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Investigating Mathematics Teachers' Content Knowledge, attitudes and beliefs at Secondary Level

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Abstract

This paper explores the content knowledge of mathematics teachers at the secondary level, their attitudes and beliefs towards mathematics instruction in Khyber Pakhtunkhwa. Researchers used a survey-type quantitative research design to execute this project. The population for this study was comprised of 692 High and Higher Secondary schools of the public sector of both the genders and 1427 mathematics teachers from seven selected districts. The sample was drawn from the population by taking 143 schools (male & female) and 355 mathematics teachers randomly applying the proportional allocation technique. A questionnaire consisting of 80items on content knowledge, attitudes, and beliefs was developed in light of the objectives of the study. The questionnaire was pilot tested to judge the reliability. The value of Cronbach's alpha was calculated to be 0.81. Experts of items developers were involved to judge the validity of the tool. Data was collected and analyzed using Chi-square (χ2) test. Findings explored that response of mathematics teachers was significant regarding their content knowledge, attitudes, and beliefs. But in some concepts of mathematics, they were not competent. Conclusions reveal that in some domains of mathematics, teachers are not competent. It is recommended that continuous refresher courses might be arranged for their capacity building.

Keywords: Content knowledge, Secondary Level, Mathematics teachers, Attitudes, Beliefs

Introduction

It has always been of great interest about the knowledge that teachers should or supposed to possess for mathematics instruction. There is a great difference of opinion among the people about the degree of that knowledge but the majority of them is agreed that teachers must know about what they are going to teach or something more else (McAuliffe, 2013). The rationale for investigating teachers' knowledge has two basics. Firstly for eligibility for the job, it is essential to assess the teacher's knowing what he is supposed to teach in the classroom. Secondly, to investigate the interest in the degree of the knowledge and type of knowledge required for teaching (Pournara, Hodgen, Adler, & Pillay, 2015).

Many research studies have been executed on exploring teachers' subject-matter knowledge for the last two centuries. The objective of these analyses has been for assessing teachers' content knowledge or their performance in the classroom (Sapire, Shalem, Wilson-Thompson &Paulsen, 2016). These research studies were related to content knowledge and its relationship with teachers' professional development and performance. Teacher's content knowledge has a closer relationship with students' achievement and outcomes. To assess the content knowledge of teachers, tests based on subject-matter are also used (Pournara, 2016). Studies aimed to investigate the correlation between the subject & pedagogical knowledge of mathematics teachers and their effect on students' achievement revealed that subject-matter knowledge was more effective on students' achievement rather than pedagogical knowledge (Ndlovu & Brijlall, 2015).

As teachers' mathematical knowledge is concerned, research scholars opine that teachers' knowledge of mathematics gained during their students' life is not enough for effective instruction of mathematics (Livy, Vale & Herbert, 2016). Teachers' mathematical content knowledge has been focused in previous studies under two headings, first the characteristics of their completed courses and

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second the nature of their subject-matter knowledge. Teachers' pedagogy to achieve and implement the curriculum relies on their competency in mathematical knowledge plus attitudes and beliefs (Siyepu, 2013). Teachers' mathematical knowledge depends upon their pedagogy of presenting it. Research has explored that mathematical knowledge plays an important role in examining students' performance and presentation of their skills. Teachers' procedural knowledge puts a pleasant effect on his style of instruction. A teacher equipped well with content knowledge does not depend much on his teaching materials and he tries out something new in the classroom (Adler, 2017). For enhancing students' achievement ratio, teachers' instructional process must be effective. Mathematical knowledge of teachers, is deeply related to the development of students, thinking in the classroom (Yilmaz, 2016).

Teachers' subject-matter knowledge touches almost all the dimensions of teaching and learning. The required knowledge for mathematics teaching and the qualities of mathematics teachers have been suggested by some mathematics institutions and organizations. National Council of Mathematics Teachers (2000) reveals that the knowledge of mathematical concepts and procedures and their relationship, techniques of mathematics reasoning, problem-solving, and effective communication of mathematics concepts on various levels of presentation, are the necessary contents of knowledge required for mathematics teaching. Aksu and Umit, (2016) portrait mathematical knowledge needed as understanding of core mathematics concepts, knowledge based on facts and procedure of problem-solving and their inter-relationship, skills of subject-matter presentation in the classroom, and understanding of the norms of reasoning and evidence. It looks consensus among the researchers that teachers must possess a deep understanding of mathematics content knowledge and its pedagogy. Also, teachers should be able to build a relationship among different concepts of mathematics before and after their level of teaching (NCT of Mathematics, 2000; National Mathematics Advisory Panel, 2008; Mewborn, 2003; Conference Board of the Mathematical Sciences, 2012). For effective mathematics teaching, it is pertinent to know and understand it, learners psyche, and teaching techniques. NCTM also describes that mathematics teachers require different sorts of knowledge such as subject-matter knowledge, understanding of curriculum, and students difficulties in the subject. Educational researchers have classified teachers' knowledge into three types, Knowledge of contents, knowledge of pedagogy, and basic pedagogical skills (Taylor, 2018).

For effective instruction, content and pedagogical knowledge have very important. Studies have shown that only subject knowledge is not enough for quality teaching. Pedagogical knowledge cannot replace the conceptual understanding of teachers (Mapolelo & Akinsola, 2015). Both pedagogical and content knowledge merge understanding of mathematics and its instruction. Teachers take advantage of such knowledge to present their topics in chronological order. Using different domains of teacher's knowledge, he/she has to decide what part of content knowledge is effective for students learning. This categorization of teacher domains of knowledge is useful for the institutions or organizations that prepare or design teaching-learning materials or develop programs of teacher education for teachers' professional development (Gokalp, 2016).

Some researchers like Roberts-Hull, Jensen and Cooper, (2015) categorize teacher knowledge into knowledge of lesson planning and mathematical content knowledge.

To investigate how mathematics teachers teach in the classroom, the most analytic way is to observe them while teaching. Data achieved from observation may be combined with the information achieved from the other sources. Teachers' interviews and their mathematical activities are also widely applied to investigate teachers' knowledge. The tasks of judging their knowledge may be assessing content knowledge of some specific topic or handling of issues in the classroom (Couto & Vale, 2014). There are also defects in evaluating teacher knowledge. It has been a great debate about teacher knowledge regarding mathematics teaching. Some articulate that strong subject-matter knowledge is essential while others argue that knowledge of teachers' pedagogical skills like understanding of students' interests, motivation, and individual differences is a necessary part of teachers' knowledge (Bansilal, Brijlall & Mkhwanazi, 2014).

Shulman (1987) has presented seven kinds of teacher knowledge which are subject knowledge, general knowledge of pedagogical skills, pedagogical content knowledge, learners' knowledge and their learning, curriculum knowledge, contextual knowledge, and knowledge regarding educational philosophies. Knowledge of subject contents is a fundamental element of

teacher knowledge that puts positive impact on students learning. All the components of teacher knowledge are deeply related to content knowledge. Teachers must present content knowledge in a way so that students may understand it. Mathematics content knowledge is decisive for achieving goals and objectives and developing students achievement effectively (Ball *et al.,* 2008; McAuliffe, 2013; Bansillal, Brrijlall & Mkhwenazi, 2014; Pourrnara, Hodgenn, Adler & Pilley; 2015, Livy, Vale & Herbert, 2016; Aksu & Umit, 2016; Pournara, 2016).

Effective mathematics teaching demands strong content knowledge on the part of teachers. Teachers having deep subject-matter knowledge of mathematics, develop their confidence for teaching to higher grades. It enables them to focus their attention on applying effective teaching strategies for students' practical understanding. TEGMA (2014) has suggested that teachers training institutions should recruit teachers with deep content knowledge of that particular subject. Prospective mathematics teachers are trained to develop their pedagogical skills of presenting content knowledge through their pre-service training. Some research studies emphasize the need of getting content knowledge for teachers to enhance students' achievement levels. Bowie and Reed, (2016), find no relation between teacher's in-service training and students' performance.

Kumar and Subramaniam, (2012) examined attitudes of mathematics teachers about their teaching on an attitude scale regarding different variables, findings revealed there was no difference among teachers attitudes concerning their gender difference. Brijlall, (2014) explored the factors that affect attitudes of 240 secondary level prospective mathematics teachers enrolled in two different pre-service training programs in Anatolian Teacher Training High School. Many factors e.g. GPA, nature of training program, gender, preference for choosing teaching profession, parents educational background, learning speed, size of the family, and monthly income. Findings disclosed that the significance of difference existed among teachers attitude scores regarding these factors. Lim and Guerra, (2013) analyzed attitudes of 958 mathematics teachers using an attitude scale on three-point ratings examining different factors. Results explored that a significant difference was found between average attitude gains of male and female teachers in favor of the female gender.

According to Wilburne and Long, (2010), various elements affect students learning in mathematics, for example, students and teachers' attitudes towards mathematics, teachers' certain beliefs about mathematics, and teachers' and students' difficulties in fostering learning. Teachers' attitudes were not investigated as compared to their beliefs. Baştürk and Dönmez, (2011) have revealed that Students' attitudes towards mathematics grow negative as they grow older but the relationship between attitudes and their learning attainment becomes stronger. Results of many previous studies show that most of the prospective mathematics teachers possess a negative attitude towards mathematics teaching (Blomberg, Sherin, Renkl, Glogger, & Seidel, 2014)). It is a matter of deep concern for teacher training institutions that how can teachers having negative attitudes develop positive attitudes among their mathematics students (Chan and Yung, 2017)). It is very difficult to change once attitudes developed negatively among the teachers and the students.

Many research studies conducted on institutions of teacher training that used constructivist instructional approach have found improvement in prospective elementary mathematics teachers attitudes towards mathematics teaching (Gibson & Van Strat, 2001; Smith, Esch, Hayes, & Plumley, 2016.; Gess-Newsome, 2015; Nilsson & Vikström 2015). As regards mathematical education, the need for the development of attitudes has been standing for a long. There is a need to develop self-confidence, motivation, and interest in the students towards mathematics by finding the ways and means. But all the findings are not consistent in this regard, Bayraktar, (2011) has pointed out that children start their student life with a positive attitude towards mathematics but as time passes it is changed into negative with growing age. Many researchers believe that for the sake of effective mathematics teaching, it is required to develop positive attitudes among the teachers. Mapolelo and Akinsola, (2015) state that to cultivate positive attitudes among university students, it is the need to improve the attitudes of their mathematics teachers. Cross, (2015) has urged the need of improving teachers' attitudes for bringing change in the curriculum and students perceptions. He viewed that mathematics teachers having less content knowledge, its pedagogy, and often shy of teaching mathematics, will not be able to produce positive attitudes in their students.

Attitudes and beliefs of teachers regarding their classroom teaching, affect directly their classroom practices. According to Furinghetti and Morselli, (2011), there exists a relation between teachers' beliefs and their actual classroom teaching. Mathematics teachers with conventional beliefs

about teaching practices stressed students' performance and completing courses rather than learning and understanding. Teachers' beliefs influence the beliefs of their students too. Beswick, (2012) found similar results between teachers and students beliefs. Nowadays teachers are directed to adopt the constructivist approach of instruction instead of the traditional method of teaching which they got during their schooling. Sweeting, (2011) has highlighted that mathematics teachers' beliefs system are conflicting with the philosophy of constructivism. According to Brown and McNamara, (2011), there are many approaches to altering the belief systems of students and teachers. The first one is related to the examination and manifestation of individual beliefs. The other is to build relationships among the beliefs of teachers and students.

Beliefs of mathematics teachers reflect their perceptions about their knowledge and understanding of mathematics. These beliefs have a deep impact on the process of making decisions regarding their courses and teaching methods (Hayes, & Plumley, 2016. Gess-Newsome, 2015 and Nilsson & Vikström 2015). Papay, (2011) views the beliefs of mathematics teachers as their personal opinion or communication regarding their abstracted concepts. Teachers share their beliefs regarding curriculum change, instruction process, knowledge of subject matter and they respond to devise fresh understanding and experience (Lannin, Webb, Chval, Arbaugh, Hicks, Taylor, (2013). Teachers' beliefs are their perceptions regarding their instructional activities and pedagogical skills (Sweeting, (2011). Previous research studies highlight the significance of differences between males and females (Furinghetti & Morselli, 2011)). Professional development courses bring positive change in the behaviors and perceptions of math teachers.

Objectives

Following were the objectives of this study.

1. To examine competency of secondary level mathematics teachers in their content knowledge
2. To explore the subject knowledge of math teachers achieved during their in-service trainings.
3. To find out the attitudes and beliefs of teachers about their mathematics instruction.

Research Methodology

A study was carried out to investigate "The Content knowledge of mathematics teachers, their Attitudes and Beliefs about their teaching mathematics.

Research Design

A survey-type quantitative descriptive research design was used to execute this study. According to Airiason and Gay (2000), "Descriptive research is concerned to collect data for testing hypotheses or to reply research questions, related to the ongoing condition of the population of the study. Classic descriptive studies are focused upon evaluation of attitudes, beliefs, opinions, demographic information, situations, and proceedings" (pp 249-250).

Population

The study population included all 1427 mathematics teachers of 692 public sector high and higher secondary schools of both genders from seven identified districts of Khyber Pakhtunkhwa (Source: EMIS Annual School Census Report 2017-2018 Govt. of KPK).

Sample

From the above population, 143 schools of the public sector and their 355 relevant secondary teachers were chosen randomly as a required sample using the proportion allocation technique which was 20% of the target population. Airasian and Gay, (2000) propose 306 as a sample size for the population of 1500 which shows that sample is suitable for this study.

Research Instruments

For achieving the set objectives and studying related literature, three survey-type questionnaires were prepared for collecting data from the teachers, teaching to secondary classes. One questionnaire was prepared comprising of 30 items for exploring content knowledge of the relevant teachers on three points rating. The other questionnaire consisting of 30 items was developed to know the professional development of secondary level math teachers on a three-point rating scale and the third one having 20 items was designed to examine the attitudes and beliefs of concerned teachers about mathematics teaching.

Items on content knowledge were subdivided into six domains i.e. Quadratic equations and variations, Partial Fractions, Sets and Statistics, Trigonometry and Geometry, Matrices and Logarithms, and Algebraic expressions. Ives, (2009), views a questionnaire as "The main tool which is frequently applied to collect data from various spotted areas is a questionnaire." A questionnaire

consists of a formal, written, set of closed-ended and open-ended questions that are asked from every participant in the study.

Validation and pilot testing of research instruments

For validity testing, these questionnaires were shared with the experts of items development for seeking their opinion and their proposed corrections and suggestions were incorporated. These questionnaires were pilot tested on 30 local secondary level math teachers. Some items were discarded and the final draft was prepared for administration.

Reliability of questionnaire

For measuring the reliability of prepared tools, Cronbach's alpha coefficient was determined that ranged from 0.77 to 0.89 indicating the tool to be reliable. Yin (1994) explains reliability as it represents the problems concerning data collecting and steps of procedure presenting identical results constantly. It shows the interior uniformity within the items of the developed instrument.

Table 1

Reliability of data collection tools

S: No.	Category Level	Quantity	Alpha value
1	Content knowledge items	30	0.86
2	Professional development items	30	0.89
3	Teachers' attitudes and beliefs	20	0.77
	Total	80	0.87

Data Collection

To collect data from the respondents, both the survey questionnaires were handed over to the relevant teachers and they were given the necessary instructions to fill them. Some teachers were also provided self-addressed envelopes who were not present on that day due to some engagements for returning questionnaires through the mail. Maximum questionnaires were received from the respondents on the same day after their completion. Teachers were asked that their information will be used only for research purposes and will be kept confidential.

Analysis of Data

Data collected from the respondents were arranged in tabular form of Excel sheets in frequencies. Numerical values to these frequencies were assigned as under.

Great Extent =03 marks, Some Extent= 02 marks, Not at all=01 mark

Statistical Package for data analysis of Social Sciences was utilized to apply the necessary Chi-square test for analyzing the responses achieved from the two questionnaires.

Results and Discussion

The objective of data analysis was to interpret the results and draw necessary findings, conclusions, and recommendations for this study. The significance of difference was calculated in form of p-value.

Table 2

Teacher response about Content Knowledge in following mathematical domains

Serial No.	Contents of subject knowledge	Response frequency			$\bar{x}$	$\chi 2$	sig.
		N.A.	S.E.	G.E.			
1	Knowledge of concepts and procedure of "Quadratic equations and variations".	6 1.6%	55 15.4%	294 82.8%	2.84	163	0.00
2	Knowledge of concepts and problem solving of "Partial Fractions".	13 3.6%	69 19.4%	274 77.2%	2.74	540	0.00
3	Knowledge of concepts and problem-solving procedure of "Sets and Statistics".	7 2.0%	115 32.4%	233 65.6%	2.63	216	0.00
4	Knowledge of concepts and procedure of problems solving on "Trigonometry and Geometry".	7 2.0%	151 42.5%	197 55.5%	2.53	166	0.00
5	Knowledge of concepts and procedure of problems solving on "Matrices and Logarithms".	8 2.3%	120 33.8%	227 63.9%	2.61	202	0.00
6	Knowledge of concepts and procedure of problems solving on "Algebraic statements"	6 1.7%	154 43.4%	195 54.9%	2.53	167	0.00

Competency of mathematics teachers regarding the stated contents shows the significance of their response ($\chi 2=163$, $p<0.05$) as shown in the above table and 83% of teachers viewed themselves

as competent in the content knowledge of the above-mentioned areas in the secondary level mathematics course.

The above table shows the significance of competency level of mathematics teachers related to knowledge of partial fractions i.e. ($\chi 2$=540, p<0.05), and a substantial portion of teachers (G.E. =77%) rated themselves competent concerning contents of item # 2.

Numerical values of item No. 3 show the significance of teachers' response in the competency area of sets & statistics i.e. ($\chi 2$=216, p<0.05), and a substantial portion of teachers (66%) viewed themselves well up in the knowledge of the above-mentioned areas.

Measurements of category No. 4 show the significance of teachers' response possessing the knowledge of the concepts of Trigonometry and Geometry i.e. ($\chi 2$ =166, p<0.05) and a substantial portion (GE =56%) rated themselves at home in the knowledge of given areas.

Statistical values of category No. 5 indicate the significance of teachers' response in the subject knowledge of the given concepts i.e. ($\chi 2$= 202, p<0.05), and a substantial portion (GE = 64%) viewed themselves well up in the above-mentioned content areas.

Category No.6 of the above table reveals the significance of teachers' views in content knowledge of Algebraic expressions i.e. ($\chi 2$=167, p<0.05), consequently a large portion of math teachers (GE=55%) rated themselves competent in doing exercises on Algebra.

Table 3

Teachers' response about In-service professional development of Content Knowledge in secondary level mathematics

Serial No.	Competency in in-service professional development	Observed frequency			$\bar{x}$	$\chi 2$	sig.
		N.A.	S.E.	G.E.			
1	In-service professional development on Quadratic equations and Variations	81 22.8%	206 58.0%	10 2.8%	1.8	198	0.00
2	In-service refresher courses on contents of Partial fractions	95 26.8%	190 53.5%	12 3.4%	1.7	160	0.00
3	In-service professional development on topics of Sets and Statistics	134 37.7%	153 43.1%	10 2.8%	1.6	121	0.00
4	In-service professional development on concepts of Trigonometry and Geometry	153 43.1%	131 36.9%	13 3.7%	1.5	17.5	0.07
5	In-service professional development on problem-solving of Matrices and Logarithms	147 41.1%	139 39.2%	11 3.1%	1.5	8.3	0.06
6	In-service professional development on solving different exercises of Algebra.	171 48.2%	112 31.5%	14 3.9%	1.5	4.3	0.06

Readings of item No. 1 in the above table show the significance of mathematics teachers' response regarding their in-service professional development on the given content areas i.e. (chi-square=198, p<0.05), and 58% of teachers opted for focusing to some extent. 16.4% of teachers responded that they never participated in any refresher course of mathematics while 23% of teachers were of the view that these contents were not included in their professional development.

Statistical values of item No. 2 of the above table show the significance regarding teachers' response on in-service training on partial fractions ($\chi 2$=160, p<0.05) and 54% of respondents opted for some extent whereas 27% of teachers responded that they were not provided professional development on these contents.

Readings of item No. 3 in the above table disclose the significance of teachers response related to professional development on Sets and Statistics i.e. (chi-square=121, p>0.05) while 43% of teachers opted the choice of some extent and 38% of teachers responded that they were not provided in-service training on these contents.

Statistical values of statement No. 4 explore the significance of teachers responses regarding their in-service training on trigonometry and geometry ($\chi 2$=17.5, p>0.05). 43% of teachers responded that these contents were not included during their PD trainings while 37% of teachers went for the choice of some extent.

Measurements of category No. 5 in the above table reveal that teachers' response related to in-service PD on Matrices and Logarithms was calculated non-significant i.e. ($\chi 2$=8.3, p>0.05) because 41% of teachers responded that these concepts were not touched at all during their PD trainings. Whereas 39% of teachers responded that they were provided PD on these contents to some extent.

Statistical analysis of item No. 6 in the above table indicates that teachers response regarding their in-service PD on Algebraic Expressions was measured non-significant ($\chi 2=4.3$, $p>0.05$) because 48% of teachers responded that they were not involved in PD on these contents and 32% teachers responded that they were provided PD on these concepts to some extent.

Table 4

Significance of teachers' response regarding their attitudes about teaching mathematics

Item No.	Statements	Response frequency					$\bar{x}$	$\chi 2$	p
		SDA	DA	UD	A	SA			
1	I teach with professional confidence and a warm heart.	21 5.9%	37 10.4%	23 6.5%	226 63.7%	48 13.5%	3.6	429	0.00
2	I am confident about the methods of teaching mathematics.	26 7.3%	42 11.8%	20 5.6%	170 47.9%	97 27.3%	3.7	224	0.00
3	Time passes quickly when I am teaching mathematics in practice sessions.	51 14.4%	2 0.6%	33 9.3%	114 32.1%	155 43.7%	3.9	218	0.00
4	Generally, I feel secure about the idea of teaching mathematics.	5 1.4%	89 25.1%	43 12.1%	136 38.3%	82 23.1%	3.5	138	0.00
5	I have always done well in mathematics classes.	17 4.8%	8 2.3%	1 0.3%	196 55.2%	133 37.5%	4.1	440	0.00
6	Mathematics makes me feel inadequate.	37 10.4%	85 23.9%	214 60.3%	16 4.5%	3 0.8%	2.6	414	0.00
7	I am not the type of person who could teach mathematics very well.	142 40.0%	169 47.6%	14 3.9%	12 3.4%	18 5.1%	1.8	340	0.00
8	Teaching mathematics does not scare me at all.	171 48.2%	173 48.7%	3 0.8%	6 1.7%	2 0.6%	1.5	479	0.00
9	I have generally done better in mathematics courses rather than the other courses.	93 26.2%	102 28.7%	5 1.4%	130 36.6%	25 7.0%	2.6	161	0.00
10	I am not sure about what to do when I am teaching mathematics.	136 38.3%	166 46.6%	15 4.2%	35 9.9%	3 0.8%	1.8	314	0.00
11	At school, my friend teachers always come to me for seeking help in mathematics.	134 37.7%	127 35.8%	6 1.7%	75 21.1%	13 3.7%	2.1	207	0.00
12	I generally get a sinking feeling if I come across a hard problem while teaching.	111 31.3%	169 47.6%	46 13.0%	18 5.1%	11 3.1%	2.0	257	0.00
13	I have hesitated to take courses that involve mathematics.	147 41.4%	171 48.2%	7 2.0%	22 6.2%	8 2.3%	1.7	370	0.00

It is evident from the numerical values of the above table that teachers showed their strong positive behavior towards the mentioned statement with (A=64% & SA=13.5%) indicating the significance of response ($\chi 2=429$, $p<0.05$).

Numerical values of statement # 2 indicate the significance of mathematics teachers responses (A=48%, SA=27%) and ($\chi 2=224$, $p<0.05$), showing their agreement with the mentioned statement.

It is evident from the measurements of statement # 3 that teachers presented their positive perception about the given statement (A=32%, SA=44%) showing significance of response ($\chi 2=218$, $p<0.05$).

Statistical analysis of statement # 4 in the above table clears that teachers showed a positive attitude about the given statement (A=38%, SA=23%) indicating significance of response as ($\chi 2=138$, $p<0.05$).

It is obvious from the above readings of statement # 5 that teachers showed their positive perception regarding the given statement (A=55%, SA=38%) giving the significance of response ($\chi 2=440$, $p<0.05$).

Numerical measurements of statement No. 6 in the above table show that maximum teachers were undecided about the mentioned statement (UD=60%, SDA=10%) showing the significance of response ($\chi 2=414$, $p<0.05$).

It is clear from the readings of statement # 7 in the above table that maximum teachers showed their disagreement about the given statement (DA=48%, SDA=40%) giving significance of response ($\chi 2=340$, $p<0.05$).

Statistical analysis of statement # 8 shows that maximum teachers disagreed with the given statement (DA=49%, SDA=48%) giving the significance of response ($\chi 2=479$, $p<0.05$).

It is clear from the calculations of statement # 9 that a major portion of the teachers upheld their agreement with the stated attitude (DA=29%, SDA=26%) showing significance of response ($\chi 2=160$, $p<0.05$).

It is clear from measurements of statement # 10 in the above table that a great part of the teachers gave their disagreement with the stated attitude (DA=47%, SDA=38%) indicating significance of response ($\chi 2=314$, $p<0.05$).

Numerical analysis of statement # 11 represents that a great part of the teachers rejected the mentioned attitude statement (DA=36%, SDA=38%) giving significance of response ($\chi 2=207$, $p<0.05$).

It is obvious from the numerical values of statement # 12 that a huge part of the teachers rejected the given attitude statement (DA=48%, SDA=31%) measuring it as significance of response ($\chi 2=256$, $p<0.05$).

Statistical analysis of statement # 13 of the above table clears that a great part of mathematics teachers disagreed with the mentioned attitude statement (DA=48%, SDA=41%) showing significance of response ($\chi 2=369$, $p<0.05$).

Table 5

Beliefs of mathematics teacher about their teaching

Item No.	Statements	observed frequency					$\bar{x}$	χ^2	p
		SDA	DA	UD	A	SA			
14	I believe that mathematics is exciting and interesting to teach.	24 5.8%	34 9.6%	9 2.5%	246 69.3%	42 11.8%	3.6	547	0.00
15	I believe that mathematics is one of the subjects that I like the most.	17 4.8%	15 4.2%	2 0.6%	230 64.8%	91 25.6%	4.0	514	0.00
16	I believe that I get never tired of teaching mathematics in the classroom.	32 9.0%	20 5.6%	9 2.5%	190 53.5%	104 29.3%	3.8	327	0.00
17	In my belief, mathematics is a boring subject.	65 18.3%	80 22.5%	5 1.4%	153 43.1%	52 14.6%	3.1	162	0.00
18	I believe that Good mathematical knowledge makes it easier to learn other subjects.	101 28.5%	147 41.4%	32 9.0%	60 16.9%	15 4.2%	2.2	161	0.00
19	I believe that teachers teach effectively those concepts that require mathematical reasoning.	102 28.7%	112 31.5%	103 29.0 %	21 5.9%	17 4.8%	2.2	128	0.00
20	I believe that I get no satisfaction from teaching mathematics.	111 31.3%	133 37.5%	3 0.8%	99 27.9%	9 2.5%	2.5	207	0.00

It is obvious from the readings of item No. 14 that a great part of the teachers had a positive perception about the given belief statement (A=69%, SA=12%) showing the significance of response ($\chi 2=547$, $p<0.05$).

Numerical analysis of the calculations of item #15 reveals that a major portion of the teachers upheld their opinion of agreement about the given belief statement (A=65%, SA=26%) giving significance of response ($\chi 2=514$, $p<0.05$).

Measurements of statement # 16 in the above table indicate that a majority response of the relevant teachers (A=54%, SA=29%) was measured significant ($\chi 2=327$, $p<0.05$) showing their agreement with the given belief statement.

Statistical analysis of calculations of statement # 17 reveals positive belief perception of the teachers about the given statement (A=43%, SA=15%) giving their significance of response ($\chi 2=162$, $p<0.05$).

It is clear from the measurements of statement # 18 in the above table that a majority response of mathematics teachers (DA=41%, SDA=29%) was calculated significant ($\chi 2=161$, $p<0.05$) regarding their disagreement about the mentioned belief statement.

Calculated values of statement # 19 in the above table indicate that a major portion of the teachers disagreed with the given statement (DA=32%, SDA=29%) giving significance of response ($\chi 2=128$, $p<0.05$).

Numerical values of statement # 20 in the above table show that and a great part of the teachers showed their disagreement about the mentioned belief statement (DA=38%, SDA=31%) representing the significance of response ($\chi 2=207$, $p<0.05$).

Findings

These findings were drawn from the interpretations of the above tables.

The majority of mathematics teachers viewed themselves as experts in the mathematical contents of quadratic equations and variations. Also, the majority of the respondents viewed themselves as weak in teaching the contents of partial fractions. A substantial portion of the teachers rated themselves an experts in subject knowledge of sets and statistics. The majority segment of relevant teachers viewed themselves strongly related to the content area of trigonometry and geometry.

A major part of secondary teachers was found expert having content knowledge of matrices and logarithms. In the field of concepts of algebraic expressions, a great segment of the relevant teachers was found an expert in the given areas. Professional support about the contents of quadratic equations and variations was not provided to the substantive portion of mathematics teachers. A small portion of mathematics teachers was not provided any type of professional development in mathematics. Mathematical knowledge of proper & improper fractions was not delivered to the majority of mathematics teachers during their in-service training. Professional support regarding the content knowledge of sets and statistics was not provided to the maximum of mathematics teachers during their in-service training. Professional development on the contents of trigonometry and geometry was not provided to a majority of mathematics teachers. Professional support in terms of the contents of matrices and logarithm was not delivered to the maximum of mathematics teachers. In-service training was not provided to a majority of mathematics teachers regarding the contents of algebraic expressions.

Mathematics teachers' response regarding the development of attitude towards mathematics instruction was significant. Mathematics teachers' response related to their beliefs about mathematics instruction was too significant.

These findings also verify the results of recent past research studies conducted in this regard (Gibson & Van Strat, 2001; Smith, Esch, Hayes, & Plumley, 2016; Gess-Newsome, 2015; Nilsson & Vikström 2015).

Conclusions and Discussion

The following conclusions were drawn in light of the above findings.

The secondary school mathematics teachers possess sufficient knowledge of the subject-matter of the current curriculum of secondary level mathematics. Mathematics teachers do not feel competent in the contents of partial fractions, statistics, and practical geometry. Regular professional development on their subject-knowledge is not arranged regularly to develop their content knowledge. Some math teachers have not participated in any refresher course so far on mathematics learning. Mathematics teachers have shown positive perceptions about their attitudes towards mathematics instruction. Teachers have also positive belief perception of classroom math teaching. There exists no particular difference between the attitudes and beliefs of math teachers belonging to rural and urban localities.

In light of the above-derived findings and conclusions, the following recommendations are suggested for the future.

a) Since there is a weakness on the part of mathematics teachers in some domains of secondary level content knowledge, so necessary steps may be taken to make them competent in those

areas.

b) In the process of curriculum revision, concerned mathematics teachers might be involved and necessary professional development might be provided to them in this regard.

For the capacity building of mathematics teachers, all of them may be provided equal opportunities of attending refresher courses.

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Annexure

Table

Total Secondary school mathematics teachers of Govt. Higher secondary and secondary schools of selected population

S: No.	Name of District	Total High and Higher secondary schools						Grand Total	Total math teachers
		Urban			Rural				
		Male	Female	Total	Male	Female	Total		
1	Abbottabad	6	5	11	65	41	106	117	242
2	Battagram	1	0	1	34	7	41	42	87
3	Haripur	9	5	14	57	48	105	119	245
4	Kohistan	0	0	0	30	2	32	32	66
5	Mansehra	2	1	3	84	43	127	130	268
6	Nowshehra	15	3	18	57	28	85	103	212
7	Peshawar	32	25	57	53	39	92	149	307
	Total	65	29	104	380	208	588	692	1427

Source: EMIS Annual School Census Report 2017-2018 Govt. of KPK