

To Study the Basic Concepts of Monte Carlo Simulation by Calculating the Value of π

Hamdullah Khan^{1*}, Anees ur Rahman², Fazli Manan³

¹Department of Physics, Balochistan University of Information Technology Engineering & Management Sciences Quetta, ²Centre for Nuclear Medicine and Radiotherapy, Quetta,

³Department of Electronics, Quaid-i-Azam University, Islamabad.

Abstract

The Monte Carlo Simulation is a basic tool in computational and theoretical physics by which complicated and difficult physical, statistical and mathematical problems are solved which cannot be solved by usual analytical means. In this paper the basic concepts of Monte Carlo techniques are discussed and used by calculating the value of π by Monte Carlo simulation and generating random numbers with the help of a computer program (Mathematics).

Key words: Monte Carlo Method, Simulation, Random numbers, Probability, C++.

*Corresponding Author's email: hamdullah.khan@buitms.edu.pk

INTRODUCTION

The Monte Carlo method was discovered in 1940 by John Von Neumann, Stanislaw Ulam and Nicholas Metropolis during their research on the nuclear weapon project (Manhattan project) in the Los Alamos National Laboratory. Its name relates to the Monte Carlo Casino, a well known Casino of which where Ulam's Uncle was a member.

Monte Carlo is a technique which involves using random numbers and probability statistics to solve problems (Wittwer J). Monte Carlo methods are computer algorithm depending upon the repeated random sampling to compute their results for the physical, statistical and mathematical problems. These methods are used when complicated problems cannot be solved by conventionally analytical methods (Harvill and pipes, 1970, Vose, 2008).

Monte Carlo Method has applications equally in physical sciences, nuclear physics, space physics, fluid mechanics, telecommunication, engineering, computation biology, applied statistics, games, finance and business etc. They are widely used as well in mathematics to calculate multidimensional definite integrals with complicated boundary conditions. There is difference between a simulation, Monte Carlo method and Monte Carlo simulation. A simulation is a fictitious

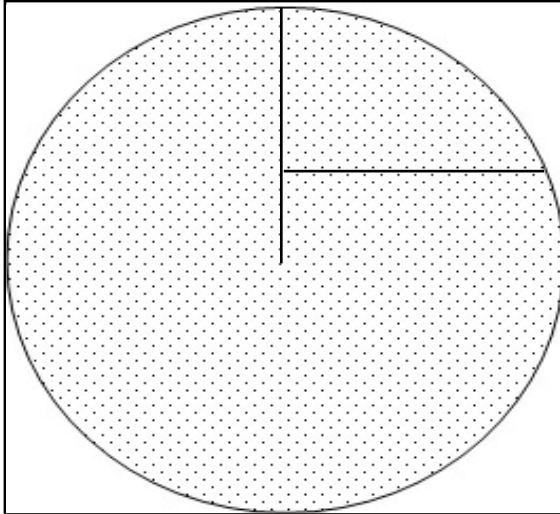
representation of reality, a Monte Carlo method is a technique which is used to solve a mathematical and statistical problem and a Monte Carlo simulation uses repeated sampling to determine the properties of some action (Wikipedia). For example if we toss a coin once, head or tail outcome can be used to simulate the tossing of coin. This can be done by drawing one pseudo random uniform variable from the interval $[0, 1]$ is simulated the toss of coin. If the value is less than or equal to 0.5, designate the outcome as head, but if the value is greater than 0.5 the outcome as tails. This is simulation but not Monte Carlo simulation. The area of an irregular shape inside in a unit area of square and computing the ratio of hits within the irregular figure to the total number of darts thrown. This is a Monte carol method but not a simulation. If we select to draw a large number of pseudo random uniform variable from the internal $[0, 1]$ and assigning values less than or equal to 0.5 as heads and greater than 0.5 as tails is a Monte Carlo simulation of the behavior of tossing a coin (Woller, 1996).

MATERIALS AND METHODS

Monte Carlo (MC) methods stochastic technique which based on the use of random numbers and probability statistics to

investigate problems. It has applications in many fields.

The methodology to calculate the value of π by using this technique is discussed as below by considering the square and an inscribed circle in it as below.



2r

Figure:1 The circle of radius r is inscribed in the square of side $2r$. Let us consider a circle inscribed in a square of having side $2r$ and radius of circle is r . The ratio of area of a circle to the square is

$$\frac{\pi(r)^2}{(2r)^2} = \frac{\pi}{4}$$

If we imagine the player of dart who hits the darts to a square and counts that of hits and missing. It is obvious that area of the square and circle is proportional to the No. of hits. In other words

$$\frac{\text{Number of darts hitting shaded area of circle}}{\text{Number of darts hitting entire square}} = \frac{\pi r^2}{4 r^2} = \frac{\pi}{4}$$

$$\pi = \frac{4 * \text{Number of darts hitting shaded area of circle}}{\text{Number of darts hitting Entire Square}}$$

If each dart thrown hits somewhere inside the square, the ratio of hits in the shaded area to throws will be one fourth the value of π . If we do this experiment it takes a very large number of throws to get a proper value of π . To perform this procedure a computer program has been written in C++ language. The algorithm of this program is as under,

```
#include<iostream.h>
#include<stdio.h>
#include<conio.h>
```

```
#include<stdlib.h>
#include<iomanip.h>
#include<math.h>
void main(void)
{
    clrscr();
    FILE*stream;
    float r,s,x,y,dist,z;
    int i,k,n,range,min,max;
    cout<<"enter min and max,try"<<"\n\n";

    cin>>min>>max>>n;
    cout<<"# of Iteration"<<setw(12)<<"# of
    hits"<<setw(16)<<"value of pi";
    cout<<"\n\n";

    stream=fopen("MCData.txt","w+");
    fprintf(stream,"# of iteration  # of hits    value
    of pi");

    fprintf(stream,"\n\n");

    range=(max*100)-(min*100);
    k=0;
    for(i=1;i<=n;i++)
    {r=rand()/10%(range+min);
    s=rand()/10%(range+min);
    x=r/100;
    y=s/100;
    dist=sqrt(x*x+y*y);
    if(dist<=1&&dist>=0)
        k=k+1;
    z=4.0*k/i;
    cout<<i<<setw(18)<<k<<setw(16)<<z;

    fprintf(stream," %d          %d %f",i,k,z);
    fprintf(stream,"\n\n");
    cout<<"\n\n";
    }
    fclose(stream);
    getch();
}
```

In this program random numbers are generated. If we suppose our circle has radius 1.0unit, then for each throw we can generate two random numbers as x and y coordinates which are used to calculate the distance from the origin (0,0) using the pythagora's theorem $x^2 + y^2 = 1$. If the distance from the origin is less than or equal to 1.0 then it means that value lies within the shaded area and counts as hit. Repeat

this process thousands of times and we get an estimated value of π . The accuracy of this value depends on number of iterations and the quality of random numbers generated (Woller, 1996).

RESULTS AND DISCUSSION

For the scientific program execution, three inputs are entered, one is minimum, second maximum value and third is number of iterations (4). From minimum and maximum values the range is determined and the computer codes generates random number between 0 and 1. In the first iteration these random numbers are checked the condition $x^2 + y^2 \leq 1$, if this condition satisfied it means our first throw hit is within circle otherwise it is outside from circle and count as a missing (5). Then we try second, third and repeatedly try to get hits and miss of throws. Finally by calculating the probability of hits and multiplying it by 4 we get estimated value of π i.e. 3.142. By giving the inputs min =0,max=1 and number of attempts 1000 the outputs of some values are illustrated as below.

# of iteration	# of hits	value of pi
1	1	4.000000
2	2	4.000000
3	3	4.000000
4	4	4.000000
5	5	4.000000
6	6	4.000000
7	6	3.428571
8	7	3.500000
9	8	3.555556
10	9	3.600000
11	10	3.636364
12	11	3.666667

13	12	3.692308
14	13	3.714286
15	13	3.466667
16	14	3.500000
17	14	3.294118
18	15	3.333333
19	15	3.157895
20	16	3.200000
636	499	3.138365
637	500	3.139717
638	501	3.141066
639	502	3.142410
640	503	3.143750
641	503	3.138845

Thus there are two main reasons which compel to use Monte Carlo method because of their anti aliasing properties and their ability to approximate quickly an answer that would be very time-consuming to find out the answer too if we were using methods to determine the exact answer. The second point refers to the fact that Monte Carlo method is used to simulate problems that are too difficult and time-consuming to use other methods for.

CONCLUSION

By the Monte Carlo method and simulation, different types of complicated physical, mathematical and statistical problems can be solved. This technique has been used for the calculations of value of π successfully by using computer scientific program. Monte Carlo simulation can be improved by powerful computer, proper computer codes, random numbers and number of iterations.

ACKNOWLEDGEMENT

We are thankful to Syed Zaheer Abbas Gilani Assistant professor Physics department BUIITEMS for his help in using C++ computer language.

REFERENCES

- Harvill LR and Pipes LA. (1970).Applied Mathematics for Engineers And Physicists. 3rd Edition,New York, Mc Graw. Hill Inc.
- Monte Carlo method –Wikipedia, the free encyclopedia
- Woller T. 1996).The Basics of Monte Carlo Simulation,
<http://www.chem.unl.edu/zeng/joy/mclab/mcintro.html>
- Wittwer J. Monte Carlo Simulation Basics,<http://www.vertex42.com/ExcelArticles/mc/monteCarloSimulation.html>.
- Vose D. (2008). Risk Analysis, A Quantitative Guide (3rd. Ed). John Wiley & Sons.