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Writing Equations in Algebra: Investigation of Students' Misconceptions

* Dr. Zarina Akhtar (Corresponding Author)

** Abdur Rashid, Lecturer

*** Dr. Sajjad Hussain

Abstract

Algebra is blamed as an abstract way of thinking in mathematics learning. Writing equations from given text is a skill. Logical and step by step noting information help to write the correct equation. It is transferring text to symbols. This study aimed to investigate secondary school student's misconceptions in writing equations in algebra. 414 secondary school students enrolled at 4 Islamabad Model Schools of sector Islamabad was the sample of the study. A multiple-choice test based on 4 items was used as an instrument. The test has 4 statements and 4 options from which one is the correct answer and others are related to some misconceptions of students. Test was administered during their class. Student's response patterns were observed. Frequency percentages and ratios were used to analyze data. Two types of misconceptions "arithmetic process" and "missing reference" were identified in student's responses whereas the other three misconceptions "pro-numeral as unit", "addictive nature" and "changes" were not reported by students' responses. It is concluded that the main reason behind these misconceptions seems related to classroom practice. Students try to translate words into the arithmetic process to write an equation by given text. Further, they do not focus on reference terms given in text and they ignore/miss it while writing equations. It is recommended to the teachers they need to clear the concept given in word problems and give teach them with logic what is meant by which word in the problem. There is a need for focused classroom activities to overcome these misconceptions.

Keywords: Algebra, Writing Equations, Secondary Students, Misconceptions

Introduction

There is a movement all over the world to make algebra more concrete. Teachers and experts have designed special activities, software, and printed material for the students' concrete learning of algebra. It is found that students have a poor understanding of algebraic concepts which leads to a negative attitude towards algebra as they move to higher grades (Mullis, Martin, Foy, & Hooper, 2016). Despite every effort, students still found difficulty in understanding the abstract concepts of algebra. The reason elaborated by Foster (2007) was the abstract ideas of algebra are taught without understanding. To provide a learning experience to students for the understanding of the concept of algebra teachers have to know how students perceive the concept and what type of misconceptions they may build upon the knowledge they have learned. Rodrigues and Thacker (2019) described many teachers possess limited knowledge and understanding of mathematical concepts. They have misconceptions about different concepts and they transfer these to students in their classes during teaching. This is why it is very important to study students' thinking and identify which misconceptions they may have and feedback-back to the teachers for improving students learning of algebra.

The current study is designed to investigate students' misconceptions about writing equations. The items used in this study were based on the early work done by MacGregor and Stacey (1995). In their study, they presented some algebraic word problems to the sample students and study their mental models used in handling these algebraic problems. They found the majority of the students did not attempt to use algebra. These items were again presented by MacGregor and Stacey (1996) to students and analyze their responses for writing equations. The finding of study showed although the students understand all the information presented in a problem statement; they are unable to integrate it into an equation. The analysis of student's responses showed some responses were careless mistakes

* International Islamic University, Islamabad, Pakistan
** Department of Education, Shaheed Banazir Bhutto University Sheringal, Upper Dir, Pakistan
*** Department of Education, University of Swat

but some were errors based on their cognitive problems and some based on intellectual problems. Student's responses were carefully studied and those word problems were then converted to multiple-choice questions. The alternatives used were based on some logical errors misconceptions. These responses showed students have some problems in converting statements to mathematical expressions. Keeping in view the students' problem current study has been designed to investigate student's misconceptions in writing a linear equation. The statements which were previously used by MacGregor and Stacey (1996) were selected to construct the test which was based on four multiple-choice items and students were required to select one option. One was the correct answer and others were associated with some misconceptions regarding writing equations the same instrument was used to study students' misconception.

Literature Review

Writing an equation is a way of expressing a word problem into mathematical form which leads towards the solution of the problem. Many research studies elaborated on students' problems about writing equations from information presented in words. A series of research studies Clement, Narode and Rosnick (1981), Clement (1982), Kieran (1992), MacGregor and Stacey (1993, 1995 and 1996), Rosnik (1981), has discussed students' different problems in writing equations and proposed different reasons for their problems. One common reason considered as students don't have algebraic thinking patterns. Algebraic thinking is an important skill that must be learned by students at early grades but algebraic thinking is not guaranteed to develop algebraic learning (Mustafa, Sail, Ismail, & Tasir, 2018).

Ostmeyer and Nebraska (2009) found students apply two strategies; one is the "think-aloud strategy" and focus on the "keywords" strategy for writing equations from a given word problem. They found that the think-aloud strategy is working for a short time students forget it till they face a new situation. Some other researchers has identified students different problem in writing equation. The one most common is; when students are asked to write/formulate an equation they tried to translate or map a statement of a problem into an equation by replacing keywords with mathematical symbols from left to right. Students this way of responding was named as syntactic strategy or word matching approach by Clement (1982). Herscovics (1989) studied it in a different way he described this strategy was based on rules for arranging symbols in an expression. The syntactic translation was a commonly used procedure by students for writing equations from natural language which is a cause for reversal error. The reversal error was defined by Clement (1982) as, it is observed when students tried to translate problem from picture to equation, data tables to the equation, and equation to sentence. The Cocking and Chipman (1988) studied students’ problems and described it as "direct sequential word-for-symbol mapping" (p.30) is a cause of reversal order they did not mention any other cause. Nicaud, Bouhineau, and Gelis (2014) discussed syntax and semantics in algebra. They found students have less knowledge and meaning of algebraic concepts so they have problems. This is elaborated by Semuel, Mulenga, and Angel (2016) as students may be exposed to the situation they are familiar with for formulating equations and teachers need to emphasize the situation for transition from arithmetic to algebra.

The other way of writing equations from verbal data is static comparison as described by Clement (1982) the equation is used to represent an association of related groups rather than equal numbers. Thus when students incorrectly translate 'there are six times as many students as professors' as $6S=P$ according to MacGregor and Stacey (1993) students are not symbolizing a sequence of words instead they represent a group of 6 students associated with one professor. The equal sign represents the association, not the equality, and the letter S and P are labels for students and professors. This is another misconception called misinterpretation of algebraic letters students' label algebraic letters as abbreviated words or labels for objects. Many research studies are evident of this misconception for example Booth (1984), Kuchemann (1981). Students misunderstand the structure and meaning of the equation and the significance of the equal signs. The chance of misunderstanding of the statement of problem may be reduced for formulating equations if students understand letters stand for numbers and not for objects.

Students’ misconceptions studied by Clement, Narode and Rosnik (1981) were “reversal error”, “the lots approach”, “focusing on a total by introducing extraneous variable” and “vertical rule symbolization in data table”. The reversal error misconception has been discussed in above paragraph. The lots approach behavior pattern was observed when students' tried to solve word problems involving ratios in terms of 'lots'. The students symbolize a correct conceptualization of the problem with incorrect equations.

The "focus on total" misconception was observed when students were asked to symbolize students and professor problem with an equation. Some students responded as 6S + P = T or 6P + S =T. It appears from their response that the relationship between the number of professors and the number of students only makes sense if some kind of sum is included in the equation. Clement et al. (1981) tagged it as a natural consequence of the way the students' ideas are structured. Equations so face dif tables in writing an equation from data table called vertical rule symbolization in data table.

Some students have misconceptions when they transform words to symbols. This appears while writing equations where sum of three quantities is equal to number. This situation was pointed out by Stacey and MacGregor (2000). They said students may have “additive nature” misconception when they write an equation where three terms have been added instead of adding the components provided they use new symbols for the parts and add something to explain the situation.

Literature highlighted some students instead of selecting correct equation they may have a focus on "arithmetic process" and have "arithmetic process" misconception. It is also evident from the previous research conducted by MacGregor and Stacey (1995) that studied the mental model approach to find answer first then tried to write an equation solution process the information for solution but it shows they do not appreciate the power of algebra. Adstudents'h, and Astidu-Addo (2015) studied students’ errors in students' word problems and highlighted students inability to comprehend and interpretation of sentences in-order to proceed to the process of encoding skill.

Many students focus on those things which they think are necessary for the solution and they use the "pro-numeral as a unit" of measurement. The other misconception component is about the selection of components parts. They chose only those components which are different from the reference term rather than choosing the expression for each component part. They may have focus on the term and ignore the "changes". It is named as "changes" misconception.

Some students do reverse from the above description they correctly formulate the components related to the reference term but ignore the reference term itself. This misconception is named as “missing reference term” misconception.

In current study five misconceptions described in literature were the main focus i.e. “arithmetic process”, “missing reference”, “pro-numeral as unit”, “additive nature” and “change”. The multiple questions constructed have one correct option and others were reflecting these misconceptions. The given statements were based on sum of three quantities for solving the question.

Objective of the Study

1. To identify students’ responses to misconceptions given word problems.
2. To investigate students misconception while designing the equation from given word problem.

Research Methodology

This study examines students' misconceptions about writing equations. The instrument was a test. It contains four problem statements with four options for each item. The options were taken from the early student responses studied by MacGregor and Stacey (1996). They gave word problems to students and study student thinking behind incorrect responses. These incorrect responses were further associated with different misconceptions as stated above. The validity of the test was measured through experts from the mathematics field. They were school teachers teaching to secondary classes and teacher trainers. All the options were explained to them as misconceptions related to specific word problems. After their judgment, the instrument was finalized. The population of the study was secondary school students enrolled in Islamabad Model Schools. Total 414 secondary school students from class 7th, 8th, 9th, and 10th were selected as samples from two schools. The sample of the study was 89 from class 7th, 126 from class 8th, 131 from class 9th, and 68 from class 10th students. They responded to these items. The instrument was pilot tested and found appropriate for this age and grade level students. The test was administered to the students in their classes. The collected responses were analyzed through frequency, percentage. The students' incorrect responses were analyzed with 'ratio' as their response patterns. The responses occurred in greater ratio were considered as their answers and studied concerning the misconceptions identified by MacGregor and Stacey. This study investigated student's incorrect responses and found which type of misconceptions they possess. The questions are presented in Figure 1.

Item 1 The perimeter of a triangle ABC is 44 cm. If mAB=2*x* cm, mBC=14 cm and mAC= *x* cm By using your algebra knowledge which of these equations would you write first? $44x - 14x = 30$ $x = \frac{44-14}{3}$ $x + 2x + 14 = 44$ $x + y + z = 44$	Item 2 The sum of angles of a triangle is 180 degrees. If the $m< A = 2a$, $m < B = 103^0$ and $m < C = a$ By using your algebra knowledge which of these equations would you write first? $a + b + c = 180$ $a + 2a + 103 = 180$ $a = \frac{180-103}{3}$ $180a - 103a = 77$
Item 3 A bus took people on a 3-day tour. The distance traveled on day 2 was 85 km further than on Day 1. The distance traveled on Day 3 was 125 km further than on Day 1. The total distance covered was 1410 km. By using your algebra knowledge Which of these equations would you write first? $x + 85 + x + 125 = 1410$ $x + (x + 85) + (x + 125) = 1410$ $x + 85 + 125 = 1410$ $x = \frac{1410-85-125}{3}$	Item 4 Some friends collected some storybooks. If Ahmad has 3 times as many as Ali. Amna has 10 less than Ali. Altogether the three of them have 100 storybooks. By using your algebra knowledge Which of these equations would you write first? $3x - 10 = 100$ $3x + (x - 10) = 100$ $x = \frac{100+10}{5}$ $x + 3x + (x - 10) = 100$

Figure 1: Items 1, 2, 3, and 4 from Mathematics book

Data Analysis

Items 1, 2, 3, and 4 were multiple-choice questions and the options were designed in a way that one is the correct option and others are associated with above-defined misconceptions. The students were required to select one option from the given for respective statements. The selected options may help to identify students' misconceptions in writing equations. The options 'B', 'C', 'D, and 'C' of items 1, 2, 3, and 4 respectively have different style as compared to other options of the items. In Figure 1 these options are presented in a 'professional' style to keep them similar as in the original instrument but onward from this section it is presented in 'linear' style because of the format and space in tables. The items were multiple-choice questions it seems easy for students to respond to them, 96% of students responded to these items and 4% have not attempted. Students' response on each item is presented in Table 1 to Table 4.

Table 1: *Students' response on item 1*

Options	Frequency	Percentage
A. $44x - 14x = 30$	48	12%
B. $x = (44-14)/3$	85	21%
C. $x + 2x + 14 = 44$*	267	64%
D. $x + y + z = 44$	5	1%
Not Attempted	9	2%
Total	414	100%

* Correct option

Table 1 shows students responses on each option of item 1. The overall correct response was 64%. The option "B" was selected by 21% of students. It seems the focus of these students is on solving equations rather than describing situations by available information. They are interested in the arithmetic process which leads to the answer. The other students' don't know the power of algebra which can help them in describing and solving more difficult situations.

Table 2: *Students' response on item 2*

Options	Frequency	Percentage
A. $a + b + c = 180$	30	7%
B. $a + 2a + 103 = 180$*	259	63%
C. $a = (180 - 103)/3$	84	20%
D. $180a - 103a = 77$	29	7%
Not Attempted	12	3%
Total	414	100%

* Correct option

Table 2 indicates 63% of students were correct on item 2. 20% of students selected option "C" who has focused on solving equation they have selected the option which is more arithmetic and leads towards the solution of an equation.

Table 3: *Students' response on item 3*

Option	Frequency	Frequency
A. $x + 85 + x + 125 = 1410$	61	15%
B. $x + (x + 85) + (x + 125) = 1410$*	151	36%
C. $x + 85 + 125 = 1410$	92	22%
D. $x = (180-103)/3$	95	23%
Not Attempted	15	4%
Total	414	100%

* Correct option

Table 3 shows item 3 appears difficult 36% of students were correct on it. The option "D" and "C" were two other options which seems an answer to 23% of students. The students selected option "D" as the correct option was inclined towards the arithmetic process they want to get answer. This may be what they are practicing in the classroom or the statement of the item is not clear to them. Teachers need to pay attention to students this problem. The option "C" was selected by 22% of students as a correct option and these students miss to add the changes that occurred on each day.

Table 4: *Students' response on item 4*

Options	Frequency	Percentage
A. $3x – 10 = 100$	65	16%
B. $3x + (x – 10) = 100$	148	36%
C. $x = (100 + 10)/5$	47	11%
D. $x + 3x + (x – 10) = 100$*	138	33%
Not Attempted	16	4%
Total	414	100%

* Correct option

The values in Table 4 indicate 33% of students selected option "D" which is correct and 36% of students selected option "B". It seems the students who selected option "B" have correctly designed each component according to the given reference but miss to include the reference term itself while formulating the equation. The other group of 16% of students who have selected option "A" may have a problem in understanding the concept of more or less and they have missed the changes that occurred.

The instrument consists of four items and every item has four options. Students' have to select one option which is correct for them. Students have selected different options as is evidenced by Table 1 to Table 4. Students' way of responding to each item shows their response pattern. Although students can respond in 256 ways if they select options randomly for all four items they have not responded on all items. 394 students' have responded on all four options of all items.

Table 5 indicates students' response patterns based on 394 students responses who have responded on all four options of items and students not attempted responses were not included in the analysis of Table 5.

Table 5: *Students' response pattern on items 1, 2, 3, 4*

Patterns	1	2	3	4	Freq	Ratio
Experts	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + (x + 85) + (x + 125) = 1410$	$x + 3x + (x – 10) = 100$	65	42.2
Pattern 1	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + (x + 85) + (x + 125) = 1410$	$3x + (x – 10) = 100$	33	21.4
Pattern 2	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + 85 + 125 = 1410$	$3x + (x – 10) = 100$	21	13.6
Pattern 3	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + 85 + x + 125 = 1410$	$3x + (x – 10) = 100$	15	9.7
Pattern 4	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + 85 + 125 = 1410$	$x + 3x + (x – 10) = 100$	12	7.8
Pattern 5	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x = (1410 - 85 - 125)/3$	$3x + (x – 10) = 100$	12	7.8
Pattern 6	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + 85 + 125 = 1410$	$3x – 10 = 100$	12	7.8
Pattern 7	$x = (44 - 14)/3$	$a = (180 - 103)/3$	$x = (1410 - 85 - 125)/3$	$x = (100 + 10)/5$	11	7.1
Pattern 8	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + 85 + x + 125 = 1410$	$3x – 10 = 100$	9	5.8
Pattern 9	$x + 2x + 14 = 44$	$a + 2a + 103 = 180$	$x + 85 + x + 125 = 1410$	$x + 3x + (x – 10) = 100$	8	5.2
Pattern 10	$x = (44 - 14)/3$	$a = (180 - 103)/3$	$x = (1410 - 85 - 125)/3$	$x + 3x + (x – 10) = 100$	7	4.5

Table 5 indicates students' response patterns for each item. The frequency of each response pattern shows its occurrence in data and ratio is the relationship between the observed and expected frequency of a response when selected at random. The 10 most common response patterns are presented in Table 5.

Discussion

Student's responses for each item were separately analyzed in the above tables. Table 5 showed the overall response patterns and thinking style of students. The 'Expert' was the most common response pattern. 65 students (15.7% of the total sample) showed this response pattern and they were correct on all items. They have a clear understanding of the information presented and the required task.

33 students showed response pattern 1 they were correct on 1, 2, and 3 item and incorrect on item 4 where they selected option "B" instead of "D". These students miss the reference term in item 4. The ratio of this response pattern was 21. It seems these students have a missing reference term misconception.

The students showed response patterns 2, 4, 5, and 6 were correct on items 1, 2 but incorrect on items 3 and 4 it shows their scattered thinking about the options of items 3 and 4. The students displayed response pattern 3 miss the reference term in both items (3 and 4). It shows they have a missing reference term misconception. The group of students showing response pattern 7 have focus on the arithmetic process while responding on all four items. It showed they have arithmetic process misconception. These students' have focused on getting answers instead of writing an equation.

It supports the findings of MacGregor and Stacey (1995). The reason may be they practice the same in their classes. Teachers need to address students with this response pattern. It could be said other way and support the findings of Adu, Assuah, and Asiedu-Addo (2015) that students cannot comprehend and interpret the information given in the problem. These findings also support the findings of Uçar Sarımanoğlu (2019) as it was found that because of lack of knowledge about arithmetical operations student has inability for making connections between variables. They believe that variables are used as numbers.

The students displayed response pattern 10 focus on the option which reflect their 'arithmetic process' misconception for item 1, 2, and 3, and they have selected the correct option for item 4. In other response patterns, students' way of responding is not consistent they showed one style of thinking on item 3 and different on item 4. These types of responses can be identified as careless responses.

Conclusion

Writing equation from the given information is a mathematical skill. It is a way of translating words into symbols. The students' success depends on writing an equation correctly as it leads to a correct solution to the problem. If some information is missed or interpreted wrongly it leads to an incorrect solution. The finding helps to conclude that only 16% of students have a clear understanding and can write correct equations the others have "arithmetic process" misconception. The findings further indicate students have a "missing reference term" misconception. The data does not confirm other predicted misconceptions "pro-numeral as unit", "additive nature" and "changes" instead it shows some careless responses.

It is therefore recommended to teachers focus on students "arithmetic process" and "missing reference term" misconceptions and tries to have more practice about writing equation instead of solving equations.

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