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Changes in Students' Beliefs: A Case of Mathematics

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Abstract

Investigation of change in 8th grade students' beliefs about mathematics in Punjab through the intervention of collaborative teaching (CT) was the main purpose of the study. Semi-structured interviews were conducted at the start, middle, and end of intervention. The researchers developed an interview protocol and validated it through expert opinion and piloting. The data were analyzed using descriptive coding, percentages and line graph. A clear positive shift was found in the beliefs of students occurred about mathematics in collaborative settings of teaching.

Keywords: collaborative teaching, mathematics, beliefs, Pakistan

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Introduction

Helping students in achieving learning outcomes of mathematics is one of the core objectives of school educators. Studies in the field of mathematics have shown that one's beliefs whether about oneself or about a particular subject significantly impact the academic achievement, cognition and learning behavior (Krosnick, 2007). Perkins, Adams, Pollock, Finkelstein, and Wieman (n.d.) and Ragland (2011) have found strong positive relationship between students' beliefs and their conceptual ability of the particular subject. Cobb (1986) stated that beliefs are very important factor which direct human behavior and affect the perceptual understanding. So in order to improve the process of students' learning of mathematics, the beliefs of students about the nature of mathematics and other related aspects of mathematics, are the two main components which always have been the focus of attention of mathematics educators.

According to Cobb (1986) beliefs are the personal assumptions of an individual about the nature of any phenomena or reality. Likewise in the words of Kloosterman, Raymond, and Emenaker (1996), as cited by Ragland (2011), beliefs are "the personal assumptions from which individuals make decisions about the actions they will undertake" Beliefs are considered to be those assumptions and suppositions about things, people or concepts which are held by an individual or group as to be true (Anderson & Silva, 2009).

The affective domain of McLeod was elaborated and re-conceptualized by Ragland (2011) who investigated the students' beliefs in two specific aspects i.e., mathematics and its teaching. The findings showed the mathematical beliefs of the students stating it as interesting, difficult, boring, and useful and memorization demanding subject, etc. He further found that majority of the beliefs about mathematics are strongly related with students learning and achievement in mathematics. Taylor (2009) also suggested that instruction and particular curricula can cause change in students' beliefs and make their performance better, even in short time span.

Many instructional approaches are used by the mathematics' teachers in the class rooms. Countries like USA, UK, China, Australia, Canada and other developed countries are using collaborative teaching model in delivering instructions and many studies from these countries have shown positive effect of collaborative teaching on students' academic achievement (Mcduffe, Scruggs, & Mastropieri, 2007). Cook and Friend (1995) as cited in Glaeser (n.d.) defined collaborative teaching as "it is a style

of interaction between at least two co-equal parties voluntarily engaged in shared decision making as they work toward a common goal." Moreover, it is a process where two or more teachers plan, present, and assess classroom instruction. Their role varies based on the lesson activities and teachers' specific strengths in instructions. CT shows fruitful results when it is carried out with vigilant planning and considerate decision making. CT may change the beliefs of students about mathematics in the context of Pakistan.

Objective of the Study

Keeping in view the importance of CT as a fruitful instructional approach the objective of this study was to investigate the changes in students' beliefs about mathematics through collaborative teaching settings.

Methodology

An experimental research design, Solomon Four Group, was applied to investigate students' beliefs about mathematics through CT. The experiment was conducted in a public school of district Sargodha of Punjab province. Researchers faced many difficulties in the selection of a public school such as obtaining consent from the headmaster, number of enrolled students required for the experiment, and availability of the equal qualification of the teachers.

Semi-structured interviews were conducted from the students participating in the experiment as an experimental group. Bryman (2004) viewed that use of semi-structured interviews to explore any idea is useful. These interviews were conducted face to face with subjects individually because subjects sometime do not express the information due to uncertainty of confidentiality (Cohen, Manion, & Morrison, 2007). The interviews with the students were conducted three times during the study i.e. start, middle, and at the end of the intervention of collaborative teaching. The total number of students in the experimental group was 59. Due to students' absentees and time constraints, only 45 students were interviewed at the start of experiment. Same students were interviewed in the middle and end of intervention. All interviews were conducted personally by the researchers.

Students were assured of the secrecy of the information provided by them and that it will not be shared with their mathematics teachers, parents and the headmaster. Interviews were recorded by videotaping (Creswell, 2002) as Fraenkel and Wallen (2006) argued that researcher can't avail what he wants from interview if he does not capture what the interviewee actually says. An interview protocol,

consisting of six questions about students' mathematical beliefs in the collaborative teaching setting, was developed by the researchers. It was validated through expert opinion. The three experts (one bilingual and two Subject matter experts (SMEs)) gave their opinion on beliefs included in the protocol and about the language used in the protocol. The interviews were conducted in Urdu, the National language of Pakistan. Therefore, the researchers first translated the interview protocol into Urdu language. The protocol was piloted on fifteen students studying mathematics at 8^{th} grade. The approximate average time for each interview was five minutes. The data collected through interviews were first transcribed and translated from Urdu to English language. The translation was also critically reviewed by two language experts. They highlighted some grammatical mistakes in the translated version and did some rephrasing.

Results

The findings of the study about the changes in beliefs of students about mathematics as a subject have been presented in Table 1 showing progressive transferal in the beliefs of students at the start, middle, and end of CT intervention. It shows that the students' belief changed with the progression of the experiment. The interview schedule was designed around six belief themes. The proceeding section presents the description of change in students' beliefs under each of the six themes in terms of narrations, qualitative data, and quantitative data by students.

Difficulty Level of Mathematics Subject

Table 1 shows that at the start of experiment, 58% of the students believed that mathematics is a difficult subject for them. This percentage declined to 46% in the middle and 23.5% at the end of the intervention. In the beginning of the experiment, one of the students said, "It seems to be a difficult subject to me. Sometimes I do understand questions but most of the time I can't." In the middle of experiment he added to his earlier comment by attributing the difficulty of mathematics to the language problem. He claimed that "I do not understand the content written in English. If the mathematics is written and taught in Urdu then it might prove an easy subject for me." He further said that "I feel difficulty in algebra, especially with word problems. I can't understand the questions written in English." Likewise, another student said, "I like to face the difficulties. It is a difficult subject that is why I enjoy it." He also ascribed difficulties in the subject of mathematics with teaching of his mathematics teachers. He contended, "I feel it to be a difficult subject because of the difficult long questions. Our mathematic teachers do not teach us well. They mostly

leave the long and difficult questions." But at the end of intervention there was change in that students' belief about mathematics difficulty. He stated, "I feel it a little bit easy now; because now I have started to understand the concepts and I practice it a lot at home, also."

Similarly, one student believed that "mathematics is a difficult subject for me and I feel difficulty in algebra." He further shared his experience in these words: "The speed of mathematics teacher's writing on the chalk board is very fast. I cannot copy the content sharply. The teacher speedily solves the questions and clears the chalk board." At the end of experiment, the same student expressed his belief in these words, "I have no issue with learning of mathematics. I study it for two hours daily at home; it has become an easy subject for me."

Mathematics as an Interesting Subject

In the beginning of the intervention, 55% of the students had the belief that "it is not a boring subject." The percentage of this belief of students rose to 58% in the middle of the intervention; and reached to 80.4% at the end of intervention and students were found to have belief that mathematics is an interesting subject.

Before the intervention, one of the students said, It is boring subject for me especially Algebra." In the middle of experiment he said that I feel ease with Algebra section. It has become interesting for me due to performing activities to solve algebraic problems. It was too boring for me before." His belief changed at the end of experiment and he stated that "It is an interesting subject for me and I study it at least three hours, at home."

Similarly, another student said, "It is boring subject for me because of its difficulty." At the end of experiment he shared his belief in these words "Now, I can understand mathematics well which I could not understand previously. Now, it has become an easy subject for me. I don't get bored in the presence of two teachers in mathematics classroom and spend good time there."

Usefulness of Mathematics

The third belief investigated was about the usefulness of mathematics in everyday life. At the start of the intervention, surprisingly, 51.6% of students believed that mathematics is useful and employed in everyday life. This belief was speedily changed in the middle of the experiment with a percentage of 96%. Almost all the students i.e., 98%, had change to this belief by end of intervention. Most of the

students said that mathematics is only used in counting; they used the Urdu, a national language word "Hisab Kitab." Before intervention, one student believed, "There is no other use of mathematics except counting." In the middle of experiment he said that "This room in which we are sitting is made using mathematical concepts e.g. the front wall is a rectangular shape. I think it is not limited to everyday life." He defined that, "Yes. It is used in many things. It is used in shops; it is used in schools; it is used in houses; it is used in books; and it is used in banks and offices."

Relation among Mathematical Concepts

Fewer students (6.4%) were found with the belief that mathematical topics are related to each other when asked at the start of the experiment. The percentage of students rose to 40.6% in the middle of the experiment, and at the end of the experiment the percentage of the students who believed that mathematical topics and concepts are related to each other, was found to be 66.7%. The researchers also felt that before the intervention, those students who thought that the topics are related with each other had no idea about the way they are related in. The students appeared to be confused on the connection among the mathematical topics. For example, one student said that "Yes, I think the topics of mathematics are related with each other." When the researchers further explored this by asking that how they are related with each other? He replied, "I don't know much about it." But at the end of intervention that student said that "operations of algebra are connected with each other such as plus and multiplication."

Memorizing Formulae

Students' beliefs about the necessity of memorizing formulae to solve mathematical questions were changed. At the end of the experiment the percentage of those students who had this belief was 31.3%. This percentage was 96.7% in the beginning of the intervention. One student shared that "It will be difficult for me to solve mathematical questions without memorizing the formulae." In the middle of the experiment the same student said that "In some topics it is necessary to memorize the formulae first, but not always." similarly, at the end of intervention that student argued "if someone gets understanding of the mathematical concept then there is no need to memorize the formulae." In the beginning of the intervention, another student had the belief that "I have always been cramming the formulae before solving mathematical questions." The belief of that student was changed positively at the end of experiment and he stated that "Understanding is more important than memorizing the formulae because sometimes when you forget the formula even then you can solve the questions through understanding of the concept involved." Similarly, one

student said that "due to too many formulae of mathematics it is difficult for me to memorize them." Another student argued that an "Individual needs a good memory for better mathematics learning."

Studying Mathematics to Solve Mathematical Problems

In the beginning, most of the students (80%) had the belief that the purpose of studying mathematics is to find solutions to mathematical problems. This belief of the students was found to be changed in the middle of the experiment and this percentage was decreased to 62%. At the end of the experiment when students were again asked about their belief, only 37.5% of the students were having the belief that there was no purpose of mathematics other than finding the solutions to mathematical problems.

One student at the start of the intervention had the belief that "mathematics has only one purpose i.e. to solve the questions. This is what I always did in my whole academic carrier." However, at the end of the intervention the same student had the belief that "It has many purposes like preparing good problem solver and enhancing intelligence."

Another student believed that "The only purpose of mathematics is to pass exams with good grades." But his belief changed at the end of intervention and he attributed the purpose of learning mathematics for development of the ability to deal with everyday problems."

Table 1

Changes in Beliefs of Students about Mathematics

Beliefs of Students	Before	During	End
	Yes	Yes	Yes
		In Percent	
1) Math is an easy subject	42	54	76.5
2) It is an interesting subject	55	58	80.4
3) It is useful in everyday life	51.6	96	98
4) Its concepts are related to each other	6.4	40.6	66.7
5) one must memorize the formulae first to solve math problems	96.7	76	31.3
6) Math is used to find the solutions of mathematical problems only	80	62	37.5

Changes in beliefs of 8th grade mathematics students are also shown in fig. 1. The steeper declining lines show rapid positive change in the beliefs of students during the experiment e.g. usefulness of mathematics in everyday life and relationship among mathematical concepts (see fig. 1).

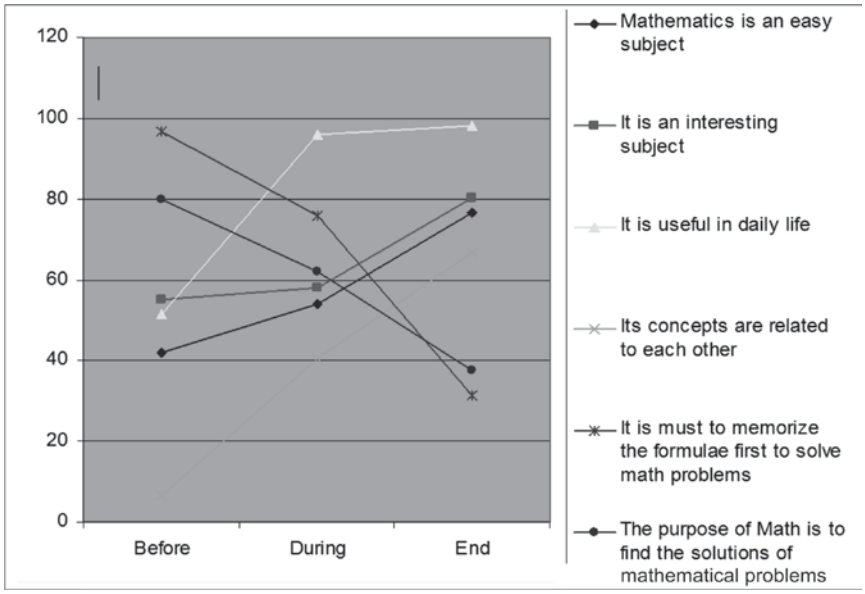

Figure 1 **Students' beliefs about mathematics**

Discussion

It is evident from literature that specific teaching practices can change students' beliefs about mathematics (Taylor, 2009). In this study, the researchers conducted semi-structured interviews of the students to explore their beliefs about mathematic in collaborative settings in the beginning, during, and at the end of experiment. CT positively changed their beliefs about mathematics.

The proficiency to solve mathematical problems is needed to develop and improve: generic ability to solve real life problems, critical thinking skills, understanding of mathematical concepts, and mathematical interests and curiosity (NCTM, 1991). Some of the tools used in teaching to develop such mathematical proficiency include brainstorming, cause and effect diagrams, flowcharting, decision matrix, heuristics, and algorithms but in Pakistan, mathematics students are practicing formula driven rather than concept driven mathematics. Similarly, mathematics teachers teach and require students to memorize the formulae to solve mathematical problems i.e. deductive method. The researchers also found that even at the end of

the experiment 31% of students held the belief that it is necessary to memorize the mathematical formulae first to solve mathematical problems. Similar were the findings by Kloosterman (2003) and Dossey, Mullis, Lindquist, and Chambers (1988). We are living in the world of mathematics. Every economist, scientist, businessperson, accountant, engineer, mechanic, farmer, shopkeeper and even street hawker requires and need to use mathematical concepts in their fields. Therefore, students of mathematics are needed to be aware of the use of mathematical concepts in everyday life. In Pakistan students and teachers focus on getting good grades in exams and do not relate the concepts with everyday life. The researchers found that nearly half of the students believed that there is limited use of mathematics in everyday life. Kloosterman et al. (1996) also found that students' believe in the limited use of mathematics in everyday life. This study found that CT entirely changed students' belief about learning of mathematics in general and in collaborative teaching settings.

This study opens new dimensions towards adaptation of new teaching approaches. It gives a move forward to the researchers and academicians in the field of math education. For the enhancement of critical thinking and better comprehensive understanding of mathematics students in Pakistan, different mathematics teaching methods like inductive method, question answer, analytical, drill and practice with different collaborative settings may be used in class rooms. These methods may change the students' mathematics learning approach from memorization to understanding and make them better learners.

Suggestions and Recommendations

1. This research study found that CT changed the beliefs of the students about mathematics. Having strong beliefs about learning does impact one's academic achievement, both cognitive and behavior (Krosnick, 2007; Ragland, 2011). Therefore, it is recommended that mathematics teachers may practice CT in classroom in order to modify students' attitude towards mathematics which may result in enhancing their academic achievement.
2. In the beginning, 93.6 % students were having the belief that mathematics concepts are not related to each other which were reduced to 34.3% by the end of the intervention. So, mathematics teachers may put more effort to relate various mathematical concepts for better and comprehensive understanding.

3. 96.7% students believed that it is necessary to memorize the mathematical formulae first in order to solve mathematical problems in the start of intervention which were reduced to 31.3% by the end. Therefore, it is recommended that mathematics teachers may use inductive method in their teaching of mathematics to bring change in students' mathematics learning approach from memorization to understanding.
4. To explore the usefulness of this method in wider context further research should be conducted on female students of public schools, private schools' students, and on different grade levels.

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