SENSITIVITY ANALYSIS OF SELECTED METHODS FOR PREDICTING REFERENCE CROP EVAPOTRANSPIRATION

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A reliable estimate of crop water requirement is one of the key design criteria required for planning, designing and operation of irrigation schemes. This, in turn, requires accurate estimtion of reference crop evapotranspiration for which a variety of equations are widely used. The sensitivity analysis of five selected methods was conducted. The senstivity coefficients were computed by approximating the derivatives by finite differences. A sensitivity coefficient (relative sensitivity)$\Delta ETo/\Delta Xi * Xi/ETo$ was calculated to compare the relaive importance of different input variables. The sensitivity coefficient defines the change in computed ETo due to change in the variable. The mean annual sensitivity coefficients were determined by applying 10 years data (1974-1983) obtained from agrometeorological stations of Thailand. The sensitivity analysis indicates that the Penman method provides more realistic estimates of the importance of the various climatological factors. The Jensen-Haise method is more sensitive to temperature as compared to other methods. Based on temperature, the Jensen-Haise is the most sensitive equation and the Radiation method is the least sensitive. Between these two extremes, in the order of most to least sensitive, are the Blaney-Criddle and the Penman equations. The results also show that there is less variation in the sensitivity coefficient values of temperature in Thailand and high variation for the values of wind velocity.

INTRODUCTION

A reliable estimate of crop water requirements is one of the key design criteria required for planning, designing and operation of irrigation schemes. This, in turn, requires accurate estimation of reference crop evapotranspiration for which a variety of equations are widely used. Sensitivity and variability of equation parameter can be used as a potential method of system identification. McCuen (1974) was one of the first workers to examine the sensitivity of various evaporation models to understand their structure, the effect of variations in meteorological measurement errors. That work has been extended and complemented by Saxten (1975) who used the combination energy budget and aerodynamic approach of Penman (1948), modified to allow for measured net radiation and turbulent diffusion theory in an analytical approach to sensitivity analysis. Beven (1979) performed a similar sensitivity study using Penman-Montieth equation for actual evaporation for grass and forest surface and a range of meteorological conditions within a broadly temperate maritime climatic regime in Britain. Beven found that the sensitivity of aerodynamic and canopy resistance parameters introduced the influence of the vegetation type to the equation. Camillo and Gurney (1984) made a sensitivity analysis of a numerical model for estimating evaporation. In this study, a sensitivity analysis of methods for predicting reference crop evapotranspiration is conducted in relation to the climatic parameters involved and prevailing climatic conditions in a specific region aimed at:

1. Investigating the sensitivity of reference crop evapotranspiration rates to determine error in input data.
2. Finding out which method should be preferred for a larger region with specific climatic conditions.

Selected methods predicting reference crop evapotranspiration: The methods selected for the study were:

a. Blaney-Criddle method (FAO)
b. Radiation method
c. Jensen-Haise method
d. Penman method, and
e. Pan evaporation method.

The complete derivation and description of these methods is given by Doorenbos and Pruitt (1977).

The sensitivity analysis was conducted by two separate but related techniques. One method is to obtain a solution from a set of variables, then increase each variable value step by step while holding all other values constant and note the change of solution. A better approach is to differentiate mathematically the equation under study in order to derive equations for the rate of change of dependent variable with respect to each independent variable. This is a more efficient method but it requires that the equation is mathematically tractable.

Scarborough (1962) shows that sensitivity equations can be developed for function:

$$N = f(\mu_1, \mu_2, \ldots\ldots\ldots\ldots, \mu_n) \qquad (1)$$

This equation denotes any function of several independent quantities μ_1, μ_2, ..., μ_n which are subject to the errors $\Delta\mu_1, \Delta\mu_2$, ..., $\Delta\mu_n$, respectively. These errors in the μ's will cause an error Δ N in the function, according to the relation:

$$N + N = f(\mu_1+\Delta\mu_1, \mu_2+\Delta\mu_2, \ldots \mu_n+\Delta\mu_n) \qquad (2)$$

To find an expression for ΔN, expand the right hand side of equation (2) by Taylor's Theorem, neglecting squares, products and higher powers in the expansion yields:

$$\Delta N = \frac{\partial N}{\partial \mu_1}\Delta\mu_1 + \frac{\partial N}{\partial \mu_2}\Delta\mu_2 + \ldots\ldots\ldots\ldots \qquad (3)$$

If one considers change or error that occurs in only one variable, all other terms would go to zero.

$$\Delta N = \frac{\partial N}{\partial \mu_1}\Delta\mu_1 \qquad (4)$$

Relative change can be defined as:

$$N\varepsilon = \Delta N / N \qquad (5)$$

$$\mu\varepsilon = \Delta\mu / \mu \qquad (6)$$

Substituting the equations (5) and (6) into (4) provides the general solution:

$$\frac{N\varepsilon}{\mu_1\varepsilon} = \frac{\partial N}{\partial \mu_1} * \frac{\mu_1}{N} \qquad (7)$$

$$Si = \frac{\partial N}{\partial \mu_1} \cdot \frac{\mu_1}{N} \qquad (8)$$

The sensitivity coefficient, Si represents that fraction of change in μ that is transmitted to change in N. So, equation (8) for non-dimensional sensitivity coefficient (relative sensitivity) is used for the sensitivity analysis of the selected methods predicting reference crop evapotranspiration.

$$Si = \partial ETo / \partial Xi * Xi / ETo \quad i = 1, ... n \quad (9)$$

Unfortunately, the equations under study are not mathematically tractable, it was, therefore, difficult to find the partial derivatives of ETo with respect to some of the independent variables. Thus, the method of finite differences is used in approximating the partial derivatives:

$$Si = \Delta ETo / \Delta Xi * Xi / ETo \quad (10)$$

RESULTS AND DISCUSSION

The sensitivity coefficients of the climatological parameters used in the estimation of refernce crop evapotranspiration rates provide a means of quantitatively examining the relative influence of change of the climatological parameters on computed evapotranspiration rates. Five selected methods as discussed earlier are used in order to estimate the sensitivity coefficient of various climatological parameters.

Using monthly values of climatological factors (temperature, relative humidity, sunshine duration, wind velocity) for the period 1974-1983, values of sensitivity coefficients of five selected methods were computed. Climatological data from 9 stations in Thailand were used. The resulting mean annual sensitivity coefficients (relative sensitivity) values are given in Tables 1 and 2.

The sensitivity coefficient values of Table 1 indicate that the Penman method is almost equally sensitive to temperature and mean relative humidity. The mean relative humidity is the most sensitive meteorological factor for Nong Plab, U Thong, Tha Phra and Bang Khen. Nong Plab is located in penisular Thailand (South). So, relative humidity may be sensitive due to cloudiness and maritime climate of the station. The Phra is situated in NE of Thailand whereas U Thong is near the border of Burma in the Central Thailand. High vegetation or forests may be responsible for the importance of humidity in these locations. At Bang Khen humidity has very slighlty larger sensitivity value than temperature. As the station is near to the Gulf of Thailand, water vapours from the Gulf may be responsible for this. Temperature is the second most sensitive factor for the four previously mentioned stations.

Temperature is a more sensitive factor than humidity for the stations of Nan, Si Samrong, Ubon Ratchathani, Chai Nat and Kho Hong. Sunshine duration is the third most sensitive factor at all stations with the highest value of 0.45 at Chai Nat. Wind velocity is the least sensitive factor for the Penman method at all 9 stations. Thus, the sensitivity values computed from the Penman method appear to be rational and provide a good indication of the relative effect of the changes in the climatic parameters.

The sensitivity coefficient values obtained from the Radiation method are the most sensitive to mean relative humidity except the stations of Ubon Ratchathani, Chai Nat and U Thong where sunshine duration is more sensitive than humidity. Temperature is less sensitive than humidity and sunshine duration and almost invariable with the location of the stations. This is due to the mathematical structure of the radiation equation where temperature is involved as a weighing factor only. Again, wind velocity is the least sensitive factor at all stations but with a greater variation of sensitivity values among the stations.

The sensitivity coefficients computed with Blaney-Criddle method reflect that temperature, regardless of location is the most sensitive climatic parameter and with the exception of Ubon Ratchathani and Chai

Table 1. Mean annual sensitivity coefficients

Method	Station	Climatic parameters			
		Wind	Sunshine	RH*	Temperature
Penman	Nan	0.082	0.37	-0.52	0.57
	Si Samrong	0.115	0.41	-0.50	0.63
	Tha Phra	0.133	0.40	-0.67	0.60
	Ubon Ratchathani	0.132	0.39	-0.50	0.57
	Chai Nat	0.146	0.45	-0.54	0.63
	U Thong	0.186	0.38	-0.70	0.60
	Bang Khen	0.118	0.39	-0.62	0.61
	Nong Plab	0.137	0.36	-0.70	0.58
	Kho Hong	0.093	0.39	-0.55	0.60
RAD	Nan	0.036	0.52	-0.61	0.34
	Si Samrong	0.055	0.55	-0.62	0.33
	Tha Phra	0.064	0.54	-0.60	0.34
	Ubon Ratchathani	0.054	0.54	-0.49	0.34
	Chai Nat	0.063	0.59	-0.53	0.33
	U Thong	0.076	0.54	-0.53	0.33
	Bang Khen	0.056	0.53	-0.60	0.34
	Nong Plab	0.064	0.51	-0.61	0.34
	Kho Hong	0.046	0.51	-0.60	0.34
BC	Nan	0.055	0.40	-0.46	0.85
	Si Samrong	0.079	0.40	-0.51	0.86
	Tha Phra	0.091	0.40	-0.51	0.85
	Ubon Ratchathani	0.080	0.41	-0.41	0.84
	Chai Nat	0.090	0.44	-0.42	0.84
	U Thong	0.131	0.38	-0.43	0.84
	Bang Khen	0.077	0.39	-0.57	0.87
	Nong Plab	0.097	0.37	-0.50	0.85
	Kho Hong	0.064	0.38	-0.51	0.86
JH	Nan		0.49		0.89
	Si Samrong		0.51		0.90
	Tha Phra		0.51		0.90
	Ubon Ratchathani		0.51		0.90
	Chai Nat		0.51		0:90
	U Thong		0.51		0.90
	Bang Khen		0.49		0.90
	Nong Plab		0.47		0.90
	Kho Hong		0.48		0.90

* = Mean relative humidity (Penman method).
Mean relative humidity (Radiation method).
Minimum relative humidity (Blaney-Criddle method).

Nat, the minimum relative humidity ranks second. This is because the formula mainly depends upon temperature. For the Blaney-Criddle method, wind velocity is also the least but more sensitive than for the radiation and less sensitive than for the Penman method. The Penman method reflects the highest sensitivity coefficient values for wind when compared with the radiation and Blaney-Criddle methods.

plies that the effect of change in the input coefficient level does not change with location. Sunshine duration is less sensitive than temperature for Thailand.

The values in Table 1 also show that based on temperature, the Jensen-Haise is the most sensitive equation and the radiation method is the least sensitive. Between these two extremes, in order of most to least sensitive, are the Blaney-Criddle and the Penman equations. There is some variation in the ranking of sensitivity from one variable to another. An interesting feature is that the least sensitive methods, the radiation and the Penman, are structured on the underlying physical principles than the other methods.

Table 2. Mean annual sensitivity coefficients: Pan evaporation method

Method	Station	Climatic parameters			
		RH mean	Evaporation	Wind	Fetch
Pan Evaporation					
	Nan	0.25	1.00	-0.025	0.014
	Si Samrong	0.26	1.00	-0.041	0.014
	Tha Phra	0.26	1.00	-0.046	0.014
	Ubon Ratchathani	0.24	1.00	-0.040	0.014
Case A	Chai Nat	0.25	1.00	-0.048	0.014
	U Thong	0.25	1.00	-0.070	0.015
	Bang Khen	0.25	1.00	-0.039	0.014
	Nong Plab	0.26	1.00	-0.036	0.014
	Kho Hong	0.26	1.00	0.041	0.014
	Nan	0.28	1.00	-0.024	-0.017
	Si Samrong	0.29	1.00	-0.040	-0.017
	Tha Phra	0.29	1.00	-0.045	-0.017
	Ubon Ratchathani	0.27	1.00	-0.040	-0.017
Case B	Chai Nat	0.28	1.00	-0.047	-0.017
	U Thong	0.28	1.00	-0.047	-0.018
	Bang Khen	0.29	1.00	-0.039	-0.017
	Nong Plab	0.29	1.00	-0.046	-0.017
	Kho Hong	0.29	1.00	-0.031	-0.017

The sensitivity coefficient values derived from the Jensen-Haise method demonstrate the importance of formulating an equation fom consideration of underlying physical principles. For the Jensen-Haise method, the temperature sensitivity coefficient is 0.90 regardless of the geographic location. A constant sensitivity coefficient im-

The high sensitivity coefficient values of any paramater of any method mean that change or error in a particular parameter will transfer a great error or change in ETo estimates. A negative coefficient for relative humidity indicates that a reduction in ETo will result from an increase in relative humidity. Thus, accurate measurements of these variables are essential to accurate prediction of reference crop evapotranspiration.

The sensitivity analysis of the Pan Evaporation method was also conducted. The results are presented in Table 2. By keeping the pan evaporation constant, the sensitivity analysis was conducted to see the effects of RH mean, wind speed and fetch on Kp values. Table 2 shows that RH mean is the most sensitive whereas wind and fetch are the second and third, respectively under the prevailing climatic conditions. The positive values of sensitivity coefficient for RH mean in this method indicate an increase in Kp values with increase in relative humidity for any given wind level and fetch. It is true because high levels of humidity will decrease the advecation and measured pan evaporation will be nearer to ETo.

The negative values of the sensitivity coefficient of wind velocity and fetch (case B) are an indication that Kp values decrease as wind speed and windward distance of dry fallow area increase. The positive values of sensitivity coefficient of fetch for case A show that with increased windward side coefficient of fetch for case A show that with increased widward side distance of green crop, Kp values increase. It mean with increased distance, air becomes cooler, more humid and air temperature is dropped due to transpiration from green crop and the advection effect is low.

The sensitivity coefficient values of mean relative humidity and fetch are almost invariant for different locations of Thailand. But, there is an appreciable change in the values of sensitivity coefficient of wind velocity with geographic location which vary from -0.025 to -0.070 that is 2.8 times of minimum value. Table 2 also indicates that fetch is not so sensitive under humid and calmer climates.

When the pan is located in dry fallow area, mean relative humidity and fetch are 12% and 21% more sensitive in case B as compared to case A. Even with humid climate, a change in pan environment significantly changes the Kp values. These results indicate that Kp (pan coefficient) is very much related to the pan environment such as nearby ground surface and the climate itself. So, the pan environment may influence Kp values very significantly in dry climates when pan is surrounded by dry surface areas. These results suggest that information about the environment is important when estimating the ETo by using the evaporation data of class A pans.

Important conclusions drawn from this study are:

1. The reference crop evapotranspiration rates are more sensitive to temperature, relative humidity and sunshine duration for the area under study.
2. The sensitivity of ETo to the wind velocity is relatively much less significant as compared to other climatological factors.
3. Although, the fetch (windward side distance of green crop or dry fallow) is the least sensitive input parameter to Pan Evaporation method, yet a significant change in the level of sensitivity under two cases (case A and case B) implies that information about the pan environment is essential for accurate prediction of ETo when using class A pan.
4. A less variation in the sensitivity coefficient value among different stations

reflect that almost similar climatic conditions prevail over the entire area under study.

5. Based on results of sensitivity analysis only, a selection of an appropriate method for the area under study with specific climatic conditions is rarely possible. But, it is felt that the Penman equation will be the most approapriate for the area under study.

REFERENCES

Beven, K. 1979. A sensitivity analysis of the Penman-Monteith actual evapotranspiration estimates. J. Hydrol. 44: 169-190.

Camillo, P.J. and R.J. Gurney. 1984. A sensitivity analysis of a numerical model for estimating evapotranspiration. Water Resources Res. 20: 105-112.

Doorenbos, J. and W.O. Pruitt. 1977. Guidelines for predicting crop water requirements. FAO Irrigation and Drainage Paper No. 24.

McCuen, R.H. 1974. A sensitivity and error analysis of procedures used for estimating evaporation. Water Resources Bull. 3: 486-497.

Penman, H.L. 1948. Natural evaporation from open water bare soils and grass. Proc. Royal Soc. Series A, 193: 120-145.

Saxten, K.E. 1975. Sensitivity analysis of the combination of evapotranspiration equation. Agri. Meteorol. 15: 343-353.

Scarborough, J.B. 1962. Numerical Mathematical Analysis. John Hopkins Press, New York, USA.

Schulz, E.F. 1976. A guideline for the application of Jensen-Haise evapotranspiration equation. FAO Watershed Management & Coordination Project, Tehran, Iran.