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LOGICAL THINKING IN MATHEMATICS: A STUDY OF SECONDARY SCHOOL STUDENTS IN PAKISTAN

Abstract

The main aim of this study was to assess performance of grade 9th students in logical thinking. A test of reasoning was administered to a sample of above 500 hundred. The results show differential performance of the students. Item wise performance with background variable as school sector shows that performance of private schools students was significantly better than students of public schools. Similarly male and urban students performed well than female and rural students respectively. Interaction analysis of gender, rural urban divide shows that some items show interaction effect by behaving differently in response to background variables.

Key words*:* Logical Thinking, Mathematics Achievement, Mathematical Thinking

1.0 Introduction:

Devlin (2001) defined mathematics as the science of patterns which also emphasizes on order, structure, pattern and logical relationship. Students in mathematics education need to develop their ability to reason and think logically. It is required by students of all discipline in general and of mathematics in particular (Frances, 1995). Mathematics education aims at developing logical thinking among students who generally think in algorithm but that is not logical thinking as there is difference between logical thinking and just thinking. Depending on mere thinking leads to memorization which ultimately does not develop logical thinking. Students without logical thinking in mathematics can handle the similar situation but are unable to do something when confronted with a different situation. Logical learning enables them to understand the situation and find a logincal solution that leads them towards logical thinking (Bako, 2009).

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Logical thinking moves from known to unknown following certain objectives rules and standards which are grammar of logic (shatnawi, 1982). Logical thinking is linked to the idea that study of mathematics can prove that certain things are true in mathematics and there are certain rules of grammar with which concepts related to mathematics can be organized (Macdonal, 1986). Logical thinking is based on certain premises and if premises have mathematical nature, it can be said that mathematical logic is working.

Study of mathematics provides meaningful understanding when understood through the use of reasoning. Level of logical reasoning increases with its consistent usage in various contexts and when it is learnt at early years. Wille (2009) stated that mathematical thinking developed logical thinking which helped the students understand the realities around them and do sensible actions.

1.1 Gender Comparison in Logical Thinking

Achievement in mathematics has been associated with reasoning and logical thinking. (Dyke and Frances, 1995). Systematic reasoning is thought to be a prominent feature of mathematics education in which one can explore, identify and justify all content related to mathematics.

Bessoondyal (2005) conducted a research in mathematics to identify gender differences. His findings showed that boys performed significantly better than girls. There were gender differences on TIMSS data as stated by (Gonzales et al 2004) which indicated that girls performed better than boys. Data showed that girls used logical thinking in mathematics more than the boys.

Study conducted by Ma'moon (2005) also found that girls performed better than boys in the use of logical thinking while solving mathematics sums. In another study on gender comparison by Cox (200), it was found that female students scored higher than the boys in mathematical operations involving logical thinking. Battista (1990) conducted a study to examine gender relationship for use of logical thinking in mathematics. The results indicated that there was no significant relationship between male and female on the use of logical thinking in mathematics at secondary school level. Various research studies (Ginsburg, Cooke, Leinwand, Noell, & Pollock, 2005; Bessoondyl, 2005; Stanley, 1982, Mamoon, 2005) showed that gender differences existed on logical reasoning at secondary level. These studies reveal that boys perform better than girls in logical thinking skills.

2.0 METHODOLOGY: *The study used survey approach and population for the study was 371000 students enrolled in 2703 secondary schools at grade 9 level in 2008.*

Sample for the Study: *A multi staged probability sampling techniques was used to draw sample from population for quantitative data collection. total sample for the survey was decided to be 500 subjects or beyond. Proportionate approach in sampling was adopted as illustrated in the table 1 given below.*

Table 1 Composition of the sample

	Sector Wise		*Gender Wise*		*Location Wise*	
	Public	*Private*	*Male*	*Female*	*Rural*	*Urban*

%age	*50*	*50*	*70*	*30*	*70*	*30*
Students in sample	*289*	*289*	*405*	*173*	*405*	*173*

2.1 Tool: *Data was collected thorough a test with six constructed response items. Content validity was ensured through expert opinion and construct validity was checked through factor analysis. Reliability of the instrument was found to be 0.90. scale wise reliability is given in table 2*

Table 2 Item wise Reliabilities Analysis for Test of Mathematical Thinking

Item	*Scale Mean if Item Deleted*	*Cronbach's Alpha if Item Deleted*
L1	*68.29*	*.905*
L2	*68.70*	*.906*
L3	*68.29*	*.905*
L4	*68.90*	*.905*
L5	*68.42*	*.905*
L6	*68.63*	*.905*

Table 2 shows that all individual items had excellent reliability value above 0.90

Therefore all items were retained for the study.

3.0 Analysis *T-test was used to make gender wise comparison in logical thinking. Table 3 below shows the results of analysis*

Table 3 Gender wise comparison of logical Thinking

	Male		*Female*		*t-value*	*Sig.(2-tailed)*	*Effect size*
Scale	*Mean*	*Std. deviation*	*Mean*	*Std. Deviation*			*Cohen's d*
Logic	*13.76*	*4.992*	*12.43*	*5.415*	*2.084*	*0.039**	*0.255*

**Mean difference is significant at P<0.05*

***Mean difference is significant at P<0.01*

To evaluate the amount of mean difference in each pair of mean scores effect size was calculated using Cohen's D effect size. Cohen's d value shows that effect size was found to be small in case of logical thinking (Cohen, 88).

To find the significance in the mean score sampled paired t-test was used comparing urban and rural students' mean scores in test of mathematical thinking, mathematics achievement and all six aspects of mathematical thinking. The results are shown in table 4 indicating that urban students mean score was better than rural students in all aspect of the mathematical thinking and

mathematics achievements. However this mean difference was not significant for induction and proofs aspect of mathematical thinking.

Table 4 Location wise comparison of logical Thinking

Scales	*Urban*		*Rural*		*t-value*	*Sig. (2-tailed)*	*Effect size Cohen's D*
	Mean	*Std. Dev.*	*Mean*	*Std. Dev.*			
Logical thinking	*13.50*	*5.044*	*12.06*	*5.113*	*-2.785*	*.006*	*0.28*

**Mean difference is significant at P<0.05*

***Mean difference is significant at P<0.01*

To further elaborates the amount of the difference Cohen's D effect size was also calculated and as the table shows a small effect size in logical (Cohen, 88). Urban students mean score in logical thinking was 13.50 respectively as compared to 12.06 for the same scale respectively by rural students.

Table 5 Sector wise Analysis of Mathematical Thinking and Mathematics Achievement

Scale	*Public*		*Private*		*t-value*	*Sig.(2-tailed)*	*Effect size Cohen's d*
	Mean	*Std. Deviation*	*Mean*	*Std. Deviation*			
Logic	*11.87*	*4.918*	*13.29*	*5.277*	*3.105*	*0.002***	*0.27*

** Mean difference is significant at P<0.05*

*** Mean difference is significant at P<0.01*

Table 5 shows that private students' mean score was better than public students in logical thinking.Cohen's D values shows that effect size is small for logical thinking (Cohen, 88). Male students mean score in logical thinking was 13.76 as compared to scores of 09.95 for the same scale respectively by female students. Table 6 shows that Eta squared values of interaction effect between gender and location for logical thinking, 0.000 which shows significant results in favour of female students.

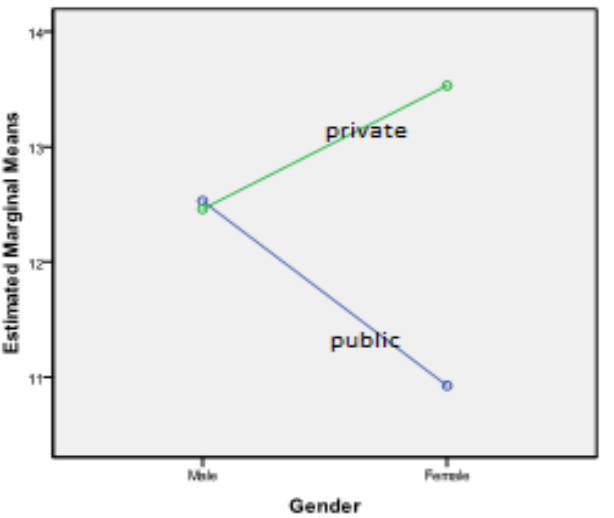
Table 6 Interaction Effect between Gender and Location

	Gender Eta^2	*Location* Eta^2	*Location & Gender* Eta^2
Logical thinking	*.001*	*.005*	*.000*

**Difference is significant at P<.05 ** Difference is significant at p<.01*

Interaction effect between gender and school location from ANOVA results show that there was no significant interaction between gender and location in logical thinking. Gender and sector were both combined as independent variables with logical thinking as dependent variables for ANOVA to know the possible interaction effect. For further elaboration estimated means were used for graphic representation. Fig.1 below shows that female students of private sector were doing better than female of public sector while male were almost the same.

Fig.1 **Estimated Marginal Means of Logical Thinking**



4.0 Discussion

Private and public schools in the sample follow the same curriculum but differ considerably in administrative autonomy and the quality of input in terms of student's enrolment.

Gender wise comparison in mean score of students shows that Males had significantly higher scores than females for Logical thinking. The superior performance by male students can be explained by high parental expectation for higher achievement and continuity of further education. Moreover being a conservative society, boys not only have opportunities to go out of their homes for private coaching, a predominant culture in Pakistan, but also can meet other educated people and their peers easily for academic discussion which possibly enhance their achievement chances. This fact is more elaborated by the disparity in rural urban achievement gap where in the cities the students can have private coaching easily. In the same way private public gap in achievement where private coaching for students of private school, with parents relatively better than parents of public school students, is a fashion and status symbol for them points to this explanation. Students interviews also shows that male students had better understanding of the questions and expressed mathematical thinking in their solution than female students. Being a male dominated society it is usual to expect male to be bold in expression both written and oral and thus male students performed better than female.

In relation to location, there were significant performance differences for Logical thinking. In

Logical thinking urban students outperformed rural students indicated by their means scores. Urban areas have often better population in terms of socio economic status and literacy rate so the students here also have the opportunities

to get private coaching which is a popular culture in Pakistan. Urban students have frequent interaction with educated people and thus gets motivated for education in contrast to the students of rural areas where they are mostly engaged with parents in farming and other jobs and thus do not get much time for study and lagged behind their urban counter parts. Students of private schools outperformed their public counterparts significantly in logical thinking. The result also showed that there was significant interaction effect between gender and school sector and logical thinking. Female students of private school were doing better than female students of public schools. On the other hand contrasting results was found in case of male students where male students in private schools did well in logical thinking. Better achievement for female students in private sectors in comparison to female students of public school is consistent with t-test analysis and can be attributed to teachers' availability, effective supervision in private schools. On the other hand male students of public schools either outperform or compare their private counterparts in logical thinking. Interaction analysis between school sector and location shows that there is no significant effect.

Students should not be discouraged during question answer process in the classroom and their faulty mathematical reasoning should be utilized for learning. Their poor reasoning should not be rewarded negatively.

This study is can replicated on a different sample particularly in other provinces of Pakistan. In this case the prospective researcher can use the same tools as used in this study or with little modifications. Similar study can also be conducted for other level of the students' e.g. primary and elementary level. In such case the researcher need to have a look on the model of mathematical thinking and develop new tools.

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