Multiple Arc Network Model for Scheduling in a Single-stage, Multi-item Compatible Process

Dr. BOKKASAM SASIDHAR
Professor at Quantitative Analysis Department, College of Business Administration
King Saud University, Riyadh, Kingdom of Saudi Arabia
Email: bbokkasam@ksu.edu.sa

Abstract

The problem of scheduling a given set of equipments in a single-stage, multi-item compatible environment, with the objective of maximizing capacity utilization is formulated as a maximal flow problem in a Multiple Arc Network (MAN). The production is usually planned against customers' orders, and different customers are assigned different priorities. The model aims to provide optimal production schedule with an objective of maximizing capacity utilization, so that the customer-wise delivery schedules are fulfilled, keeping in view the customer priorities. Algorithms have been presented for solving the MAN formulation of the production planning with customer priorities. The suitability of the algorithms has been demonstrated with small examples.

Key Words: *Scheduling, Maximal Flow Problem, Multiple Arc Network Model, Optimization.*

Introduction

A Multiple Arc Network (MAN) model of production planning in a steel mill is presented in Sasidhar & Achary (1991). A decision support system for production planning has been presented in Gunter Schmidt (19912, 1996). Considering decision support system as a tool where executive's judgment can be included along with the mathematical tool kit of the management scientist, the need to include problem management as an integral component of the decision support system for scheduling problems is indicated in Klaus Ecker et al. (1997).

Decision networks, which involve a sequence of decisions, is gaining popularity among the management scientists, as elaborated by Saksena (1982, 1985). Techniques of operations research are being used in production planning and scheduling (Charles, 2002). A stochastic production planning model for a multi-period, multi-product system, where the lead time to produce a product may be random is developed in Steven Hackman et al. (2002). A comparative study of algorithms for the flow shop scheduling problem is presented in Shiqiang Liu & Ong (2002). A review of the directions in deterministic machine scheduling theory is given in Blazewicz et al. (1998). The problem of allocating the limited capital resources to the various stages of a multistage production system, in order to improve the yield of the production stages and, at the same time minimize the annual cost is considered in George & Andreas (2002). A heuristic procedure to derive non-permutation schedules from a given permutation schedule is given in Pugazhendhi et al. (2003). The application of artificial neural networks to the problem of job shop scheduling with a scope of a deterministic time-varying demand pattern over a fixed planning horizon is given in Shan Feng et al. (2003). Since the pioneering work of Ford & Fulkerson (2010) in 1962, the use of network models and algorithms have proved to be particularly successful in different application areas.

In this paper, a MAN model for production planning in a single-machine, multi-item compatible processing environment is considered. The model aims to provide an optimal production plan, with the objective of maximizing the capacity utilization.

The Problem

The problem of finding optimal schedule for each equipment in a production process is considered, which consists of a single stage of manufacturing and which can handle different types of products, where changeover for handling one type of product to the other type incurs certain costs. For example, we can consider the production process in an engineering workshop, which can process different products using similar equipment, such as a lathe, by suitably changing the set up. A similar situation can be encountered in a steel mill, which can produce different grades and sections of steel products, by suitably changing the rolls and sequences. The machine capacity is determined by the upper limit for the quantity that can be processed for each of the products in a set up. The changeover costs increase with the number of set ups and hence to minimize the costs associated with the product changeover, the planning should be such that similar types of products should be processed successively so that the total number of changeovers and in turn the associated set up costs are minimized. The problem of cost minimization is equivalent to the problem of minimizing the number of set ups or equivalently maximizing the capacity utilization in between every set up or maximizing the total capacity utilization. Further, certain planning procedures call for priority planning where different customers are assigned different priorities.

The problem of production planning in such a situation can be formulated into MAN model and can be solved sequentially using the following algorithms:
Algorithm 1: Algorithm for maximizing flow along a MAN.
Algorithm 2: Algorithm for maximizing flow along a MAN with priority arcs.

Multiple Arc Network (MAN)

Consider the flow network N=(s,t,V,A,b) with the digraph (V,A) together with a source sεV with 0 indegree, a sink or terminal tεV with 0 outdgree with |V| vertices or nodes. A has as elements subsets of V of cardinality two called arcs together with the arc number i and with |A| arcs, i=0,1,2,3,….,n where i denotes the arc number.

We use the following notations:

(x,i,y) = ith directed arc from the vertex x to the vertex y.
b(x,i,y) ε Z+ for each (x,i,y)εA is the bound or capacity of the arc (x,i,y).
Ai(x) = {yεV/(x,i,y)εA}
Bi(x) = {yεV/(y,i,x)εA}

We assume that arcs of the form (x,i,x) for xεV do not exist in A.

Define $A(x) = \bigcup_i Ai(x)$ and $B(x) = \bigcup_i Bi(x)$

A flow f in N is a vector in R|A|, one component f(x,i,y) for each arc (x,i,y)εA.

Let $f(x,-,y) = \sum_i f(x,i,y)$

Definition

A static flow of value v from source s to sink t in the network N is a function f from A to non-negative reals such that the following equations and inequalities are satisfied.

$$\sum_{y \varepsilon A(x)} f(x,-,y) - \sum_{y \varepsilon B(x)} f(y,-,x) = \begin{cases} v(f) & \text{if } x=s \\ 0 & \text{if } x \neq s,t \\ -v(f) & \text{if } x=t \end{cases} \qquad (1)$$

$$f(x,i,y) \leq b(x,i,y) \qquad \forall\ (x,i,y)\ \varepsilon A \qquad (2)$$

$$f(x,i,y) \geq 0 \qquad \forall\ (x,i,y)\ \varepsilon A \qquad (3)$$

The static flow problem is to maximize the flow from s to t such that the flow f satisfies (1), (2), and (3).

Man with Priority Arcs

Consider the MAN N=(s,t,V,A,b). The flow in the network N is characterized by permeability of flow from source s to sink t only through arcs with like priority numbers. For any arc (x,i,y) εA, let the arc number i denote the priority number as well.

Methodology

The problem of production planning can be formulated into MAN model. We consider the following notations:

Customers	Ci	i=1,2,…..n
Products	Sk	k=1,2,…..l
Equipments	Er	r=1,2,…..p
Demand for the product Sk by customer Ci	Dki	
Delivery time for the product Sk for customer Ci	T(k,i)	
Capacity of processing product Sk by equipment Er	Akr	
Processing time for the product Sk of customer Ci	tki	

Methodology for Determining the Periods for Process Commencement

Consider the MAN N=(s,t,V,A,b) with v=|V| vertices, representing the processing stage (equipments) and products. The arcs of A include all possible customer-wise orders and all possible products that can be processed on equipments. This network will have a specific structure for a given processing unit. This network will be referred to as the PROCESSNET. A typical PROCESSNET for a process consisting of "p" similar equipments capable of processing "l" products, in a single stage, is shown in Figure 1. Periods for the process commencement is given by To=T(k,i)-tki.

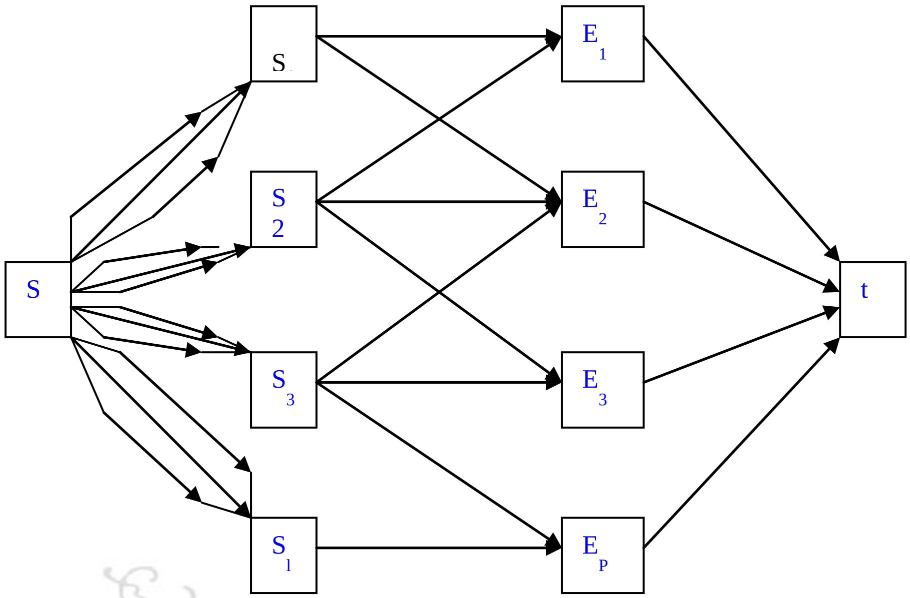

Figure 1- A typical Processnet

Methodology for Maximizing Flow Along A Man

The labeling algorithm for maximizing flow along a MAN is given below:

Algorithm 1

Step1: Start with flow f ≡ 0 as the feasible flow with v(f)=v(0)=0.
Go to Routine A.

Routine A

Step A1: Label s as (-,-, ∞).

Step A2: Take any vertex x from the set of vertices which are labeled and unscanned through.

Step A3: If (x,i,y) εA and f(x,i,y)<b(x,i,y), label y as (x+,i,ε(y)), where
ε(y)=Min.{ ε(x), (b(x,i,y)-f(x,i,y))}
If (y,i,x) εA and f(y,i,x)>0, label y as (x-,i,ε(y)), where
ε(y)=Min.{ ε(x), f(y,i,x)}
Repeat this step until no further vertices can be labeled or t is labeled.

StepA4: If t is not labeled, it suggests that there is no flow augmenting path from s to t, and hence f is the maximal flow.
If t is labeled, go to Routine B.

Routine B

StepB1: If t has label (x+,i,ε(y)), determine f'(x,i,t) = f(x,i,t)+ ε(t).
If t has label (x-,i,ε(y)), determine f'(t,i,x) = f(t,i,x)- ε(t).

Step B2: In general, at vertex xi with path of labeled vertices as
x0=s, x1, x2, ….., xi,…..,xn, xn+1=t and if xi has label (x+i-1,i,ε(xi)), determine
f'(xi-1,i,xi) = f(xi-1,i,xi) + ε(t) for all (xj,i,xj-1) backward arcs and (xj-1,i,xj) forward arcs.
If xi has label (x-i-1,i,ε(xi)), determine
f'(xi,i,xi-1) = f(xi,i,xi-1) - ε(t) for all (xj,i,xj-1) backward arcs and (xj-1,i,xj) forward arcs.

Step B3: If i>1, go to xi-1, repeat Step B2.
If i=1, repeat step B2 and go to step 2.

Step 2: Erase all labeling. Go to Routine A with flow f'.

Methodology for Maximizing Flow along a Man with Priority Arcs

The labeling algorithm for finding a maximal flow along a MAN with priority arcs, where arc number i denotes the priority number as well, is given below:

Algorithm 2

Step 1: Set i=1.
Start with a feasible flow $f \equiv 0$ and with v(f)=v(0)=0.

Step 2: Using Routines A and B of Algorithm 1, label arcs with ith priority arcs only.

Step 3: If i<n, set i=i+1. Go to step 2.
If i=n, stop.

The Scheduling Procedure

The procedure for planning and scheduling a single-stage, multi-item compatible process consists of the sequential applications of the above algorithms. For the application of the MAN with priority arcs model for scheduling the process, MAN=(s,t,V,A,b) is considered. This network will have a specific structure and this network will be referred to as the PROCESSNET. Here,

b(s,i,Sk)=Dki
b(Sk,-,Er)=Akr
For all other (x,i,y) εN, b(x,i,y)= ∞.

The scheduling algorithm is explained below:

Step 1: Consider the PROCESSNET.
Periods for the process commencement is given by To=T(k,i)-tki.
Find { Dki $\forall$ k,i such that T 0 is the same}

Step 2: Using Algorithm 2, determine the optimal flow.

Illustrative Examples

Example 1: (Scheduling when order books are full)

Consider a workshop having two similar lathes E1 and E2 which are being used for manufacturing three types of roller sets S1, S2 and S3. Let the time taken for processing each of the roller sets S1, S2 and S3 using the lathes E1 and E2 (tki) be 10, 15 and 20 weeks respectively. Assume that both the lathes are capable of manufacturing any of the products and the capacity of each of the lathes (Akr) is 400 units for each of the product during the time horizon. Consider the order position for the two customers C1 and C2 (in the same order of customer priorities) for the three roller sets S1, S2 and S3 to be as shown in Table 1. The delivery time (week number) for the product Sk for customer Ci, that is T(k,i), is given in the brackets.

Note that changeover for handling one type of roller to the other type incurs certain time and associated costs. Since the changeover costs increase with the number of set ups, to minimize the costs associated with the set ups, similar types of products should be processed successively so that the total number of changeovers are minimized. This problem is equivalent to the problem of maximizing the total capacity utilization.

Let us consider the problem of scheduling the workshop during this week (To), say, week number 10. This example illustrates the steps in scheduling for the current period.

Table 1 - Order position and delivery schedules for the two customers C1 and C2

Customer/Product	S1	S2	S3
C1	600	500	300
(20)	(25)	(30)	
C2	400	300	700
(20)	(25)	(30)	

The PROCESSNET for the workshop is as shown in Figure 2.

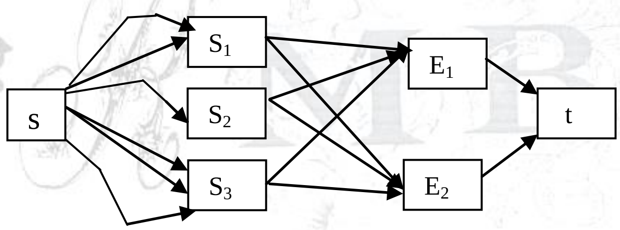

Figure 2 - PROCESSNET for the Workshop.

Applying Step 1 by considering the processing times tkr along the arcs (Sk,Er) and applying the methodology for determining the periods for process commencement as in 4.1, the demand for the product Sk by customer Ci, viz., Dki for the week number 10 will be follows:

D11=600, D12=400, D21=500, D22=300, D31=300 and D32=700.

As Step 2, applying the Algorithm 2 of maximizing flow for the same multiple arc PROCESSNET the optimal schedule is arrived at. The capacity along the ith arc from s to Sk is the order quantity Dki for the product Sk by the customer Ci, with priority i. The capacity of the arc (Sk, Er) is considered as Akr which is the capacity of processing product Sk by equipment Er. The capacities of the arcs (Er,t) are considered as infinity. Applying Algorithm 2 of maximizing flow along the MAN with priority arcs, the following maximal flow is obtained:

f* = (600, 200, 500, 300, 300, 500, 400, 400, 400, 400, 400, 400, 1400, 1000)

Thus, the optimal quantities to be supplied to the customers and the corresponding unfulfilled demands (as shown in the brackets) are shown in Table 2. Observe that there are 200 units each of unfulfilled quantities for customer C2 for the products S1 and S2. These quantities could be rescheduled with customer's concurrence.

Table 2 - Optimal supply quantities and the unfulfilled demands for the two customers C1 and C2

Customer/Product	S1	S2	S3
C1	600 (0)	500 (0)	300 (0)
C2	200 (200)	300 (0)	500 (200)

Example 2: (Scheduling when order books are not full)

As a second example, the following illustration demonstrates the applicability of the scheduling procedure in situations when the orders books are not full. The procedure provides the optimal schedule together with the buffer capacities available. This information provides guidelines to the marketing personnel for procuring immediate orders.

Consider the order position and delivery schedules for the two customers C1 and C2 (in the same order of customer priorities) for the three roller sets S1, S2 and S3 be as shown in Table 3. The delivery time (week number) for the product Sk for customer Ci, that is T(k,i), is given in the brackets.

Let us consider the problem of scheduling the workshop during this week (To), say, week number 10. This example illustrates the steps in scheduling for the current period.

Table 3- Order position and delivery schedules for the two customers C1 and C2

Customer/Product	S1	S2	S3
C1	100 (20)	250 (25)	500 (30)
C2	200 (20)	300 (25)	300 (30)

Using the same PROCESSNET for the workshop as in Figure 2 and applying Step 1, the demand for the product Sk by customer Ci, viz., Dki for the this example for the week number 10 will be follows:

D11=100, D12=200, D21=250, D22=300, D31=500 and D32=300.

As Step 2, by applying Algorithm 2 of maximizing flow along the MAN with priority arcs, the following maximal flow is obtained:
f* = (100, 200, 250, 300, 500, 300, 300, 0, 400, 150, 400, 400, 1100, 550)

Table 4 presents the optimal quantities to be supplied to the customers.

Table 4- Optimal supply quantities for the two customers c1 and c2

Customer/Product	S1	S2	S3
C1	100	250	500
C2	200	300	300

The capacities, scheduled/utilized capacities and the corresponding spare capacity available for the two equipments are shown in Table 5.

Table 5- Equipment utilization and the spare capacities available

Equipment	Capacity	Scheduled	Spare capacity available
E1	1200	1100	100
E2	1200	550	650

Further Consideration

In case the order books are not full for some of the products in a sequence, we can introduce a dummy customer with least priority having infinite orders, for each product. The scheduling of the orders for the dummy customer brings out the actual product-wise spare capacity. This serves as a guideline to the sales department in procuring further orders for particular products, so that the sequence and in turn the system capacity utilization is maximized.

Conclusion

The methodology of the system described above is in general applicable to all single-stage, multi-item compatible production processes where the process can handle different types of products involving changeover costs. Depending on the layout and process flow of the plant, a tailor-made production planning and scheduling system can be formulated using MAN techniques.

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