4}{1-2k^2+k^4}=\frac{4k^2}{\left(1-k^2\right)^2}$$

In addition: $\sqrt{\frac{1-f^2}{f^2}}=\sqrt{\frac{4k^2}{\left(1-k^2\right)^2}}=\frac{2k}{1-k^2}$;

As a result, we have:

$$\left[\frac{tg\psi+k}{1-g\psi\cdot k}-\frac{2k}{1-k^2}\right]^2\frac{1-k^2}{1+k^2}=F^2(z);\quad \left[\frac{(tg\psi+k)\left(1-k^2\right)-2k(1-g\psi\cdot k)}{(1-g\psi\cdot k)\left(1-k^2\right)}\right]^2\frac{1-k^2}{1+k^2}=F^2(z);$$

$$\left[\frac{tg\psi-k^2tg\psi+k-k^3-2k+2k^2tg\psi}{(1-g\psi\cdot k)\left(1-k^2\right)}\right]^2\frac{1-k^2}{1+k^2}=F^2(z);\quad \left[\frac{tg\psi-k^3+k^2tg\psi-k}{(1-g\psi\cdot k)\left(1-k^2\right)}\right]^2\frac{1-k^2}{1+k^2}=F^2(z);$$

$$\left[\frac{tg\psi\left(1+k^2\right)-k\left(1+k^2\right)}{(1-g\psi\cdot k)\left(1-k^2\right)}\right]^2\frac{1-k^2}{1+k^2}=F^2(z);\quad \left[\frac{\left(1+k^2\right)(tg\psi-k)}{(1-g\psi\cdot k)\left(1-k^2\right)}\right]^2\frac{1-k^2}{1+k^2}=F^2(z);$$

$$\frac{\left(1+k^2\right)^2(tg\psi-k)^2}{(1-g\psi\cdot k)^2\left(1-k^2\right)^2}\frac{\left(1-k^2\right)}{\left(1+k^2\right)}=F^2(z);\quad \frac{\left(1+k^2\right)(tg\psi-k)^2}{(1-g\psi\cdot k)^2\left(1-k^2\right)}=F^2(z);$$

$$\frac{\left(1+k^2\right)\left(tg\psi-2k\cdot tg\psi+k^2\right)}{\left(1-k^2\right)\left(1-2tg\psi\cdot k+tg^2\psi\cdot k^2\right)}-F^2(z)=0;$$

$$\frac{tg^2\psi-2k\cdot tg\psi+k^2+k^2tg^2\psi-2k^3tg\psi+k^4}{1-2k\cdot tg\psi+k^2tg^2\psi-k^2+2k^3tg\psi-k^4tg^2\psi}-F^2(z)=0;$$

$$tg^2\psi-2k\cdot tg\psi+k^2+k^2tg^2\psi-2k^3tg\psi+k^4-\ldots$$
$$\ldots-F^2(z)\cdot\left(1-2k\cdot tg\psi+k^2tg^2\psi-k^2+2k^3tg\psi-k^4tg^2\psi\right)=0;$$

$$\left[1+F^2(z)\cdot tg^2\psi\right]\cdot k^4+\left[-2tg\psi-2F^2(z)\cdot tg\psi\right]\cdot k^3+\left[1+tg^2\psi-F^2(z)\cdot tg^2\psi+F^2(z)\right]\cdot k^2+\ldots$$
$$\ldots+\left[2\cdot F^2(z)\cdot tg\psi-2\cdot tg\psi\right]\cdot k+\left[tg^2\psi-F^2(z)\right]=0;$$

$$k^4+\frac{-2tg\psi-2F^2(z)\cdot tg\psi}{1+F^2(z)\cdot tg^2\psi}\cdot k^3+\frac{1+tg^2\psi-F^2(z)\cdot tg^2\psi+F^2(z)}{1+F^2(z)\cdot tg^2\psi}\cdot k^2+\ldots$$
$$\ldots+\frac{2\cdot F^2(z)\cdot tg\psi-2\cdot tg\psi}{1+F^2(z)\cdot tg^2\psi}\cdot k+\frac{tg^2\psi-F^2(z)}{1+F^2(z)\cdot tg^2\psi}=0;$$

Finally, we have: $k^4+d_2\cdot k^3+d_3\cdot k^2+d_4\cdot k+d_5=0$ (1.9)

where: $d_2=\frac{-2tg\psi-2F^2(z)\cdot tg\psi}{1+F^2(z)\cdot tg^2\psi}$; $d_3=\frac{1+tg^2\psi-F^2(z)\cdot tg^2\psi+F^2(z)}{1+F^2(z)\cdot tg^2\psi}$;

$d_4=\frac{2\cdot F^2(z)\cdot tg\psi-2\cdot tg\psi}{1+F^2(z)\cdot tg^2\psi}$; $d_5=\frac{tg^2\psi-F^2(z)}{1+F^2(z)\cdot tg^2\psi}$; $F(z)=\sqrt{\frac{\sigma(z)}{z\cdot\gamma}}$

z_o – depth, within which the pressure increases linearly up to the required value $\sigma(z_0)$, (Figure 1.2);

z – the variable depth (Figure 1.2);

$\sigma(z)$ *- - the pressure at* z;

For the particular case $\sigma(z)=\sigma=const$, $\sigma(z)=\sigma=const$, the ordinate of the curve does not depend on the specific weight of soil - γ. Considering the pressure σ on the vertical wall at the level $z=z_0$ equals to

$\sigma = \lambda \cdot \gamma \cdot z_o$, *we note that the coefficients of equation (1.9)* γ enters only through $F(z)$. Substituting the above expressions we have $F(z) = \sqrt{\lambda \cdot \gamma \cdot z_o \cdot (z \cdot \gamma)^{-1}}$, and finally: $F(z) = \sqrt{\lambda \cdot z_o \cdot (z)^{-1}}$. Q.E.D.
Note also, that it is advisable using numerical methods for solution of (1.9).

1.3. Analytical Solution

For formation an analytical solution of (1.9) (which simplifies the analysis), we introduce the following hypothesis

$$\cos \alpha = \cos^2 \alpha \qquad (1.10)$$

The inclination angle of the wall depends on the angle of internal friction φ. Increasing of φ leads to decrease α. From the physical meaning of the problem, that the angle is in range: $-\varphi < \alpha \le 0$. It is known (Sorochan and Trofimenkov., 1985) that for a variety types of soils φ lies in the range $7^0 < \varphi \le 43^0$.
Equality (1.10) with an error not exceeding 20% is valid for $\alpha \in \{0^0, 36^0\}$, that is 83% of the range of the angle of internal friction φ. In other cases, the error may reach 37%.
We give below a table showing the acceptability of introduced hypotheses:

Table 1

α	cosα	$\cos^2\alpha$	%
0	1.000000	1.000000	0.00
1	0.999848	0.999695	0.02
2	0.999391	0.998782	0.06
3	0.998630	0.997261	0.14
4	0.997564	0.995134	0.24
5	0.996195	0.992404	0.38
6	0.994522	0.989074	0.55
7	0.992546	0.985148	0.75
8	0.990268	0.980631	0.98
9	0.987688	0.975528	1.25
10	0.984808	0.969846	1.54
11	0.981627	0.963592	1.87
12	0.978148	0.956773	2.23
13	0.974370	0.949397	2.63
14	0.970296	0.941474	3.06
15	0.965926	0.933013	3.53
16	0.961262	0.924024	4.03
17	0.956305	0.914519	4.57
18	0.951057	0.904508	5.15
19	0.945519	0.894005	5.76
20	0.939693	0.883022	6.42
21	0.933580	0.871572	7.11
22	0.927184	0.859670	7.85
23	0.920505	0.847329	8.64
24	0.913545	0.834565	9.46
25	0.906308	0.821394	10.34
26	0.898794	0.807831	11.26
27	0.891007	0.793893	12.23
28	0.882948	0.779596	13.26
29	0.874620	0.764960	14.34
30	0.866025	0.750000	15.47
31	0.857167	0.734736	16.66
32	0.848048	0.719186	17.92
33	0.838671	0.703368	19.24
34	0.829038	0.687303	20.62
35	0.819152	0.671010	22.08
36	0.809017	0.654508	23.61
37	0.798636	0.637819	25.21
38	0.788011	0.620961	26.90
39	0.777146	0.603956	28.68
40	0.766044	0.586824	30.54
41	0.754710	0.569587	32.50
42	0.743145	0.552264	34.56
43	0.731354	0.534878	36.73

Table 2a: The angle of internal friction φ_n, deg. of sandy soils.

Sandy soil	Annotation of soil characteristics	Characteristic of soil with void ratio *e*			
		0,45	0,55	0,65	0,75
Gravelly and larger		43	40	38	-
Medium-grained		40	38	35	-
Fine	φ_n	38	36	32	28
Silt	φ_n	36	34	30	26

Table 2b: Normative values of the angle of internal friction φ_n, of deg. silty-clay soils.

Soil Type and its liquid limit		Annotation of soil characteristics	Characteristic of soil with void ratio e						
			0,45	0,55	0,65	0,75	0,85	0,95	1,05
Sandy loam	$0 \le I_L \le 0{,}25$	φ_n	30	29	27	24	-	-	-
	$0{,}25 < I_L \le 0{,}75$	φ_n	28	26	24	21	18	-	-
Loam	$0 < I_L \le 0{,}25$	φ_n	26	25	24	23	22	20	-
	$0{,}25 < I_L \le 0{,}5$	φ_n	24	23	22	21	19	17	-
	$0{,}5 < I_L \le 0{,}75$	φ_n	-	-	19	18	16	14	12
clay	$0 < I_L \le 0{,}25$	φ_n	-	21	20	19	18	16	14
	$0{,}25 < I_L \le 0{,}5$	φ_n	-	-	18	17	16	14	11
	$0{,}5 < I_L \le 0{,}75$	φ_n	-	-	15	14	12	10	7

After introduction of (1.10) The resolution equation becomes:

$$\lambda = \left[tg\left(45 - \frac{\varphi + \alpha}{2}\right) + tg\alpha \right]^2 \cos^2\alpha ;$$

Substituting $\psi = 45 - \dfrac{\varphi}{2}$, then: $\lambda = \left[tg\left(\psi + \dfrac{\alpha}{2}\right) + tg\alpha \right]^2 \cos^2\alpha$.

Assuming, as before, $\cos\alpha = \pm\sqrt{\dfrac{1}{1+tg^2\alpha}}$: and choosing the sign according to the physical meaning of the problem we have: $\cos\alpha = \sqrt{\dfrac{1}{1+tg^2\alpha}}$. Thus:

$$\lambda = \left[\frac{tg\psi - tg\left(\frac{\alpha}{2}\right)}{1 + g\psi \cdot tg\left(\frac{\alpha}{2}\right)} + tg\alpha \right]^2 \frac{1}{1+tg^2\alpha}$$

Considering: $(-\varphi < \alpha \le 0)$ (1.6)

Setting: $tg\dfrac{\alpha}{2} = -\sqrt{\dfrac{1-\cos\alpha}{1+\cos\alpha}}$, we have:

$$\lambda = \left[\frac{tg\psi + \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}}{1 - g\psi \cdot \sqrt{\frac{1-\cos\alpha}{1+\cos\alpha}}} - tg\alpha \right]^2 \frac{1}{1+tg^2\alpha} = \left[\frac{tg\psi + \sqrt{\dfrac{1-\sqrt{\frac{1}{1+tg^2\alpha}}}{1+\sqrt{\frac{1}{1+tg^2\alpha}}}}}{1 - g\psi \cdot \sqrt{\dfrac{1-\sqrt{\frac{1}{1+tg^2\alpha}}}{1+\sqrt{\frac{1}{1+tg^2\alpha}}}}} - tg\alpha \right]^2 \frac{1}{1+tg^2\alpha}$$

Assuming, as before, $y' = \dfrac{dy}{dz} = tg\alpha$, we get

$$\lambda=\left[\frac{tg\psi+\sqrt{\dfrac{1-\sqrt{\dfrac{1}{1+\psi'^2}}}{1+\sqrt{\dfrac{1}{1+\psi'^2}}}}}{1-\gamma\psi\cdot\sqrt{\dfrac{1-\sqrt{\dfrac{1}{1+\psi'^2}}}{1+\sqrt{\dfrac{1}{1+\psi'^2}}}}}-tg\alpha\right]^2\frac{1}{1+\psi'^2}$$

substituting $f=\sqrt{\dfrac{1}{1+\psi'^2}}$

we have: $f^2=\dfrac{1}{1+\psi'^2}$; $\dfrac{1}{f^2}=1+\psi'^2$;

$\psi'^2=\dfrac{1-f^2}{f^2}$; $\psi'=tg\alpha=\pm\sqrt{\dfrac{1-f^2}{f^2}}$;

Then: $\lambda=\left[\dfrac{tg\psi+\sqrt{\dfrac{1-f}{1+f}}}{1-\gamma\psi\cdot\sqrt{\dfrac{1-f}{1+f}}}-tg\alpha\right]^2 f^2$; From (1.5): $\lambda=\dfrac{\sigma}{\gamma(z_o+z_1)}$;

Finally:

$$\left[\frac{tg\psi+\sqrt{\dfrac{1-f}{1+f}}}{1-\gamma\psi\cdot\sqrt{\dfrac{1-f}{1+f}}}-\sqrt{\frac{1-f^2}{f^2}}\right]^2 f^2=\frac{\sigma(z)}{z\cdot\gamma};\ \text{substituting: } F(z)=\sqrt{\frac{\sigma(z)}{z\cdot\gamma}}\ \Rightarrow\ F^2(z)=\frac{\sigma(z)}{z\cdot\gamma};$$

$$\left[\frac{tg\psi+\sqrt{\dfrac{1-f}{1+f}}}{1-\gamma\psi\cdot\sqrt{\dfrac{1-f}{1+f}}}-\sqrt{\frac{1-f^2}{f^2}}\right]^2 f^2=F^2(z);\ \text{substituting: } k^2=\frac{1-f}{1+f};$$

We express $f=f(k)$:

$$1-f=(1+f)k^2;\ 1-f=k^2+f\cdot k^2;\ f\cdot k^2+f=1-k^2;\ f(k^2+1)=1-k^2;\ f=\frac{1-k^2}{1+k^2};$$

$$\frac{1-f^2}{f^2}=\frac{1}{f^2}-1=\frac{1}{\left(\dfrac{1-k^2}{1+k^2}\right)^2}-1=\frac{(1+k^2)^2}{(1-k^2)^2}-1=\frac{1+2k^2+k^4}{1-2k^2+k^4}-1=$$

$$=\frac{1+2k^2+k^4-1+2k^2-k^4}{1-2k^2+k^4}=\frac{4k^2}{(1-k^2)^2};$$

So:

$$\sqrt{\frac{1-f^2}{f^2}}=\sqrt{\frac{4k^2}{(1-k^2)^2}}=\frac{2k}{1-k^2};$$

Finally, we obtain: $\left[\dfrac{tg\psi+k}{1-\gamma\psi\cdot k}-\dfrac{2k}{1-k^2}\right]^2\left(\dfrac{1-k^2}{1+k^2}\right)^2=F^2(z)$;

Taking the square root of the left and right part of equation we have:

$$\left[\frac{tg\psi+k}{1-\gamma\psi\cdot k}-\frac{2k}{1-k^2}\right]\left(\frac{1-k^2}{1+k^2}\right)=F(z);\ \left[\frac{(tg\psi+k)(1-k^2)-2k(1-\gamma\psi\cdot k)}{(1-\gamma\psi\cdot k)(1-k^2)}\right]\left(\frac{1-k^2}{1+k^2}\right)=F(z);$$

$$\left[\frac{tg\psi-k^2tg\psi+k-k^3-2k+2k^2tg\psi}{(1-\gamma\psi\cdot k)(1-k^2)}\right]\left(\frac{1-k^2}{1+k^2}\right)=F(z);\ \left[\frac{tg\psi-k^3+k^2tg\psi-k}{(1-\gamma\psi\cdot k)(1-k^2)}\right]\left(\frac{1-k^2}{1+k^2}\right)=F(z);$$

$$\left[\frac{tg\psi\left(1+k^2\right)-k\left(1+k^2\right)}{\left(1-g\psi\cdot k\right)\left(1-k^2\right)}\right]\cdot\left(\frac{1-k^2}{1+k^2}\right)=F(z);\quad \frac{\left(1+k^2\right)\left(tg\psi-k\right)}{\left(1-g\psi\cdot k\right)\left(1-k^2\right)}\cdot\frac{\left(1-k^2\right)}{\left(1+k^2\right)}=F(z);\quad \frac{\left(tg\psi-k\right)}{\left(1-g\psi\cdot k\right)}=F(z);$$

$$tg\psi-k=F(z)\cdot\left(1-g\psi\cdot k\right);\quad tg\psi-k=F(z)-F(z)\cdot tg\psi\cdot k;\quad F(z)\cdot tg\psi\cdot k-k=F(z)-tg\psi;$$

$$k\cdot\left(F(z)\cdot tg\psi-1\right)=F(z)-tg\psi;$$

$$k=\frac{F(z)-tg\psi}{F(z)\cdot tg\psi-1};$$

$$y'=\pm\sqrt{\frac{1-f^2}{f^2}}=\pm\frac{2k}{1-k^2};$$

$$y=\pm\int\frac{2k}{1-k^2}dz;$$

Where: $k^2=\left[\frac{F(z)-tg\psi}{F(z)\cdot tg\psi-1}\right]^2;$

$$1-k^2=1-\frac{\left[F(z)-tg\psi\right]^2}{\left[F(z)\cdot tg\psi-1\right]^2}=\frac{\left[F(z)\cdot tg\psi-1\right]^2-\left[F(z)-tg\psi\right]^2}{\left[F(z)\cdot tg\psi-1\right]^2}=$$

$$=\frac{F^2(z)\cdot tg^2\psi-2F(z)\cdot tg\psi+1-F^2(z)+2F(z)\cdot tg\psi-tg^2\psi}{\left(F(z)\cdot tg\psi-1\right)^2}=$$

$$=\frac{tg^2\psi\cdot\left(F^2(z)-1\right)-\left(F^2(z)-1\right)}{\left(F(z)\cdot tg\psi-1\right)^2}=\frac{\left(F^2(z)-1\right)\left(tg^2\psi-1\right)}{\left(F(z)\cdot tg\psi-1\right)^2};$$

$$\frac{2k}{1-k^2}=\frac{2\left(F(z)-tg\psi\right)\left(F(z)\cdot tg\psi-1\right)^2}{\left(F(z)\cdot tg\psi-1\right)\left(F^2(z)-1\right)\left(tg^2\psi-1\right)};$$

Finally:

$$\frac{2k}{1-k^2}=\frac{2\left(F(z)-tg\psi\right)\left(F(z)\cdot tg\psi-1\right)}{\left(F^2(z)-1\right)\left(tg^2\psi-1\right)};$$

Given that: $F^2(z)=\frac{\sigma(z)}{z\cdot\gamma}$ (1.11)

and differentiating the left and right side of equation (1.11), we define dz:

$$2F(z)dF(z)=\frac{1}{\gamma}\cdot\frac{d\sigma(z)\cdot z-\sigma(z)\cdot dz}{z^2}=\frac{1}{\gamma}\cdot\frac{dz\left(z\cdot\frac{d\sigma(z)}{dz}-\sigma(z)\right)}{z^2};$$

$$dz=\frac{2F(z)z^2\cdot\gamma\cdot dF(z)}{\left(z\cdot\frac{d\sigma(z)}{dz}-\sigma(z)\right)};$$

$$y=\pm\int\frac{2\left(F(z)-tg\psi\right)\left(F(z)\cdot tg\psi-1\right)}{\left(F^2(z)-1\right)\left(tg^2\psi-1\right)}\frac{2F(z)z^2\cdot\gamma}{\left(z\cdot\frac{d\sigma(z)}{dz}-\sigma(z)\right)}dF(z)\qquad(1.12)$$

Consider the special case $\sigma(z)=const=\sigma$, Then $F(z)=\sqrt{\frac{\sigma}{z\cdot\gamma}}$:

$$dz=-\frac{2F(z)z^2\cdot\gamma\cdot dF(z)}{\sigma}\cdot\frac{\gamma\cdot\sigma}{\gamma\cdot\sigma}=-\frac{2F(z)z^2\cdot\gamma\cdot dF(z)}{\sigma}\cdot\frac{\gamma\cdot\sigma}{\gamma\cdot\sigma}=-\frac{2\cdot F(z)\cdot\sigma}{F^4(z)\cdot\gamma}dF(z)=-\frac{2\cdot\sigma}{F^3(z)\cdot\gamma}dF(z)$$

$y=\mp\int\frac{2\left(F(z)-tg\psi\right)\left(F(z)\cdot tg\psi-1\right)}{\left(F^2(z)-1\right)\left(tg^2\psi-1\right)}\frac{2\cdot\sigma}{F^3(z)\cdot\gamma}dF(z)$. We give: $tg^2\psi=m$, we obtain:

$$y = \mu \frac{4\cdot\sigma}{\gamma}\int\frac{(F(z)-m)(F(z)\cdot m-1)}{(F^2(z)-1)(m^2-1)\cdot F^3(z)}dF(z) = \mu\frac{4\cdot\sigma}{\gamma}\int\frac{(F(z)-m)(F(z)\cdot m-1)}{(F^2(z)-1)(m^2-1)\cdot F^3(z)}dF(z) =$$

$$= \mu\frac{4\cdot\sigma}{\gamma}\int\frac{F^2(z)\cdot m - F(z)\cdot m^2 - F(z)+m}{(F^2(z)-1)(m^2-1)\cdot F^3(z)}dF(z) = \mu\frac{4\cdot\sigma}{\gamma}\int\frac{F^2(z)\cdot m - F(z)\cdot(m^2+1)+m}{(F^2(z)-1)(m^2-1)\cdot F^3(z)}dF(z)$$

$$y = \mu\frac{4\cdot\sigma\cdot m}{\gamma(m^2-1)}\left[\int\frac{1}{(F^2(z)-1)\cdot F(z)}dF(z) - \frac{(m^2+1)}{m}\int\frac{1}{(F^2(z)-1)\cdot F^2(z)}dF(z)+\ldots\right.$$

$$\left.\ldots+\int\frac{1}{(F^2(z)-1)\cdot F^3(z)}dF(z)\right]; \quad (1.13)$$

The integrals in (1.13), taken closed. We obtain:

$$y = \mu\frac{4\cdot\sigma\cdot m}{\gamma(m^2-1)}\left[\frac{1}{2\cdot F^2(z)} - \ln\left(\frac{F^2(z)}{1-F^2(z)}\right)+\frac{(m^2+1)}{m}\left(-\frac{1}{F(z)}+\frac{1}{2}\ln\left(\frac{1+F(z)}{1-F(z)}\right)\right)\right] \quad (1.14)$$

We introduce the notation:

$$\Omega(F(z)) = \left[\frac{1}{2\cdot F^2(z)} - \ln\left(\frac{F^2(z)}{1-F^2(z)}\right)+\frac{(m^2+1)}{m}\left(-\frac{1}{F(z)}+\frac{1}{2}\ln\left(\frac{1+F(z)}{1-F(z)}\right)\right)\right]; \quad (1.15)$$

And finally:

$$y = \mu\frac{4\cdot\sigma\cdot m}{\gamma(m^2-1)}\Omega(F(z)) \quad (1.16)$$

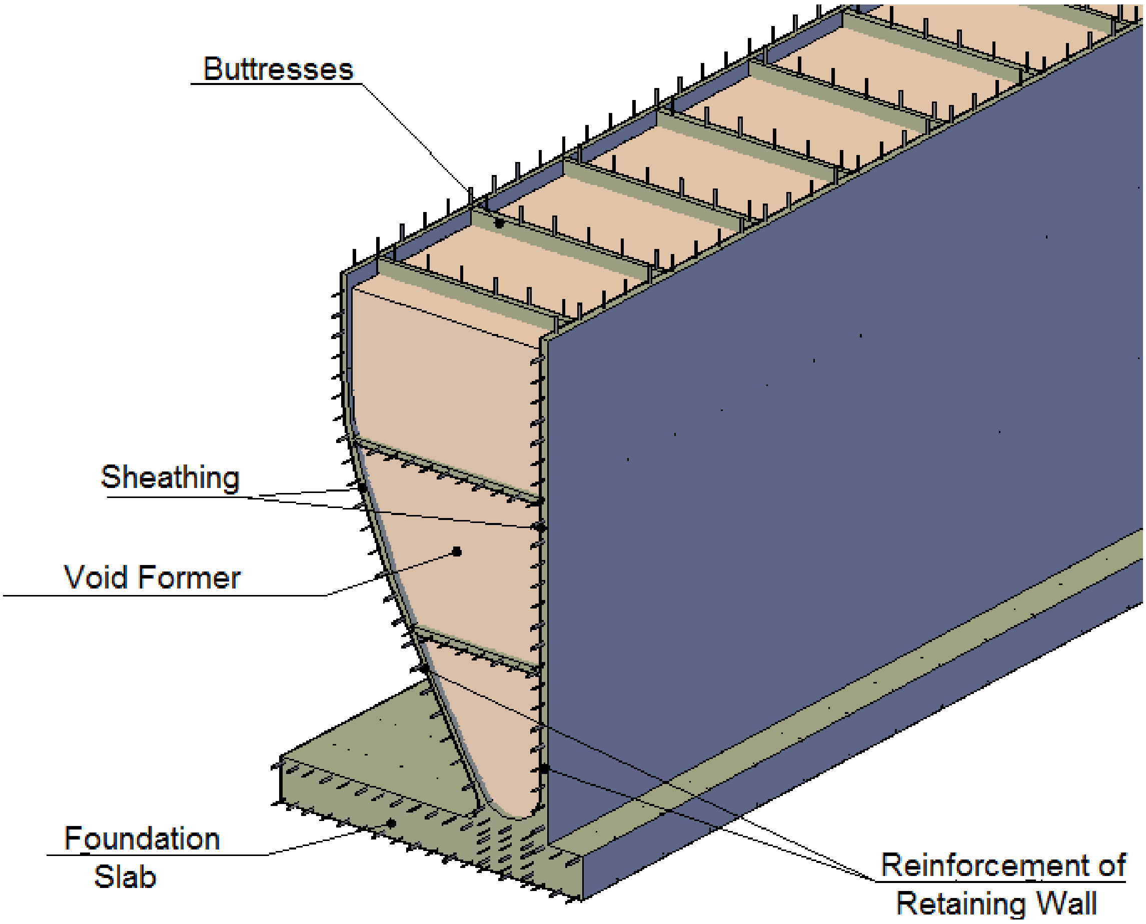


Figure 1.3. The effective retaining wall

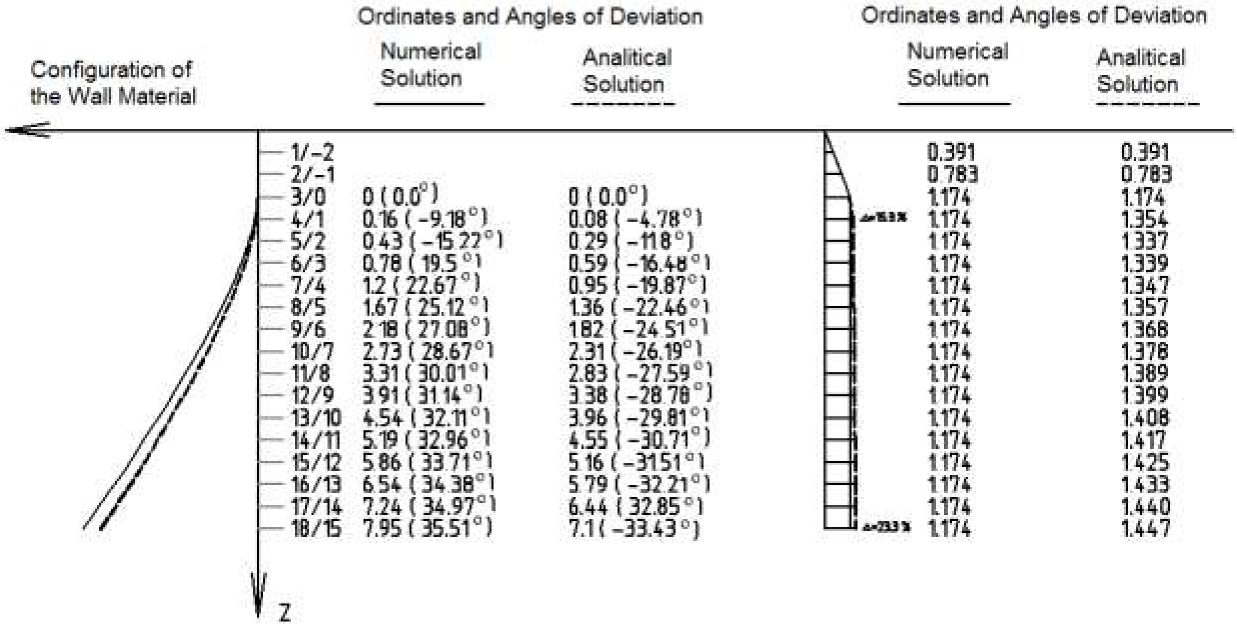
To visual demonstration of solutions we consider some examples.

Пример 1

Data:

Angle of internal friction $\varphi = 40^0$ Total depth z_{max}=18m

Iinitial depth z_0=3m ratio $\frac{z_{max}}{z_0} = 6$



	Ordinates and Angles of Deviation: Numerical Solution	Ordinates and Angles of Deviation: Analitical Solution	Ordinates and Angles of Deviation: Numerical Solution	Ordinates and Angles of Deviation: Analitical Solution
1/-2			0.391	0.391
2/-1			0.783	0.783
3/0	0 (0.0°)	0 (0.0°)	1.174	1.174
4/1	0.16 (-9.18°)	0.08 (-4.78°)	1.174	1.354
5/2	0.43 (-15.22°)	0.29 (-11.8°)	1.174	1.337
6/3	0.78 (19.5°)	0.59 (-16.48°)	1.174	1.339
7/4	1.2 (22.67°)	0.95 (-19.87°)	1.174	1.347
8/5	1.67 (25.12°)	1.36 (-22.46°)	1.174	1.357
9/6	2.18 (27.08°)	1.82 (-24.51°)	1.174	1.368
10/7	2.73 (28.67°)	2.31 (-26.19°)	1.174	1.378
11/8	3.31 (30.01°)	2.83 (-27.59°)	1.174	1.389
12/9	3.91 (31.14°)	3.38 (-28.78°)	1.174	1.399
13/10	4.54 (32.11°)	3.96 (-29.81°)	1.174	1.408
14/11	5.19 (32.96°)	4.55 (-30.71°)	1.174	1.417
15/12	5.86 (33.71°)	5.16 (-31.51°)	1.174	1.425
16/13	6.54 (34.38°)	5.79 (-32.21°)	1.174	1.433
17/14	7.24 (34.97°)	6.44 (32.85°)	1.174	1.440
18/15	7.95 (35.51°)	7.1 (-33.43°)	1.174	1.447

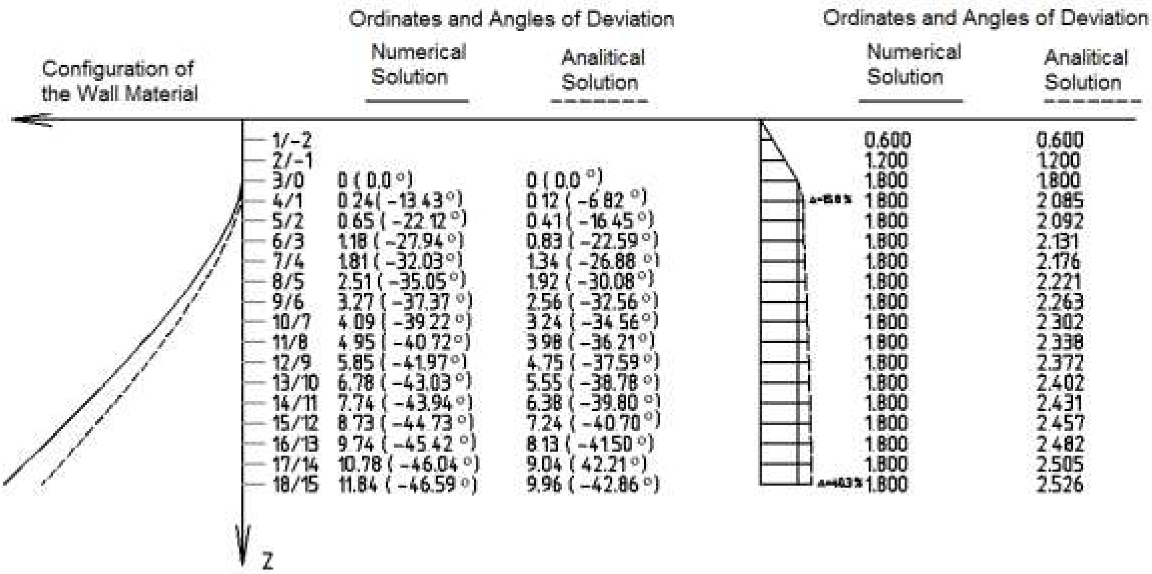
The integral error is 16.0%.

Example 2

Data:

Angle of internal friction $j = 30^0$ Total depth z_{max}=18m

Initial depth z_0=m ratio $\frac{z_{max}}{z_0} = 6$



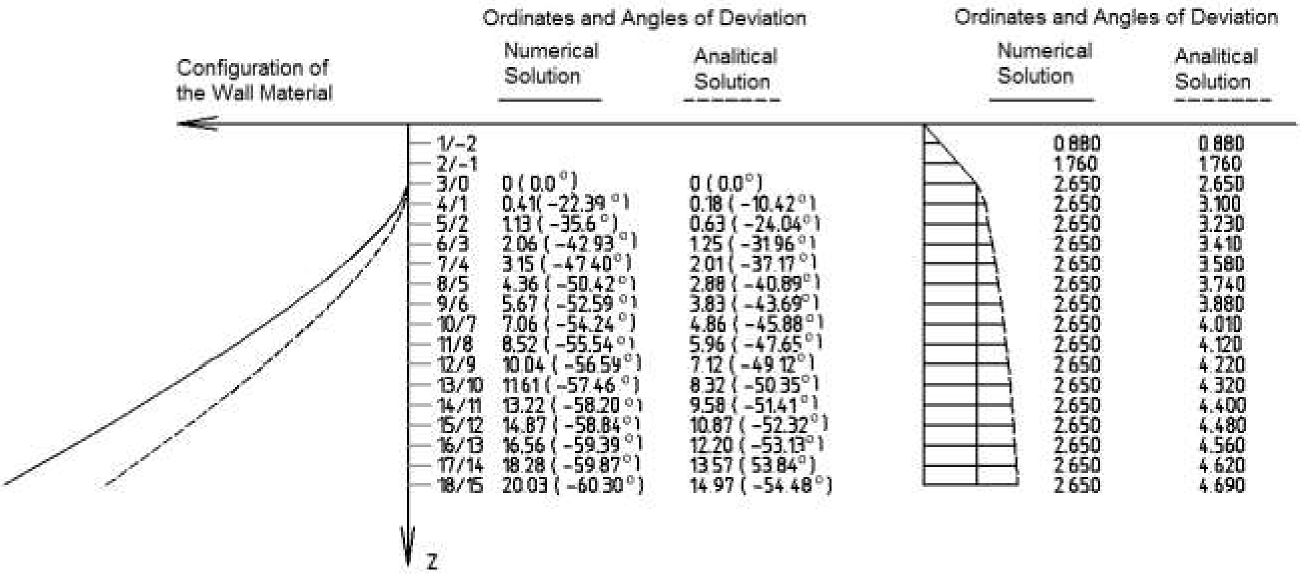
	Ordinates and Angles of Deviation: Numerical Solution	Ordinates and Angles of Deviation: Analitical Solution	Ordinates and Angles of Deviation: Numerical Solution	Ordinates and Angles of Deviation: Analitical Solution
1/-2			0.600	0.600
2/-1			1.200	1.200
3/0	0 (0.0°)	0 (0.0°)	1.800	1.800
4/1	0.24 (-13.43°)	0.12 (-6.82°)	1.800	2.085
5/2	0.65 (-22.12°)	0.41 (-16.45°)	1.800	2.092
6/3	1.18 (-27.94°)	0.83 (-22.59°)	1.800	2.131
7/4	1.81 (-32.03°)	1.34 (-26.88°)	1.800	2.176
8/5	2.51 (-35.05°)	1.92 (-30.08°)	1.800	2.221
9/6	3.27 (-37.37°)	2.56 (-32.56°)	1.800	2.263
10/7	4.09 (-39.22°)	3.24 (-34.56°)	1.800	2.302
11/8	4.95 (-40.72°)	3.98 (-36.21°)	1.800	2.338
12/9	5.85 (-41.97°)	4.75 (-37.59°)	1.800	2.372
13/10	6.78 (-43.03°)	5.55 (-38.78°)	1.800	2.402
14/11	7.74 (-43.94°)	6.38 (-39.80°)	1.800	2.431
15/12	8.73 (-44.73°)	7.24 (-40.70°)	1.800	2.457
16/13	9.74 (-45.42°)	8.13 (-41.50°)	1.800	2.482
17/14	10.78 (-46.04°)	9.04 (42.21°)	1.800	2.505
18/15	11.84 (-46.59°)	9.96 (-42.86°)	1.800	2.526

The integral error is Δ =25.0 %.

Example 3

Data: Angle of internal friction $j = 20^0$ Total depth z_{max}=18m

Initial depth z_0=3m ratio $\frac{z_{max}}{z_0} = 6$



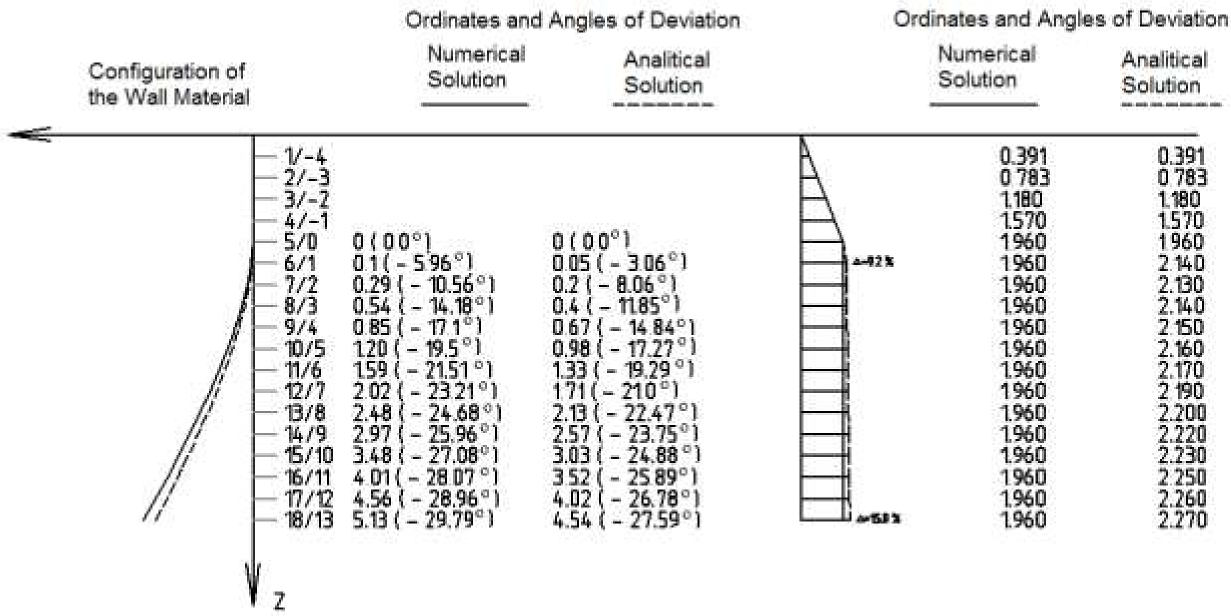
	Numerical Solution	Analitical Solution	Numerical Solution	Analitical Solution
1/-2			0.880	0.880
2/-1			1.760	1.760
3/0	0 (0.0°)	0 (0.0°)	2.650	2.650
4/1	0.41(-22.39°)	0.18 (-10.42°)	2.650	3.100
5/2	1.13 (-35.6°)	0.63 (-24.04°)	2.650	3.230
6/3	2.06 (-42.93°)	1.25 (-31.96°)	2.650	3.410
7/4	3.15 (-47.40°)	2.01 (-37.17°)	2.650	3.580
8/5	4.36 (-50.42°)	2.88 (-40.89°)	2.650	3.740
9/6	5.67 (-52.59°)	3.83 (-43.69°)	2.650	3.880
10/7	7.06 (-54.24°)	4.86 (-45.88°)	2.650	4.010
11/8	8.52 (-55.54°)	5.96 (-47.65°)	2.650	4.120
12/9	10.04 (-56.59°)	7.12 (-49.12°)	2.650	4.220
13/10	11.61 (-57.46°)	8.32 (-50.35°)	2.650	4.320
14/11	13.22 (-58.20°)	9.58 (-51.41°)	2.650	4.400
15/12	14.87 (-58.84°)	10.87 (-52.32°)	2.650	4.480
16/13	16.56 (-59.39°)	12.20 (-53.13°)	2.650	4.560
17/14	18.28 (-59.87°)	13.57 (53.84°)	2.650	4.620
18/15	20.03 (-60.30°)	14.97 (-54.48°)	2.650	4.690

integral error is $\Delta = 44.8$ %.

Example 4

Data: Angle of internal friction $\varphi = 40^0$ Total depth z_{max}=18m

Initial depth z_0=5m ratio $\frac{z_{max}}{z_0} = 3.6$



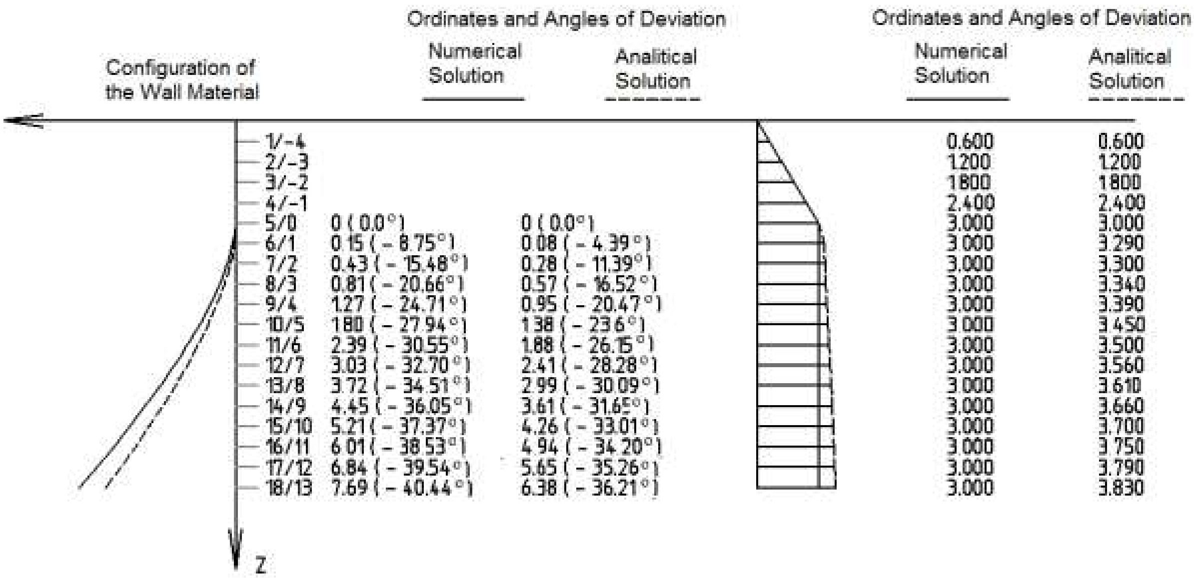
	Numerical Solution	Analitical Solution	Numerical Solution	Analitical Solution
1/-4			0.391	0.391
2/-3			0.783	0.783
3/-2			1.180	1.180
4/-1			1.570	1.570
5/0	0 (0.0°)	0 (0.0°)	1.960	1.960
6/1	0.1 (- 5.96°)	0.05 (- 3.06°)	1.960	2.140
7/2	0.29 (- 10.56°)	0.2 (- 8.06°)	1.960	2.130
8/3	0.54 (- 14.18°)	0.4 (- 11.85°)	1.960	2.140
9/4	0.85 (- 17.1°)	0.67 (- 14.84°)	1.960	2.150
10/5	1.20 (- 19.5°)	0.98 (- 17.27°)	1.960	2.160
11/6	1.59 (- 21.51°)	1.33 (- 19.29°)	1.960	2.170
12/7	2.02 (- 23.21°)	1.71 (- 21.0°)	1.960	2.190
13/8	2.48 (- 24.68°)	2.13 (- 22.47°)	1.960	2.200
14/9	2.97 (- 25.96°)	2.57 (- 23.75°)	1.960	2.220
15/10	3.48 (- 27.08°)	3.03 (- 24.88°)	1.960	2.230
16/11	4.01 (- 28.07°)	3.52 (- 25.89°)	1.960	2.250
17/12	4.56 (- 28.96°)	4.02 (- 26.78°)	1.960	2.260
18/13	5.13 (- 29.79°)	4.54 (- 27.59°)	1.960	2.270

Integral error is $\Delta = 9.5$ %.

Example 5

Data: Angle of internal friction $\varphi = 30^0$ Total depth z_{max}=18m

Initial depth z_0=5m ratio $\frac{z_{max}}{z_0} = 3.6$

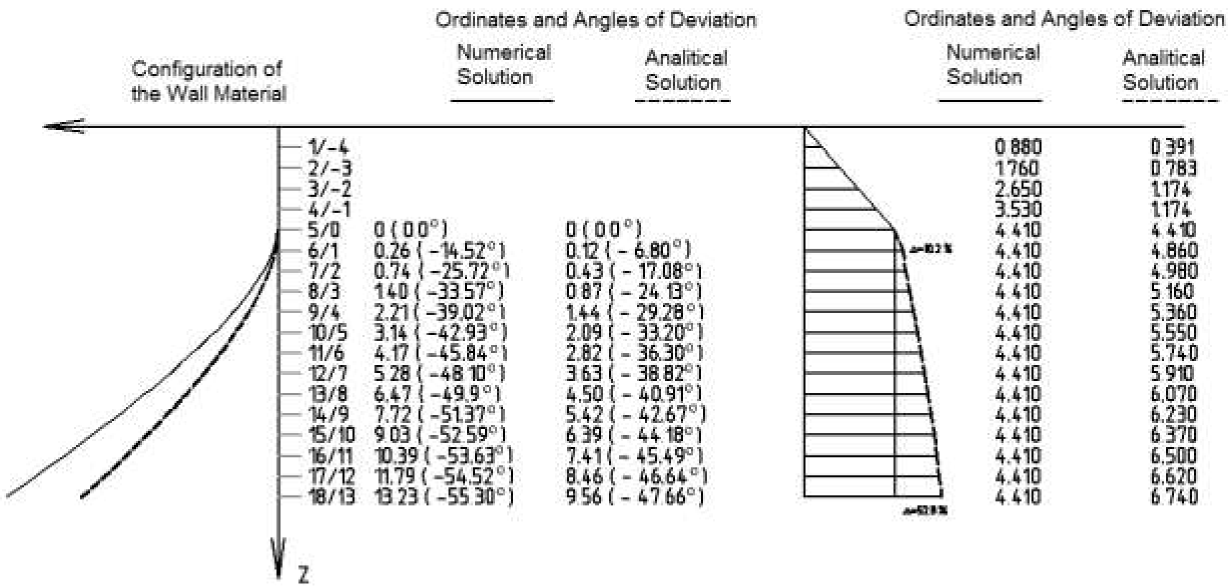


Integral error is $\Delta = 14.5$ %.

Example 6

Data: Angle of internal friction $j = 20^0$ Total depth z_{max}=18m

Initial depth z_0=5m ratio $\frac{z_{max}}{z_0} = 3.6$

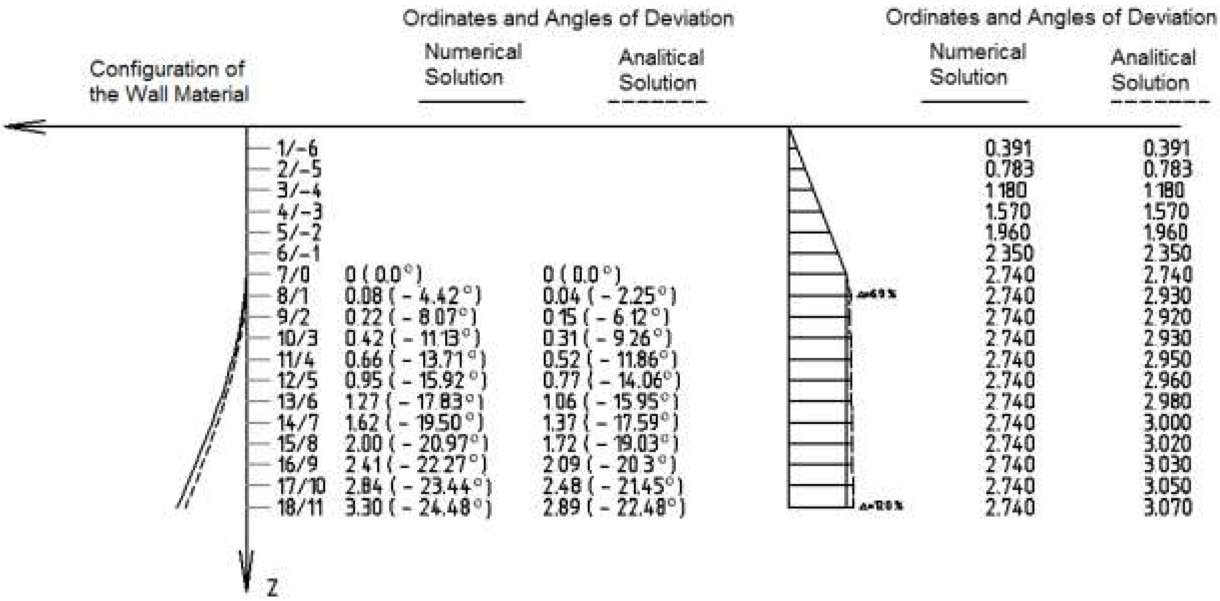


Integral error is $\Delta = 25.7$ %.

Example7

Data: Angle of internal friction $j = 40^0$ Total depth z_{max}=18m

Initial depth z_0=7m ratio $\frac{z_{max}}{z_0} = 2.57$

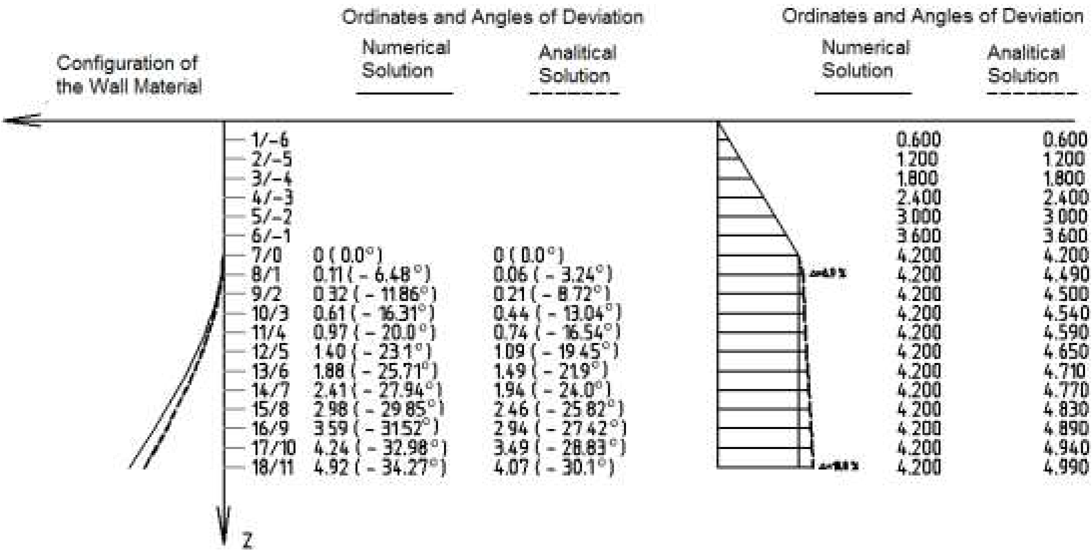


Integral error is $\Delta = 6.4$ %.

Example8

Data: Angle of internal friction $j = 30^0$ Total depth z_{max}=18m

Initial depth z_0=7m ratio $\frac{z_{max}}{z_0} = 2.57$

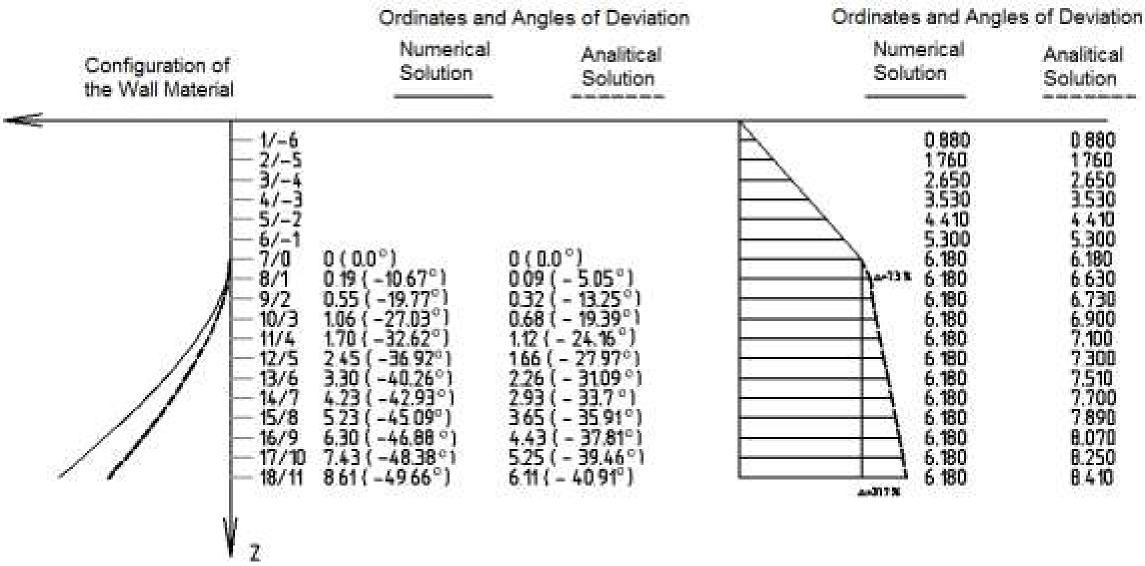


Integral error is $\Delta = 8.7$ %.

Example9

Data: Angle of internal friction $j = 20^0$ Total depth z_{max}=18m

Initial depth z_0=7m ratio $\frac{z_{max}}{z_0} = 2.57$



Integral error is $\Delta = 14.9\%$.

The error of approximation analytical solution was estimated by comparing the areas of pressure diagrams. As shown in Figure 1.3 for the majority of cases, the error does not exceed 25%.

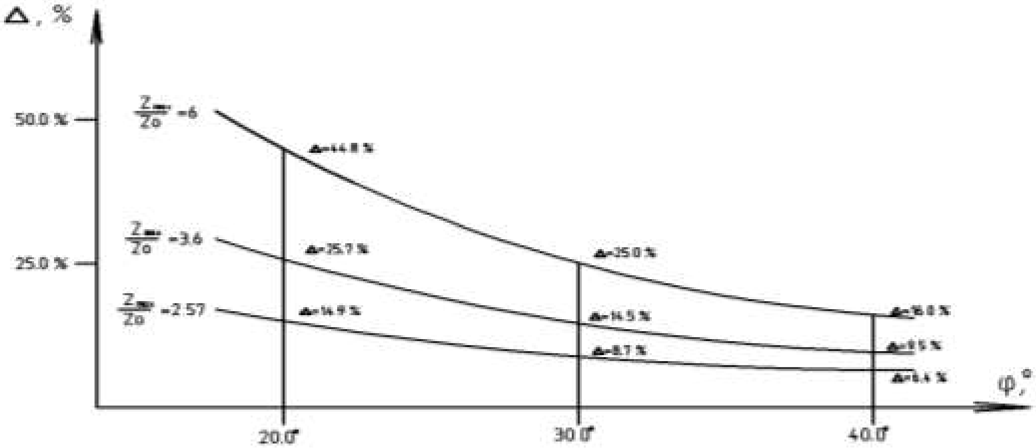


Figure1.3. Error (%) of the analytical solution depending on the angle of internal friction.

2. Formation of Energetically Equi-Strength Element of Retaining Wall.

2.1 Basic Assumptions

For simplifying, the following conditions are assumed:

Retaining wall is infinitely long caisson-type structure (Fig. 2.1). In this regard, we consider it as the plane problem;

As The I-shape element is taken as design element (Fig. 2.2), and the width of the flanges is equal to the distance between the buttresses (ribs), and the height of the cross section is equal to the total thickness of the wall (regulating parameter), the thickness of the flanges and ribs are assigned based on the technological possibility (quality requirements of vertical concreting);- During the forming of algorithm for determining the height of the section of wall element its self weight is neglected. (to reserve);

Ultimate tensile strength of reinforced concrete generated by conditional reduction value

$$R_{red}^{+} = \frac{R_{bt} + \mu R_s}{1 + \mu \cdot n} \le R_B, \qquad (2.1)$$

where $\mu = \frac{A_s}{A_B}$ - reinforcement ratio,

$A_s; A_b$ – cross sectional area of reinforcement and concrete I-beam, respectively.
$R_s; R_{bt}$ – ultimate tensile strength of reinforcement and concrete;
R_b – ultimate compressive strength of concrete (prism strength);

$$n = \frac{E_s}{E_b};$$

$E_s; E_b$ – modules of deformations of the first kind of reinforcement and concrete, respectively;

considered that the «σ-ε» diagram of concrete in tension, compression and shear are known;

Lateral (active) soil pressure $\sigma = \sigma(z)$ is represented by a trapezoid with ordinates q_1 (top), q_2 (bottom);

In the final form retaining wall is a set of composed of I-shape (box-like) elements of constant cross section with the internal cavity of variable cross section.

The sign of strain in the appointment of the ultimate strength determined on the basis of sign of parameter Lode-Nadai $-1 \le \chi_e \le +1$.

2.2. Geometrical Characteristics of Element of Wall.

Needed characteristic of cross-section, for further calculations (Fig. 2.2) are:

$$I_x = \frac{BH^3}{12} \cdot \varphi \text{ - moment of inertia;} \qquad W_x = \frac{BH^2}{6} \cdot \varphi \text{ - section modulus;} \qquad (2.2)$$

$$S_x = \frac{BH^2}{8} \cdot \eta \text{ - static moment, where: } \varphi = \alpha + 6(1-\alpha)(1-\beta)^2 \beta, \qquad (2.3)$$

$$\eta = \alpha + 4(1-\alpha)(1-\beta)\beta \qquad (2.4)$$

$$\alpha = \frac{\delta}{B}; \quad \beta = \frac{\Delta}{H}, \quad \delta, \Delta \text{ – known quantities, } \alpha \in [0,1], \quad \beta \in [0,0.5].$$

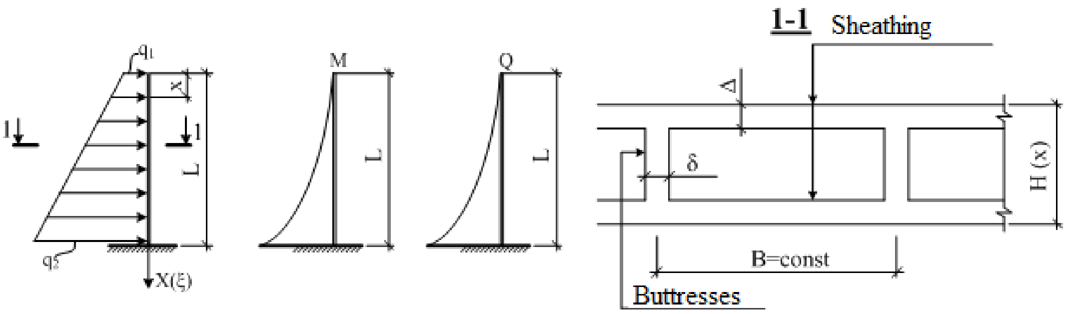


Figure 2.1. Design scheme of retaining wall

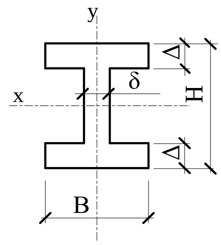


Fig.2.2. the current cross section of wall.

2.3. Internal Forces

Since, considered the plane bending, there are bending moment and shear force at sections of element (Fig. 2.1):

$$M(x) = \frac{q_2 L^2}{6} x^2 [3g + (1 - g)x], \tag{2.5}$$

$$Q(x) = \frac{q_2 L}{2} \times x [2g + (1 - g)x], \tag{2.6}$$

$g = \frac{q_1}{q_2}$, $x = \frac{x}{L}$, γ, $x \hat{I}$ $[0;1]$, L – height of wall;

M(ξ); Q(ξ) – bending moment and shear force.

2.4. Terms of Rationalization.

The height of section of wall *H(ξ)* will be searched from condition [3]:

$$\frac{1-n}{2}|s| + \frac{1+n}{2}\sqrt{s^2 + mt^2} = R^+_{red}, \tag{2.7}$$

where the parameters *ν* and *m* correspond to different criteria, and limit states are defined by the following table:

Table 2.1

№	criterion	ν	m
1	Galileo-Rankine	0	4
2	Saint-Venant	$\overline{m}$*	4
3	Coulomb	1	4
4	Mohr $D = \frac{R_{bt}}{R_b}$	Δ	4
5	Mises-Genk	1	3
*)	$\overline{m}$– Poisson's ratio		

Considering (2.2) (2.3) (2.4) (2.5) (2.6), normal and shear stresses are represented by the following dependencies:

$$s = \frac{6M(x)}{B \times j \times H^2(x)}, \tag{2.8}$$

$$t = \frac{3Q(x) \times h}{2a \times B \times j \times H(x)}, \quad \text{and} \quad B(x) = w \times H(x) = const. \tag{2.9}$$

In addition to five criteria, presented in Table 2.1, criterion introduced in [2] is considered:

$$e(x) = e_u, \tag{2.10}$$

where *e(ξ)* –- the yield value of the potential strain energy density per unit length;

$$e_u = 0{,}5c_e^2[(c_e + 1)e_{cu} - (c_e - 1)e_{tu}] + (1 - c_e^2)e_{shu}, \tag{2.11}$$

e_u – the ultimate value of the potential strai energy density per unit length,

$c_e = \frac{2e_2 - e_1 - e_3}{e_1 - e_3}$ - Lode-Nadai parameter,

$\varepsilon_1 \geq \varepsilon_2 \geq \varepsilon_3$ - *principal linear strain*

$$e_{cu} = \int_0^{\varepsilon_{uc}} \sigma_c(\varepsilon)d\varepsilon, \qquad (2.12)$$

$$e_{tu} = \int_0^{\varepsilon_{ut}} \sigma_t(\varepsilon)d\varepsilon, \qquad (2.13)$$

$$e_{shu} = \int_0^{\gamma_u} \tau(\gamma)d\gamma, \qquad (2.14)$$

$\sigma_c = \sigma_c(\varepsilon)$; $\sigma_t = \sigma_t(\varepsilon)$; $\tau = \tau(\gamma)$ - *known functions* that describe the "stress - strain" diagram for compression, tension, and shear, respectively, mainly obtained by experiment;

$\varepsilon_{uc};\varepsilon_{ut};\gamma_u$ - *The ultimate compression, tension, and shear strain of concrete, respectively.*

Equation (2.10) determines the conditional energetically equi-strength element, as it is performed only in some points of the cross section.

Substituting (2.8) and (2.9) into (2.7), and after some transformations we obtain:

$$H^4(x) - a_2H^2(x) - a_3 = 0, \qquad (2.15)$$

where the coefficients $a_i(i=2, 3)$ are given by:

$$a_2 = \frac{6(1-\nu)M(x)}{B\cdot\varphi\cdot R_{red}} + \frac{9(1+\nu)^2 m\cdot h^2 Q^2(x)}{16B^2\cdot\varphi^2\cdot\alpha^2\cdot R_{red}^{+2}}, \qquad (2.16)$$

$$a_3 = \frac{36M^2(x)\nu}{B^2R_{red}^{+2}\varphi^2}, \qquad (2.17)$$

Ignoring the negative values of H(ξ), as inconsistent with the physical meaning of the problem, in view of (1.16, 1.17), we obtain

$$H(x) = \sqrt{\frac{a_2}{2} + \sqrt{\frac{a_2^2}{4} + a_3}} \qquad (2.18)$$

In turn, for criterion (3.10), current height of the cross section of the wall is formed on the basis of the iterative procedure [2]:

$$H_{ij}(x) = H_{i(j-1)}(e_t)\left(\frac{e_i}{e_u}\right)^P, \qquad (2.19)$$

where j – number of iteration,

i – number of section,

$P \in [0;1]$ - the parameter which describes rate of convergence of the iterative process.

Refinement of the heights of section will be continued until performance of limitations:

$$\left|H_{ij} - H_{i(j-1)}\right| \leq \delta, \qquad (2.20)$$

where δ – given accuracy.

Determination of components of the stress-strain state (SSS) is performed by using Program Complex (PC), "LIRA" (Gorodtsly et al., 2003). Gorodtsly et al

The numerical solution is illustrated by the graphs shown in Figures 2.3-2.8.

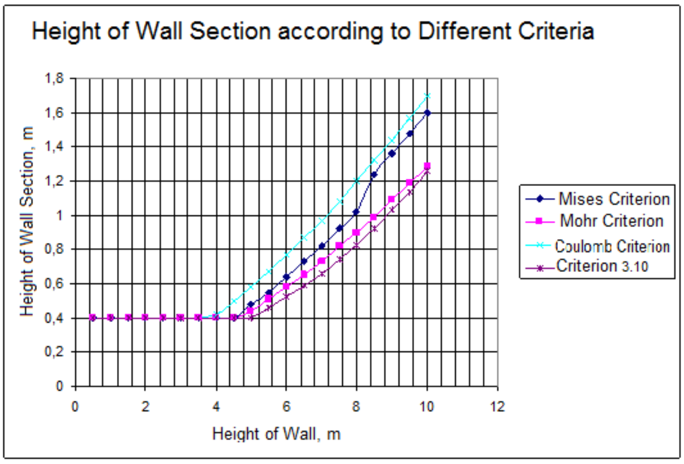


Fig.2.3 Dependence "height of wall section - height of wall," determined by various criteria.

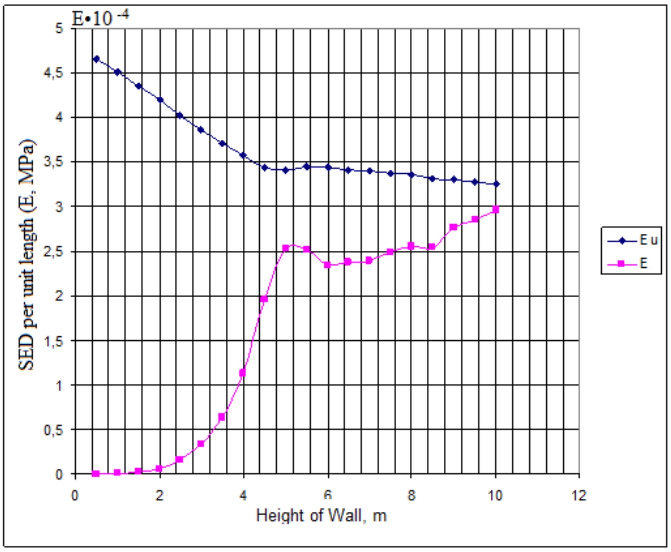


Fig.2.5. Dependence of the "SED per unit length - height of wall," as defined by Mohr: e_u- ultimate SED at a point, *e*- actual SED at the same point.

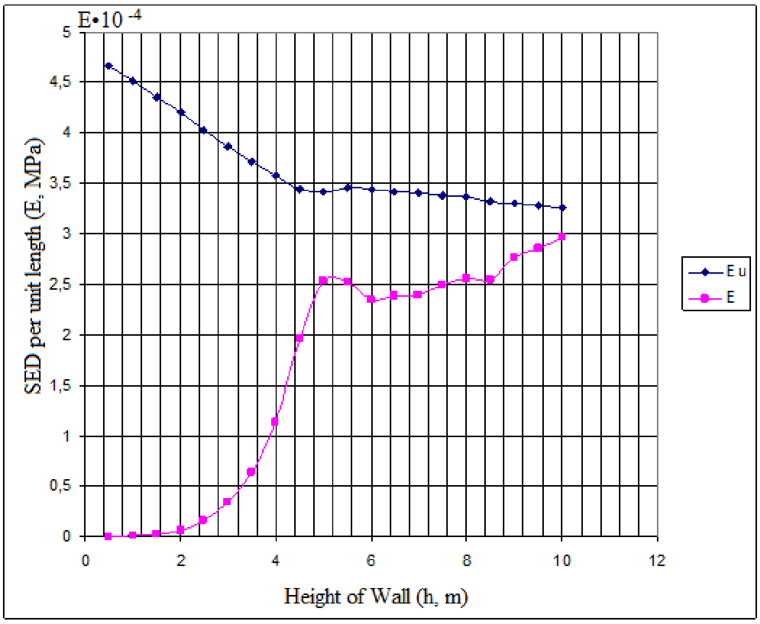


Fig.2.6. Dependence of the "SED per unit length - height of wall," as defined by energy criterion: e_u- ultimate SED at a point, *e*- actual SED at same point.

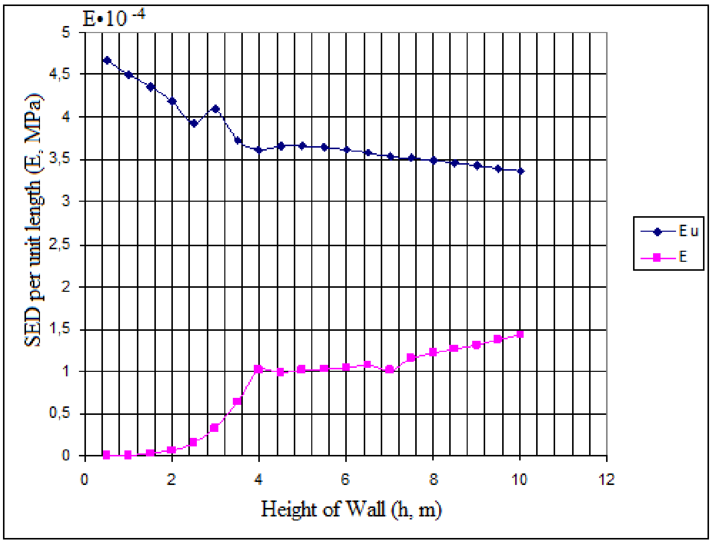


Fig.2.7. Dependence of the "SED per unit length - height of the wall," as defined by Coulomb's law: e_u- ultimate SED at a point, *e*- actual SED at same point.

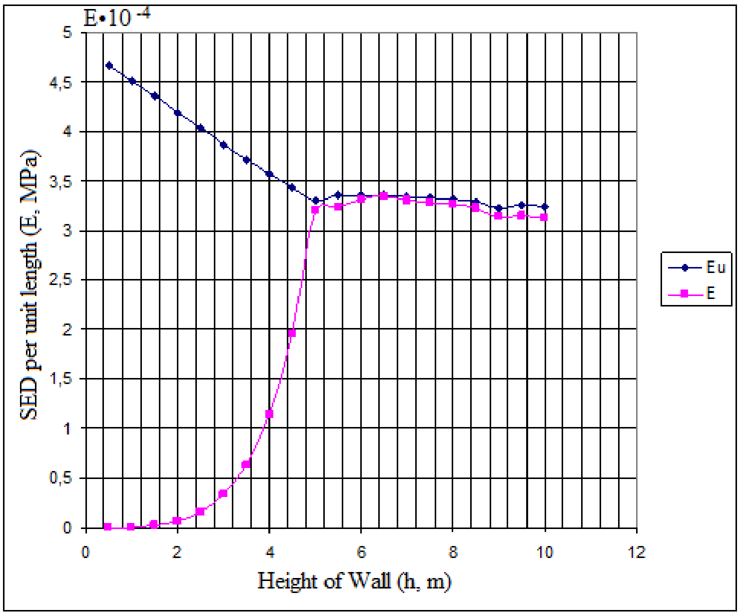


Fig.2.8. Dependence of the "SED per unit length - height of wall," as defined by (3.10): e_u- ultimate SED at a point, *e*- actual SED at same point.

Analysis of the achieved results allows the following conclusions:
Criteria (2.7) approximately define the same height of wall section (the difference does not exceed 23.5%). The SED is distributed along element ununiformly;

- The criterion (2.10) defines an energetically equi-strength element, but due to this, material saving is about 20% in relation to the criteria (2.7);
- Criteria Galileo-Rankine and Saint-Venant determine the height of wall section does not performance conditions of $e_u > e$, which is unacceptable.

Thus, isoenergetic SSS of structure causes the most acceptable distribution of material and it's effective service in structure.

3. Features of the direct design anchor retaining wall.

We will consider the anchor retaining wall (Fig. 3.1). Leaving unchanged its earlier hypothesis and the composition of the internal parameters (Fig. 2.2), we introduce a new external parameter, ie, the force in pre-tensioning (prestressed) anchor. In the case of inclined anchor the vertical loading of wall is neglected. Tensile force in which is equal to $P_{opt}^{tot} = P_{opt} / \cos \overline{\overline{b}}$

where $\overline{\overline{b}}$ - angle between anchor and horizontal axis.

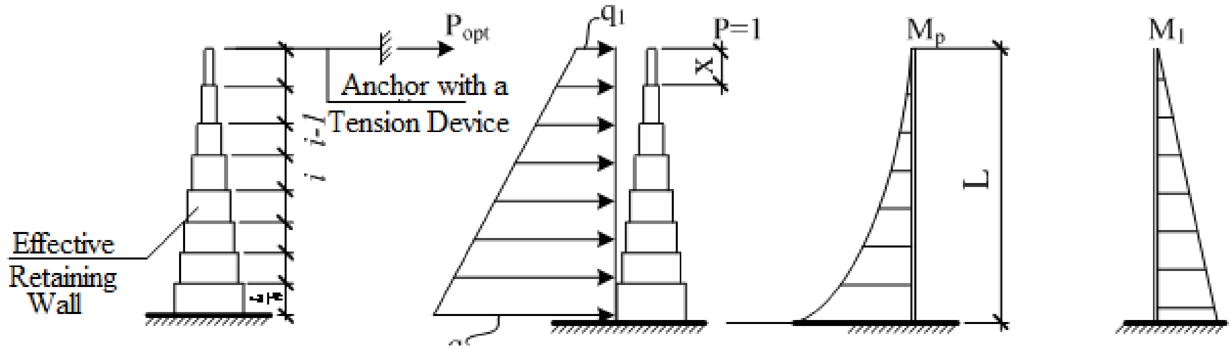


Figure 3.1. Design scheme of anchor retaining wall

Solution of problem tends to construction of energetically equi-strength wall with additional support in form of horizontal prestressed anchor. Technologically, making the advanced anchoring does not cause major difficulties Based on the ideology of formation of rational structures, presented in (Shmukler and Klimov, 2008, Vasilkov 2008)) we assume, that the external parameter, approximately, can be determined from the condition:

$$U = \inf U(a^n) \qquad n = 1,2,..., \tag{3.1}$$

where n - number of variants of comparison,

$a \in M$, M – *set of permissible values* of the pretensioning force of anchor,

U – potential strain energy (PSE).

At the same time, we introduce the assumption of unimodality of the function U.

Given, that we considered strain of plane bending for the PSE, we have:

$$U = \frac{1}{2}\int \frac{M^2(x)dx}{EI} \tag{3.2}$$

Since, rationalized wall is an element of variable cross section, equation (3.2) takes the form:

$$U = \frac{1}{2}\sum_{i=1}^{N}\int_{\frac{(i-1)L}{N}}^{\frac{iL}{N}} \frac{M^2(x)dx}{(EI)_i}, \tag{3.3}$$

where N – number of segments (sectors) of wall along it's height,

i – текущий номер участка,

$\frac{L}{N}$ - length of segment (uniform partition), L – height of wall,

$[EI]_i$ – bending stiffness of the i-th segment, $M(x)$ – bending moment.

In this case,

$$M(x) = M_g(x) + P_{opt} \cdot x, \tag{3.4}$$

where $M_g(x) = -ax^3 - bx^2$, $\quad a = \frac{q_2 - q_1}{6L}$; $\quad b = \frac{q_1}{2}$;

P_{opt} – rational value of force in anchor.

Substituting (3.4) into (3.3), after integration we obtain:

$$U = \frac{L^7}{2N^7}\sum_{i=1}^{N}\frac{1}{(EI)_i}\left\{\frac{a^2}{7}\left[i^7 - (i-1)^7\right] + \frac{abN}{3L}\left[i^6 - (i-1)^6\right] + \right.$$
$$+\frac{N^2(b^2 - 2aP_{opt})}{5L^2}\left[i^5 - (i-1)^5\right] - \frac{bP_{opt}N^3}{2L^3}\left[i^4 - (i-1)^4\right] + \tag{3.5}$$
$$\left. + \frac{P_{opt}^2 N^4}{3L^4}\left[i^3 - (i-1)^3\right]\right\}$$

we find force P_{opt} from the condition $\frac{\partial U}{\partial P_{opt}} = O$, and then differentiating (2.5). by P_{opt} to define:

$$P_{opt} = \frac{3L^4 \sum_{i=1}^{N} \frac{1}{(EI)_i} \left\{ \frac{2aN^2}{5L^2}\left[i^5 - (i-1)^5\right] + \frac{bN^3}{2L^3}\left[i^4 - (i-1)^4\right] \right\}}{2N^4 \sum_{i=1}^{N} \frac{1}{(EI)_i}\left[i^3 - (i-1)^3\right]} \qquad (3.6)$$

In the particular case $q_1=q_2=q$, a $N=1$. $P_{opt} = 0{,}375qL$, (3.7)

which coincides with the result, obtained in [5]. Since the system is statically indeterminate to the first degree

$$P_{opt} = P_f + P_{ps}, \qquad (3.8)$$

Where P_f – selftensile force, P_{ps} – pre-tensioning force.

Hence, the required value of pre-tensioning force of anchor is equal to $P_{ps} = P_{opt} - P_f$, (3.9)

selftensile force defined by force method

$$P_f = -\frac{\Delta_{1p}}{\delta_{11}}, \qquad (3.10)$$

$$\delta_{11} = \frac{L^3}{3N^3} \sum_{i=1}^{N} \frac{1}{(EI)_i}(3i^2 - 3i + 1)$$

$$\Delta_{1p} = -\frac{L^5}{N^5} \sum_{i=1}^{N} \frac{1}{(EI)i} \left\{ \frac{a}{5}\left[i^5 - (i-1)^5\right] + \frac{Nb}{4L}\left[i^4 - (i-1)^4\right] \right\}.$$

The primary structure of force method and moment diagrams are shown in Figure 3.1. Finally

$$P_f = \frac{3L^2}{N^2} \times \frac{\sum_{i=1}^{N} \frac{1}{(EI)_i} \left\{ \frac{a}{5}\left[i^5 - (i-1)^5\right] + \frac{bN}{4L}\left[i^4 - (i-1)^4\right] \right\}}{\sum_{i=1}^{N} \frac{1}{(EI)_i}\left(3i^2 - 3i + 1\right)} \qquad (3.11)$$

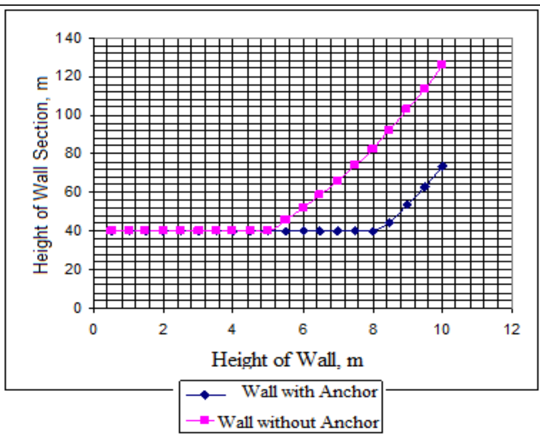
Comparing the expressions (3.6) and (3.11) can be noted that $P_{opt} = P_f$, (3.12)

and as result $P_{ps} = 0$;

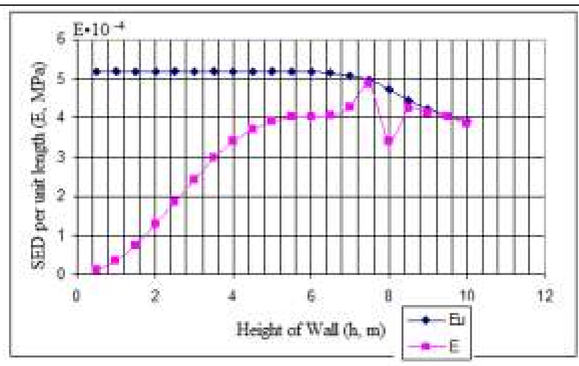
This result is very interesting and shows that in the case of condition (3.1) pre-tension of anchor is not required.
The general solution is an iterative procedure consisting of two cycles. The external cycle implements a consistent change in the force of pre-tensioning of anchor until performance of condition:

$$P_{ps}^{m} - P_{ps}^{m-1} \le \varepsilon, \qquad (3.13)$$

where m - number of external iteration, ε - given accuracy.
As initial approximation, is taken distribution of heights of wall sections, which found for cantilever system by (1.19) (1.20). Further, the internal iteration cycle is executed, generated by (1.10) (1.19) (1.20). The analysis of system was done by using PC "LIRA". The results of formation of geometry of wall by (1.10) are shown in Fig. 3.2



3.2. Dependence "height wall of section - height of wall," as defined by (3.10) in the wall with anchor: *eu* - ultimate SED, *e* - actual SED.



3.3. Dependence "SED per unit length - height of wall," as defined by (3.10) in the wall with anchor: e_u - ultimate SED, e - actual SED.

As seen from the graphs, the anchor reduces the height of section of equally strength wall by 41.5%, and changes, the qualitative nature of its height.

4. Numerical Verification and Analysis of SSS of the Proposed Structure.

Investigation of SSS of the proposed structure of retaining wall was realized using finite element (FE) modeling. Actually the calculations were carried out in LIRA environment, version 9.6R8.

Characteristics of the model are as follows:

- Type of a FE - zero Gaussian curvature shell element;
- size of FE - 25 × 25 cm;

load is uniform distributed, at depth $z_o=3m$, is equal to $11.74\ kN/m^2$ and is applied to the back surface of the wall;

- design scheme of cross-section of wall shown in Figure 4.1.

The results of analysis are illustrated by fields of displacements and internal forces (Fig. 4.2-4.6)

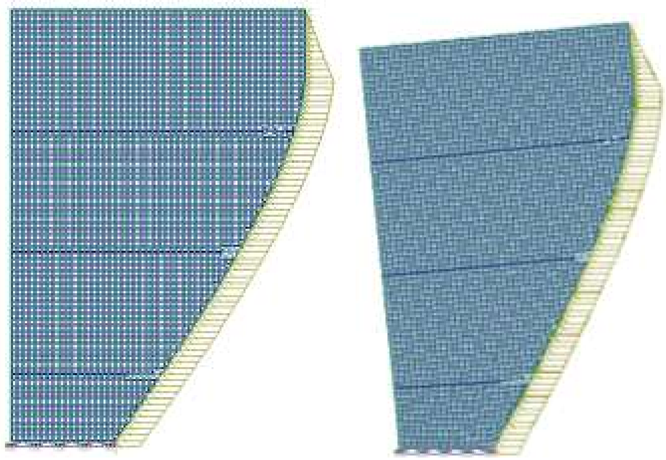


Fig. 4.2. Deformed scheme of buttress, combined with transformed pressure diagram.

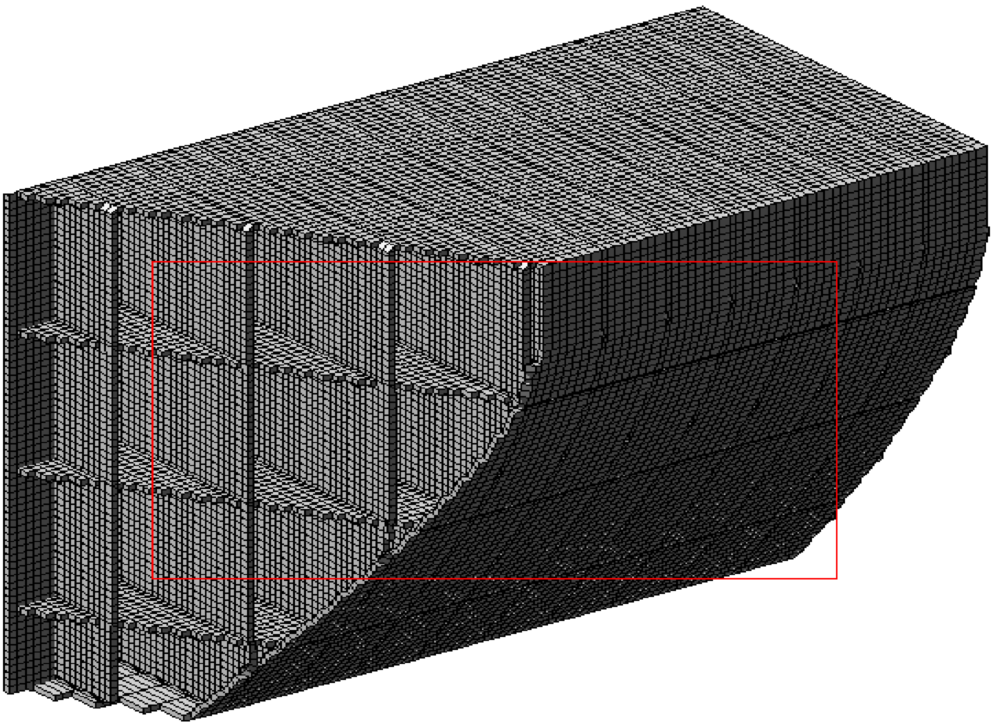


Fig. 4.1. Finite-element model of retaining wall

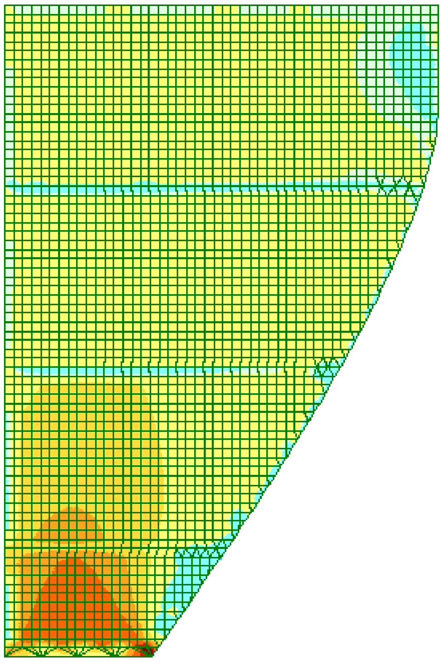
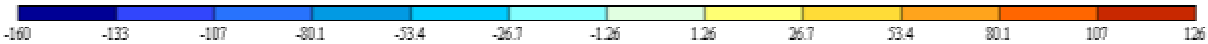


Fig. 4.2. Distribution of shear stresses in walls-diaphragms.



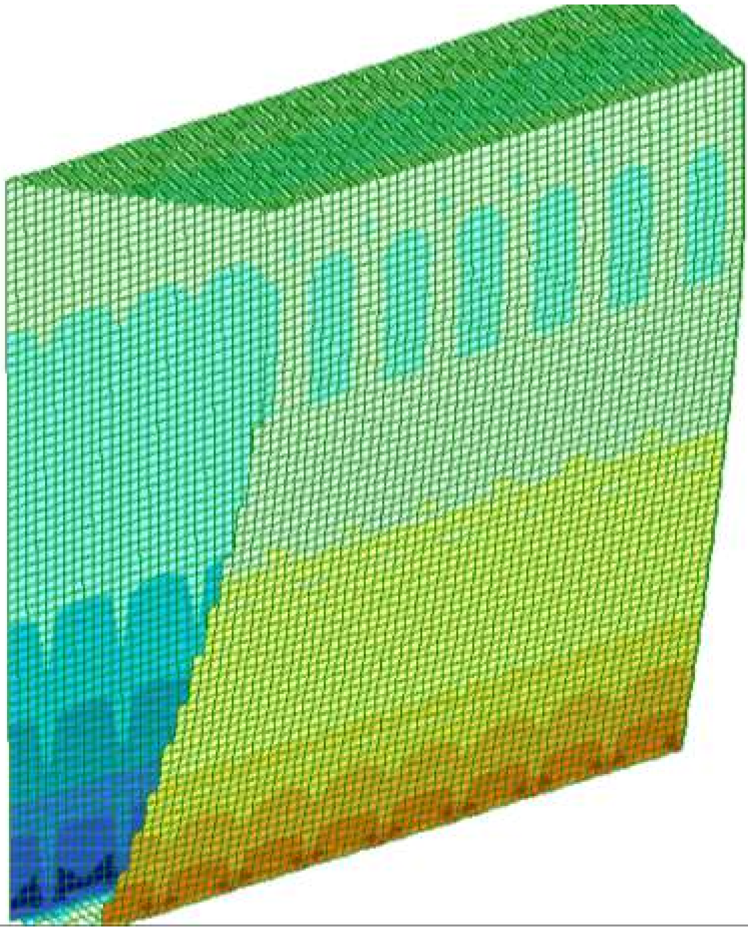


Fig.4.3. Distribution of Nx

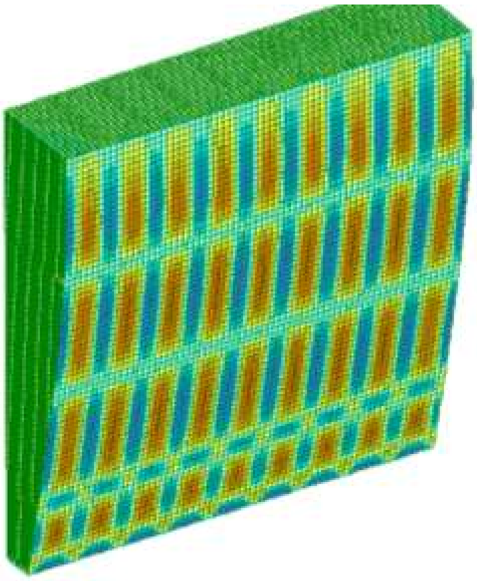


Fig.4.4. Distribution of moments My

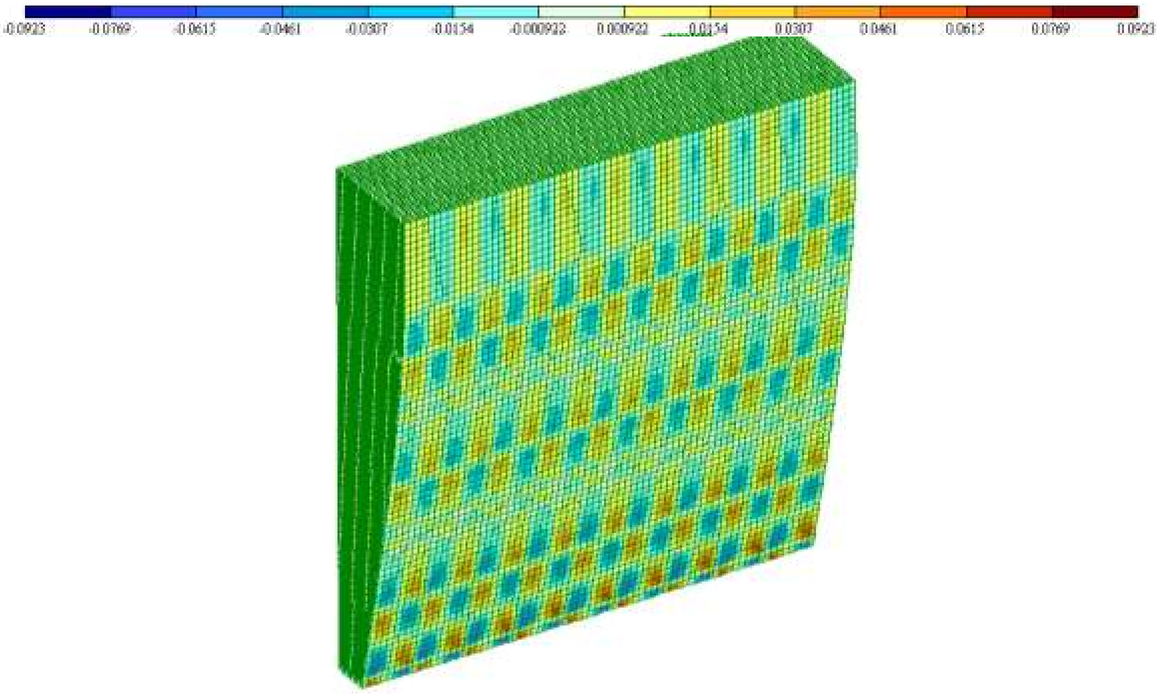


Fig.4.5. Distribution of torque Mxy

Analysis of displacement and distribution of internal forces (Fig.4.2-4.6) allows confirming representativeness of assumed hypotheses and assumptions, and appropriateness of performance management of characteristics of system, realizing the given distribution of components of SSS in the considered structure.

5. Technological Features of the Construction of Considered structures.

In this paper we consider a way to reduce weight (material consumption) of retaining walls, by means of creation of internal voids, which are performed using the removable void formers (RVF) or expendable void formers (EVF). Void formers can be made of plywood, chipboard, plastic, foam, lightweight concrete, and etc.
The main advantage of RVF is their re-use, and disadvantage - the possibility of the collapse of walls of freshly molded structure in low strength stage of concrete, or the complexity of removing. For the case of EVF, on the contrary, the disadvantages are their one-time use, and advantage - the absence of extraction operation.
It should be noted, that with the increment of degree of rationality of structures (within the accepted criteria), the use of RVF from hard materials (plywood, metal, etc.) is practically, impossible by reason of complexity of disassembly.

If there is thermal insulation requirement for retaining wall, while the effectiveness of EVF can be increased, which made of materials with low thermal conductivity, such as polystyrene.
In addition, for compaction of concrete mix should be used vibroformwork or self compacted concrete with high workobility.
Testing of technology solutions implementing the ideas of RVF and EVF was carried out during the construction of retaining walls of recreation hotel complex in Kharkiv.
The management of construction of retaining wall was carried out on flow diagram in which the process of construction was divided into streams (reinforcing, void formers and formwork instalation, concreting, formwork dismantling). For the increasing concreting process, wall was subdivided into the segments about 50m^3 per shift (8 hours) and, respectively, the minimum wall area=50x0.52=26m^2 (0.52- used concrete (m^3) in 1m^2 of wall). If height of story was 3.4m, the length of the segment was 26/3.4≈7-8 meters. The individual steps of the construction of an effective retaining wall of height about 15m are shown in Fig. 5.1. Worth noting that, an office center (eight-story building) is located at a distance of four meters from this retaining wall. Patterns of retaining wall construction (on floor, work carried out in one shift) with RVF and non-EVF are shown in Fig. 5.2 and 5.3, respectively.

Fig. 5.1. Stages of construction of effective retaining wall.

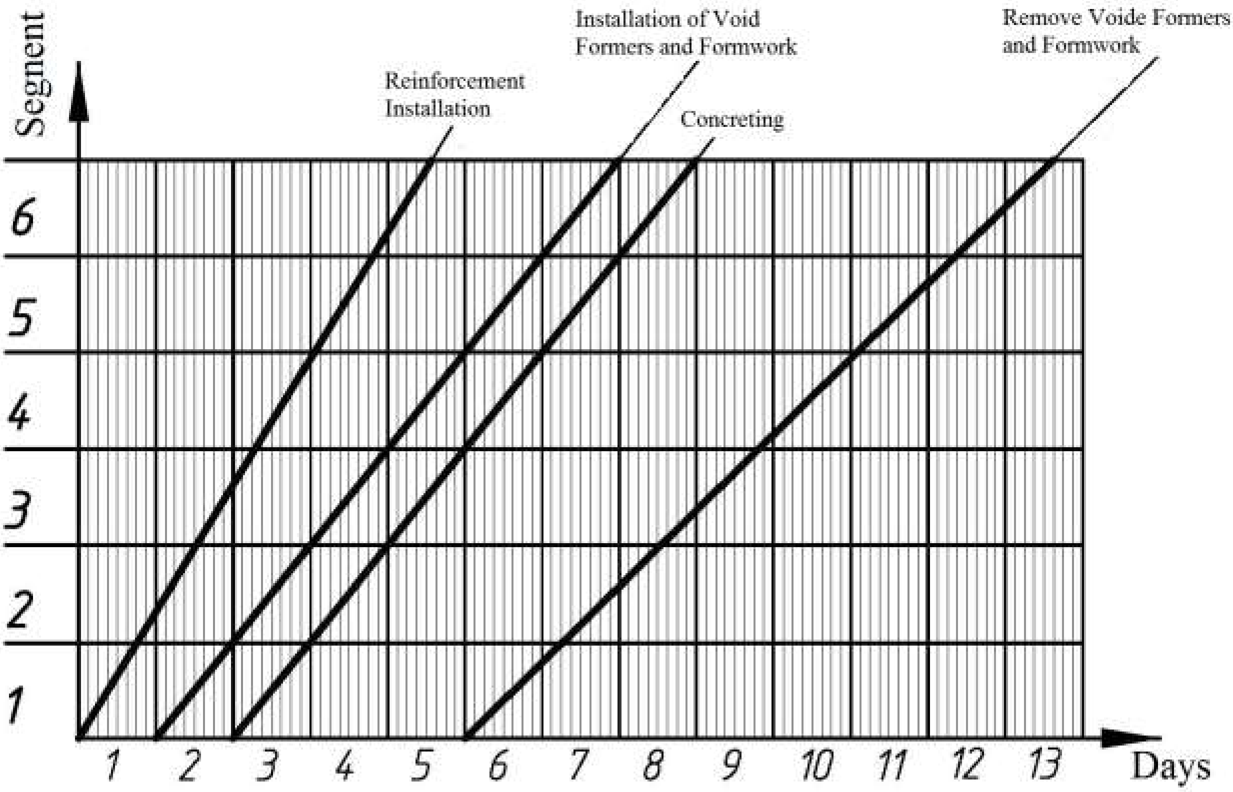


Fig. 5.2. Cyclogram for wall with RVF

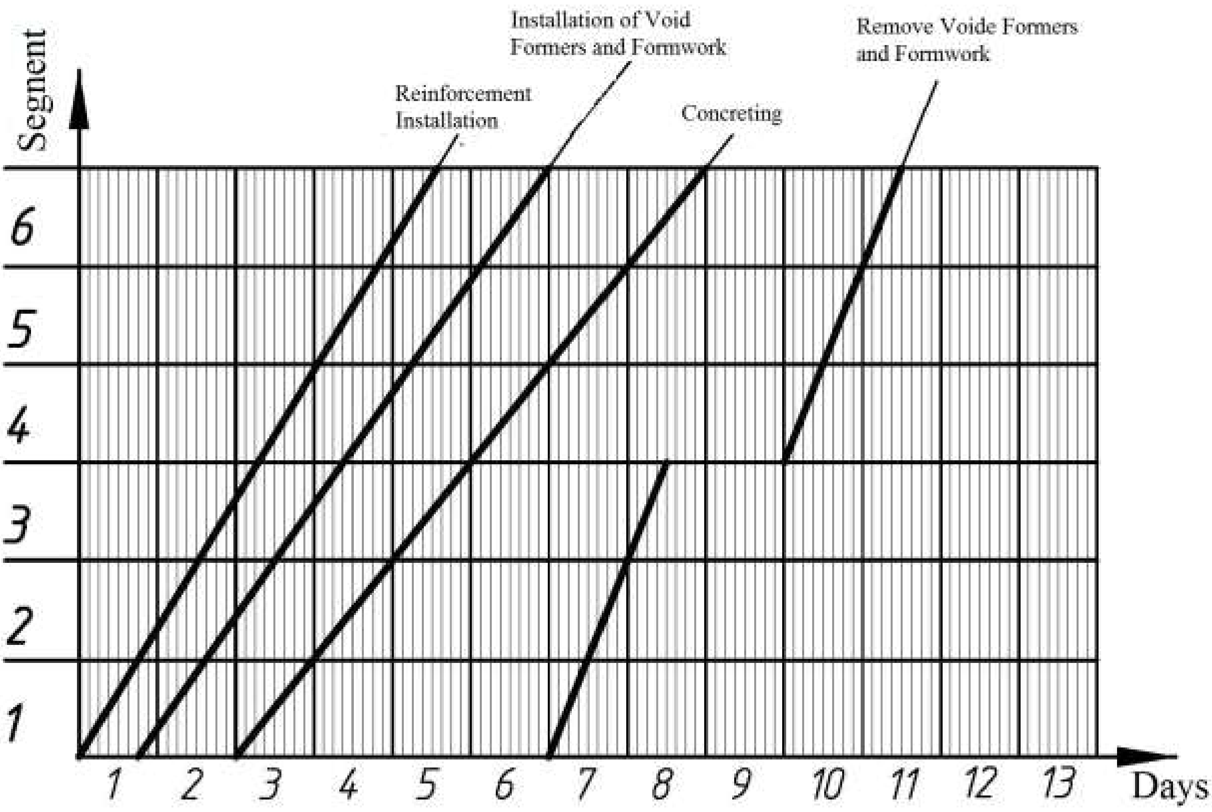


Fig. 5.3. Cyclogram for wall with EVF

Indicators of construction of retaining walls with RVF and EVF are shown in Table. 5.1.

Table 5.1 The laboriousness of the construction of retaining walls in the man-hours per 1m^2 of wall

work	RVF from inventory formwork	Styrofoam EVF
Reinforcement Installation	0,95	
Void Former Installation	0,75	0,34
Formwork Installation	0,5	
Concreting	2,3	
Void Former Removing	0,5	–
Dismantling of formwork	0,32	
Total	5,32	4,41

Approximate cost of construction of retaining walls are shown in Table 5.2. To compare the cost characteristics of technology of RVF and EVF we assumed that the cost of concrete delivery=106UAH/m^3, reinforcement=1000UAH/t, polystyrene =400UAH/m^3, 1 man-hour = 12UAH, crane rent=63UAH/hr, formwork rentals 0.5=UAH/m^{2}day.

Table 5.2 Approximate cost of construction of retaining walls UAH per 1m^2 of wall (not including consignment expenses and value-added tax)

Work	RVF from inventory formwork	Styrofoam EVF
Concret	0,52x106=55	
Reinforcement	0,049x1000=49	
Styrofoam	–	0,6x40=24
Construction of wall according to table 5.1	5,32x12=64	4,41x12=53
Crane Rental	0,16x63=10	
Formwork Rental	4,9x5x0,5=12	2x5x0,5=5
Total	190	196

Analysis of data in Table 5.2 shows that walls with EVF are costly compared with wall with RVF about $\frac{194-190}{190}\times 100\% = 2.1\%$, however, the laboriousness of construction of second wall exceeds the laboriousness of construction of first. Just heat and sound insulating characteristics of the first several times higher than second, due to the presence of polystyrene void formers. The economic feasibility of EVF increases with decrease in volume of internal voids, and consequently, the consumption of polystyrene.

Thus, for optimal (complex geometry) structures of retaining walls, for the vast majority of cases it is advisable to use EVF made from effective materials such as styrofoam.

CONCLUSION

Consideration of strain of "retaining wall - soil" together, increases the correctness of the models, by providing the features of resistance this biagregata. Representation of similar structure (retaining wall) in the form of finite element determines the possibility of direct and indirect problems of design. In turn, the dirct (optimization) approach allows us to create a structure with rational and high competitive characteristics.

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