Pak. J. Engg. & Appl. Sci. Vol. 11, July., 2012 (*p.* 47-59)

Gradually Varied Flow Computation in Series, Tree Type and Looped Compound Channel Networks

Ghulam Nabi[1], H. Reddy[2], Habib-ur-Rehman[3]

1. Center of Excellence in Water Resources Engineering, University of Engineering and Technology, Lahore, Pakistan Email: gnabi60@hotmail.com
2. Post Doctoral Research Fellow Civil and Environmental Engineering C114, 300 Main Street, University of South Carolina, Columbia SC 29208 Email prashanthreddyh@gmail.com
3. Department of Civil Engineering, University of Engineering and Technology, Lahore, Pakistan

Abstract

The computations for compound channel section are difficult in most of the case when the channels are connected with each other in different forms such as tree type of looped network. The computation at the bifurcation points are difficult in the sense that change in discharge has to satisfy the continuity and momentum equations. In the iteration procedure the computations some time become unstable due to magnification of the error. . An algorithm is presented to compute the water surface profiles in steady, gradually varied flow of open channel having compound cross section. The methodology is more general and suitable for application to compound and trapezoidal channel cross sections. The algorithm is capable of calculation of water surface profiles in all types of channel network i.e., series channel, tree type network and looped network. In this method the energy and continuity equation are solved for steady, gradually varied flow computations. The Newton Raphson method has been used for the solution of resulting non linear equations. The results have verified with the physical model for channel network. The observed and computed results are in good agreement.

Key Words: Gradually varied flow; compound channel networks; Steady flow; Newton Raphson method; Flood plain

1. Introduction

The flow in channel network is usually steady gradually varied flow most of the time. The canal sections are designed on different approaches such as regime theory, tractive force method and combination of both. The computations of gradually varied flow in a channel network help to access the efficiency of the canal network and irrigated area under different flow conditions.. A number of numerical techniques are available for computation of gradually varied flow (Chow 1959, Chaudhry 2008). The standard step method based on single step calculation is well suited for single and series channels. The gradually varied flow computation is also required for the channel network or system of the channel interconnected. Such a system exists in Indus Basin Irrigation system, where the rivers are connected with each other through link canals. The irrigation canals are feed by different source of water at different locations i.e a canal may be supplemented by another canal to irrigate more agriculture area for crop production.

The open channel network also occur braided river channel, (In braided river the channel is divided into small channels and these channels are connected with each other forming the channel network) divided shipping channels and interconnected storm water system. Although most of the research work has been carried out for unsteady flow in channel networks (Choi and Molinas 1993; Kutija 1995), Wylie (1972) developed an algorithm to compute flow around a group of islands in which total length of channel between two nodes is treated as single reach to calculate the loss of energy and node energy is used as variable. Reddy and Bhallamudi (2004) presented an algorithm for computation of gradually varied flow in cyclic looped channel networks.

Wylie (1972) developed an algorithm to compute the flow around a group of islands, in which the total length of the channel between two nodes is treated as a single reach to calculate the loss of energy and the node energy is used as a variable. In this method, the channel is not divided into several reaches as in a finite difference method. A reach is defined as the portion of the channel between two finite-difference nodes. Chaudhry and Schulte (1986) presented a finite difference method for analyzing steady flow in a parallel channel system. Their formulation is in terms of the more commonly used variables, flow depths and discharges. Schulte and Chaudhry (1987) later extended their method for application to general looped channel networks. In their method, a channel i in the system is divided into several reaches, N_i. The continuity and the energy equations can be written in terms of flow depths, and flow rates for all the reaches, resulting in a total of $2\sum_{i=1}^{i=M}(N_i)$ equations because there are N_i+1 nodes in any channel i and there are M channels in the system (a channel reach has two nodes one at start of the reach and second at the end of the reach). Additional 2*M* equations, required for closing the system, are obtained from the boundary conditions and the compatibility conditions at the junctions. The system of nonlinear simultaneous equations resulting from the above formulation is solved using the Newton–Raphson iteration technique. This requires inversion of the system Jacobian for every iteration step. In this formulation, the size of the Jacobian increases if the number of reaches in each channel is increased to increase accuracy.

Sen and Garg (2002) developed an efficient solution technique for one dimensional steady and unsteady flow in general channel network system having trapezoidal and cross section.

Problem definition

In the previous studies for channel network, Schulte and Chaudhry (1987), Reddy and Bhallamudi (2004), Naidu et al. (1997) the trapezoidal channel cross-section was considered. However due difference in hydraulic and geometric characteristics between main channel and floodplains the computations become complex. In this study equations were derived for compound channels in tree type network or looped network. The developed model was also extended to a channel network having more than one source of discharge inflow to the channel network. It can work for dendritic, looped, divergent, or any combination of such networks.

Methodology

The governing equations for water profile calculations include energy equations at different location is given as

$$z_2 + y_2 + \frac{\alpha_2 Q_{t_2}^2}{2gA_{t_2}^2} = z_1 + y_1 + \frac{\alpha_1 Q_{t_1}^2}{2gA_{t_1}^2} \qquad (1)$$

where z_1, z_2 are elevations of the main channel invert levels; y_1, y_2 are depth of water at cross sections; $\propto_1, \propto_2$ are velocity weighting coefficients; Q_{t_1}, Q_{t_2} are total discharge at sections 1 and 2; A_{t_1} and A_{t_2} are total flow areas at sections 1 and 2; h_e is energy head loss between two sections. Detailed description of derivation of energy equation is given in Chaudhry (2008).

The energy head loss between two cross sections is comprised of friction losses and expansion or contraction losses. Expansion and contraction losses are neglected the equation for energy loss is as follows:

$$h_e = \Delta x \bar{S}_F \qquad (2)$$

Average friction slope between two cross sections may be written as

$$\bar{S}_F = \frac{S_{F_1} + S_{F_2}}{2} \qquad (3)$$

where Δx = reach length; $\bar{S}_F$ = average friction slope between two sections, S_{F_1} and S_{F_2} are friction slopes at sections 1 and 2.

The discharge formula can be written as following

$$Q = K\sqrt{S_F} \qquad (4)$$

The conveyance is calculated within a sub element from the following equation:

$$K = \frac{1}{n} AR^{\frac{2}{3}} \qquad (5)$$

Total flow area (m^2), total discharge and total conveyance of the compound channel (Fig. 1) can be written as:

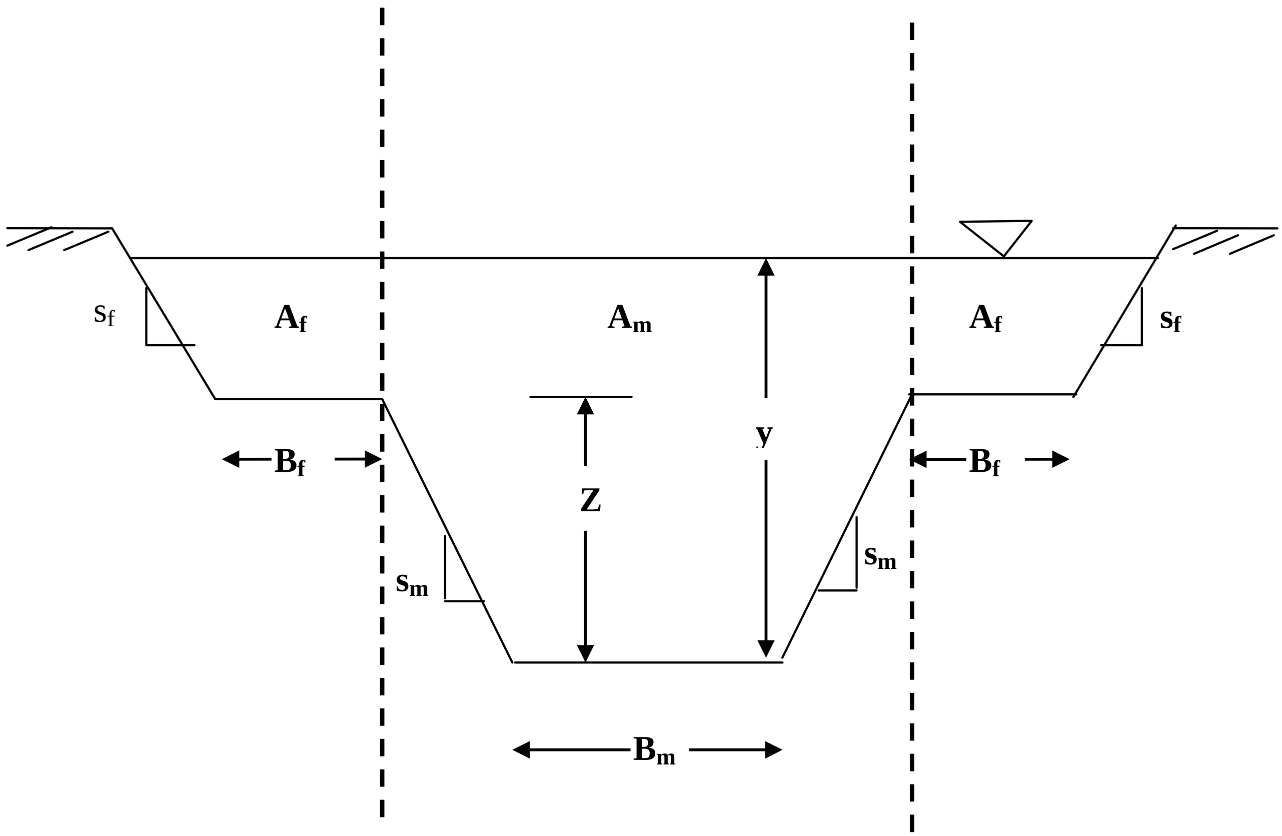


Fig 1 Schematic of compound cross section

$$Q_t = 2Q_f + Q_m \tag{6}$$

$$A_t = 2A_f + A_m \tag{7}$$

$$K_t = 2K_f + K_m \tag{8}$$

Where Q_t, Q_f and Q_m are total discharge, floodplain discharge, and discharge in main channel respectively. Similarly, A_t, A_f and A_m are total flow area of the compound channel, left or right flood plain flow area and flow area of main channel.

The velocity coefficient $\propto$ is computed based on the conveyance in the three flow sub elements

$$\propto = \frac{A_t^2}{K_t^2}\left[\frac{2K_f^2}{A_f^2} + \frac{K_m^2}{A_f^2}\right] \tag{9}$$

Where K = conveyance for sub element; n = mannings roughness coefficients for subelement; A = flow area for subelement; R = hydraulic radius for subelement. Subscripts f and m refer to the flood plain area and main channel section, other parameters are explained above.

For simplicity Equation (1) may be expressed as following.

$$z_2 + y_2 + \frac{\alpha_2 Q_{t_2}^2}{2g} = z_1 + y_1 + \frac{\alpha_1 Q_{t_1}^2}{2g} \tag{10}$$

where $a = \frac{\propto}{A_t^2}$

Areas, wetted perimeters and hydraulic radiuses of the sub elements of the compound channel can be formulated as following. The floodplain flow area either left or right is computed as

$$A_f = \left(B_f + \frac{s_f\,(y-z)}{e}\right)(y-Z) \tag{11}$$

Flow area in main channel

$$A_m = -s_m Z^2 + 2s_m yZ + B_m y \tag{12}$$

Wetted perimeter for the flood plain either left or right side of the main channel.

$$P_f = B_f + \sqrt{s_f^2 + 1}\,(y - Z) \tag{13}$$

Wetted perimeter for main channel

$$P_m = B_m + 2Z\sqrt{s_m^2 + 1} \tag{14}$$

Hydraulic radius of floodplain area either left or right side of main channel

$$R_f = \frac{A_f}{P_f} \tag{15}$$

Hydraulic radius of main channel

$$R_m = \frac{A_m}{P_m} \tag{16}$$

Continuity equation between two sections can be written as

$$Q_{t1} - Q_{t2} = 0 \tag{17}$$

Solution Algorithm

Consider a system having M channels, where each channel may have different cross section, Manning's n, bottom slope, etc. Each channel is subdivided into N_i reaches (where i refer to the channel number), with the first section numbered as 1 and last section numbered as N_i+1. Flow rate Q_t and depth y are two unknown variables at each section. The total number of unknowns in the entire channel network are equal to $2\sum_{i=1}^{i=M}(N_i + 1)$.

To solve the problem each section will have two equations continuity and energy equation, if a channel have N_1 reaches number equation will be $2N_1$, the remaining two equations will be provided as boundary conditions. In present study for a single channel the upstream boundary condition is known discharge and downstream boundary condition is known water depth. As for the channel network, the continuity, energy, junction and boundary equations comprises of the system of equations to solve unknown variables in the entire channel network. The continuity and energy equations are written for each of the N_i reaches of a channel i as.

$$F_{i,1} = z_{i,2} - z_{i,1} + y_{i,2} - y_{i,1} + \frac{1}{2g}$$

$$\left(\alpha_{i,2} Q_{t_{i,2}} \left|Q_{t_{i,2}}\right| - a_{i,1} Q_{t_{i,1}} \left|Q_{t_{i,1}}\right|\right) +$$

$$\left(\frac{\Delta x_i}{2}\right)\left(\frac{S_{F_{i,2}} + S_{F_{i,1}}}{2}\right) = 0 \tag{18a}$$

$$F_{i,2} = Q_{t_{i,1}} - Q_{t_{i,2}} = 0 \tag{18b}$$

$$F_{i,2} = z_{i,2} - z_{i,2} + y_{i,2} - y_{i,2} + \frac{1}{2g}$$

$$\left(\alpha_{i,2} Q_{t_{i,2}} \left|Q_{t_{i,2}}\right| - a_{i,2} Q_{t_{i,2}} \left|Q_{t_{i,2}}\right|\right) +$$

$$\left(\frac{\Delta x_i}{2}\right)\left(\frac{S_{F_{i,2}} + S_{F_{i,2}}}{2}\right) = 0 \tag{18c}$$

$$F_{i,4} = Q_{t_{i,2}} - Q_{t_{i,3}} = 0 \tag{18d}$$

$$\vdots$$

$$S_{F_{i,N_i}+1} + S_{F_{i,1}}) \tag{18e}$$

$$F_{i,2}\,N_t = Q_{t_{i,N_i}} - Q_{t_{i,N_i+1}} = 0 \tag{18f}$$

In $F_{i,1}$, i is channel number, 1 is equation number for the first reach. Similarly a set of equations was formulated for all the channels of the network, thus totaling $2\sum_{i=1}^{i=M} N_i$ equations. Remaining $2M$ equations are supplemented by the boundary conditions and junction equations. Boundary conditions and junction equations presented by Shulte and Chaudhry (1987) are given below.

Junction equations

The available equations at any channel junction are equal to the number of channels joining at that junction. Either two upstream channels joining with a downstream channel or one upstream channel joining with two downstream channels gives three equations. In which two equations are energy equations and one equation is continuity equation. The energy losses and the differences in the velocity heads have been neglected at the junction. The continuity equation and the two energy equations at junction (J1) of one upstream channel and two downstream channels shown in Fig 2a may be written as: Continuity equation for the junction:

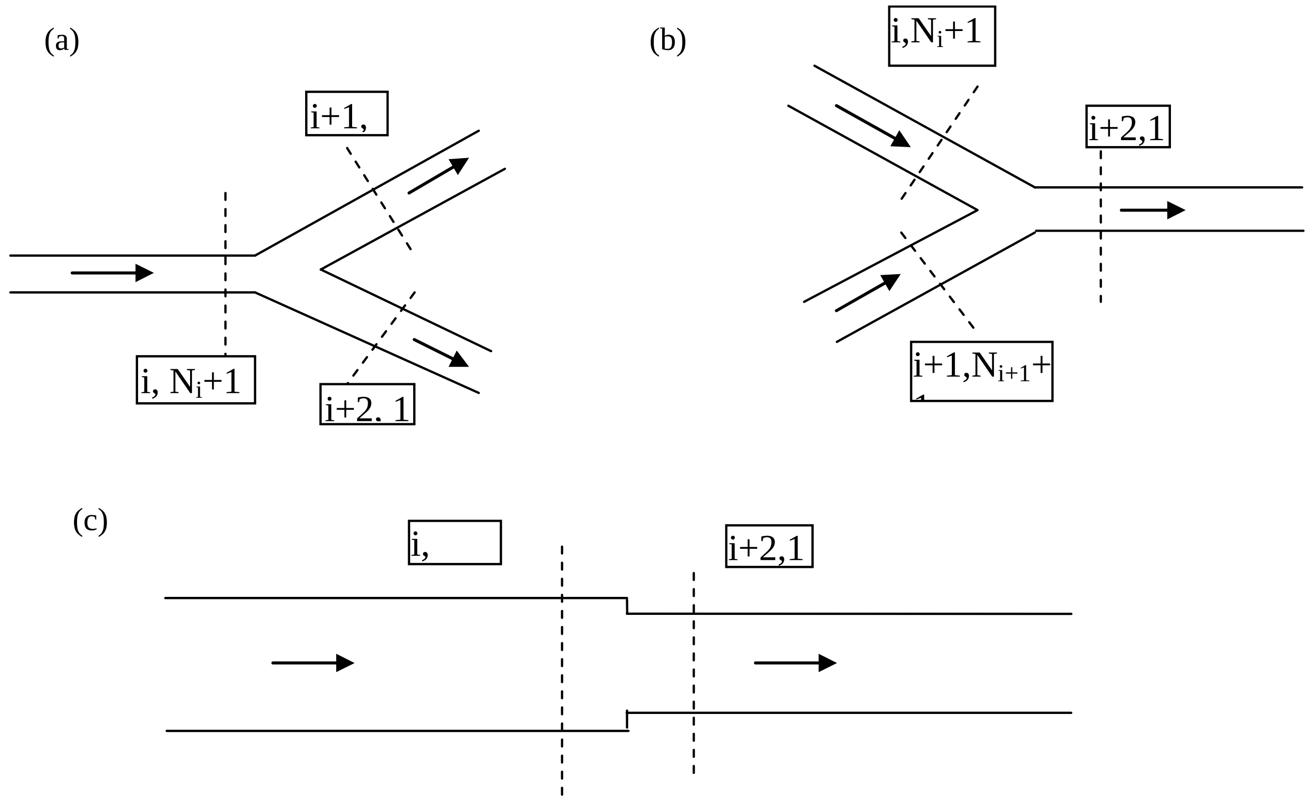


Fig.2 Channel junctions (a) two d/s channels and one u/s channel (b) two u/s channels and one d/s channel (c) two channels in series

$$F_{j_{1,1}} = Q_{t_i,N_{i+1}} - Q_{t_i,N_{i+1,1}} - Q_{t_{i+2,1}} = 0 \quad (19)$$

First energy equation for the junction

$$F_{j_{1,2}} = y_{i,N_{i+1}} - y_{i+1,1} = 0 \quad (20)$$

Second energy equation for the junction

$$F_{j_{1,3}} = y_{i,N_{i+1}} - y_{i+2,1} = 0 \quad (21)$$

At the junction ($J2$) of two upstream channels and one downstream channel shown Fig.2b the following three equations are available.

Continuity equation for the junction

$$F_{j_{2,1}} = Q_{t_i,N_{i+1}} + Q_{t_{i+1},N_{i+1}+1} - Q_{t_{i+2,1}} = 0 \quad (22)$$

First energy equation for the junction

$$F_{j2,2} = y_{i+1} - y_{i+1,}\, N_{i+1} + 1 = 0 \quad (23)$$

Second energy equation for the junction

$$F_{j_{2,3}} = y_{i,N_{i+1}} - y_{i+2,1} = 0 \quad (24)$$

Similarly at the junction ($J3$) of two series channels where different sections join as shown Fig. 2c, the following two equations are available.

$$F_{j_{3,1}} = Q_{t_i,N_{i+1}} - Q_{t_{i+1,1}} = 0 \quad (25)$$

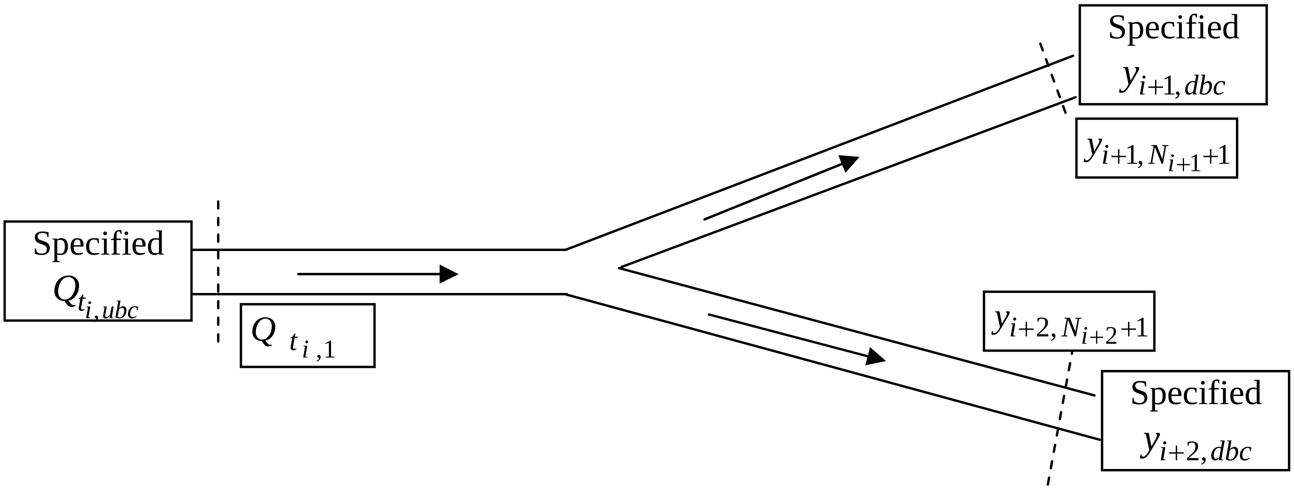
$$F_{j_{3,2}} = y_{i,N_{i+1}} - y_{i+1,1} = 0 \quad (26)$$

Boundary Conditions

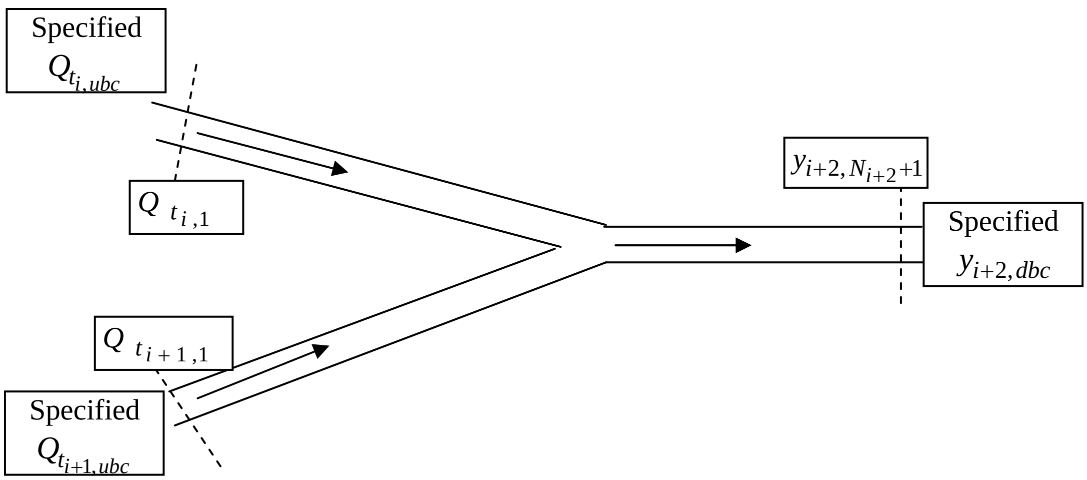
The flow was assumed as subcritical in all channel networks, for subcritical flow, the end conditions consist of a specified flow depth y_d at all the pendant nodes and a specified discharge Q_{tu} at the inflow nodes of the network as shown in Fig 3.

For one upstream channel and two downstream channels (Fig 3a) the upstream and downstream boundary conditions may be written as:

Upstream boundary condition



(a) One entry channel and two pendant channels



(b) Two entry channels and one pendant channel

Fig 3 Specification of boundary conditions

$$F_{BC3} = y_{i+2,dbc} - y_{i+2,N_{i+2}+1} = 0 \tag{27}$$

First downstream boundary condition

$$F_{BC2} = y_{i+1,dbc} - y_{i+1,N_{i+1}+1} = 0 \tag{28}$$

Second downstream boundary condition

$$F_{BC3} = y_{i+2,dbc} - y_{i+2,N_{i+2}+1} = 0 \tag{29}$$

For a junction with two upstream channels and one downstream channel the upstream and downstream boundary conditions may be written as

Upstream boundary condition

$$F_{BC1} = Q_{t_{i,ubc}} - Q_{t_i} = 0 \tag{30}$$

$$F_{BC2} = Q_{t_{i+1,ubc}} - Q_{t_{i+1,1}} = 0 \tag{31}$$

Downstream boundary condition

$$F_{BC3} = y_{i+2,dbc} - y_{i+2,N_{i+2}+1} = 0 \tag{32}$$

As described above the energy equation, continuity equation, junction compatibility condition and boundary conditions are solved using Newton Raphson method. For this method partial derivatives of flow variables are required for all the equations. The derivatives are formed as a Jacobian matrix for iterative solution. The following derivatives for different equations may be obtained as following.

Partial derivatives of energy equation

For each energy equation, there are four nonzero partial derivatives, namely the partial derivatives with respect to flow depth and with respect to the discharge at the section under consideration as well as partial derivatives with respect to the corresponding variables for the adjacent section.

Thus for an energy equation, $F_{i,k}$, between section *j* and *j+1* of channel *i*, the following non zero partial derivatives are obtained.

$$\frac{\partial F_{i,k}}{\partial y_{i,j}} = -1 - \frac{\partial \alpha_1 Q_{t_{i,j}}{}^2}{\partial y_{i,j}} + \frac{\Delta x_i}{2}\frac{\partial S_{f_{i,j}}}{\partial y_{i,j}} \quad (33)$$

$$\frac{\partial F_{i,k}}{\partial y_{i,j+1}} = 1 + \frac{\partial \alpha 2}{\partial y_{i,j+1}}\frac{Q_{t_{i,j+1}}{}^2}{2G} + \frac{\Delta x_i}{2}\frac{\partial S_{f_{i,j+1}}}{\partial y_{i,j+1}} \quad (34)$$

$$\frac{\partial F_{i,k}}{\partial Q_{t_{i,j}}} = 1 - \frac{\partial \alpha_1}{2g}\partial Q_{t_{i,j}} + \frac{\Delta x_i}{2}\frac{\partial S_{F_{i,j}}}{\partial Q_{i,j}} \quad (35)$$

$$\frac{\partial F_{i,k}}{\partial Q_{t_{i,j+1}}} = \frac{\partial \alpha_2}{2g}\partial Q_{t_{i,j+1}} + \frac{\Delta x_i}{2}\frac{\partial S_{F_{i,j+1}}}{\partial Q_{i,j+1}} \quad (36)$$

$$\frac{\partial S_{f_{i,j}}}{\partial Q_{i,j}} = \frac{2Q_{t_{i,j}}}{K_{t_{o,j}}} \quad (37)$$

$$\frac{\partial S_{F_{i,j+1}}}{\partial Q_{i,j+1}} = \frac{2Q_{t_{i,j+!}}}{K_{t_{i,j+1}}} \quad (38)$$

Following are the partial derivates of the terms *a* and S_F with respect to *y* in equations (33) to (36) by using the chain rule.

$$\frac{da}{dy} = \frac{da}{dA_f}\frac{dA_f}{dy} + \frac{da}{dA_m}\frac{dA_m}{dy} + \frac{da}{dR_f}\frac{dR_f}{dy} + \frac{da}{dR_m}\frac{dR_m}{dy} \quad (39)$$

$$\frac{dS_F}{dy} = \frac{dS_F}{dA_f}\frac{dA_f}{dy} + \frac{dS_F}{dA_m}\frac{dA_m}{dy} + \frac{dS_F}{dR_f}\frac{dR_f}{dy} + \frac{dS_F}{dR_m}\frac{dR_m}{dy} \quad (40)$$

Individual terms in the equations (39) and (40) are presented below

$$\frac{dA_f}{df} = B_f + s_f(y = z) \quad (41)$$

$$\frac{dA_m}{dy} = B_m + 2Zs_m \quad (42)$$

$$\frac{dR_f}{dy} = \frac{\sqrt{S_f^2+1}\left(\frac{S_f)Z^2+y^2)}{2} - s_f Zy\right) + B_f^2 + B_f S_f(y-Z)}{P_f^2} \quad (43)$$

$$\frac{dR_m}{dy} = \frac{B_m + 2Zs_m}{P_m} \quad (43)$$

$$\frac{dS_F}{dA_f} = -\frac{4Q_t^2 R_f^{2/3}}{n_m(2K_f + K_m)^3} \quad (45)$$

$$\frac{dS_F}{dA_m} = -\frac{2Q_t^2 R_m^{2/3}}{n_m(2K_f + K_m)^3} \quad (46)$$

$$\frac{dS_F}{dR_f} = -\frac{8A_f Q_t^2}{3R_f^{1/3} n_f (2K_f + K_m)^2} \quad (47)$$

$$\frac{dS_F}{dR_m} = -\frac{4A_m Q_t^2}{3R_m^{1/3} n_m (2K_f + K_m)^2} \quad (48)$$

$$\frac{da}{dA_f} = -\frac{2R_f^{2/3} n_m (4A_f R_f^2 n_m^2 + 3A_m R_m^2 n_f^3) - 2A_m R_f^2 R_m^{2/3} n_f n_m}{(2A_f R_f^{2/3} n_m + 3A_m R_m^{2/3} n_f)} \quad (49)$$

$$\frac{da}{dA_m} = -\frac{2R_m^{2/3} n_f (3A_f R_f^2 n_m^3 + A_m R_m^2 n_f^3) - 2A_f R_f^{2/3} R_m^2 n_m n_f^2}{(2A_f R_f^{2/3} n_m + A_m R_m^{2/3} n_f)} \quad (50)$$

$$\frac{da}{dR_f} = -\frac{4A_f A_m n_f n_m \left(R_f R_m^{2/3} n_m^2 - \frac{R_m^2 n_f^2}{R_f^{1/3}}\right)}{(2A_f R_f^{2/3} n_m + A_m R_m^{2/3} n_f)} \quad (51)$$

$$\frac{da}{dR_m} = -\frac{4A_f A_m n_f n_m \left(R_m R_f^{2/3} n_f^2 - \frac{R_f^2 n_m^2}{R_m^{1/3}}\right)}{(2A_f R_f^{2/3} n_m + A_m R_m^{2/3} n_f)} \quad (52)$$

$$\frac{da}{dR_m} = -\frac{4A_f A_m n_f n_m \left(R_m R_f^{2/3} n_f^2 - \frac{R_f^2 n_m^2}{R_m^{1/3}}\right)}{(2A_f R_f^{2/3} n_m + A_m R_m^{2/3} n_f)} \quad (52)$$

Numerical experimentation revealed that $2A_m R_f^2 R_m^{2/3} n_f n_m^2$ can be neglected without compromising the accuracy of the computations. However the terms $4A_f R_f^2 n_m^2$, $3A_f R_f^2 n_m^2$, can be neglected provided $5A_f < A_m$ and $4R_f < R_m$.

Partial derivatives of continuity equation

Here the subscript k refers to the equation number and its values is not identical to that of j. Likewise continuity equation $F_{i,k+1}$, the only non zero partial derivatives are those with respect to the discharges of the adjacent sections, i.e.,

$$\frac{\partial F_{t_{i,k+1}}}{\partial Q_{t,i,j}} = 1 \quad \frac{2F_{i,k+1}}{\partial Q_{t,i,j+1}} = -1 \tag{53}$$

Partial derivatives of junction equations

The Partial derivatives at junction of two upstream channels and one downstream channel are given as following.

$$\frac{\partial F_{J1,_1}}{\partial Q_{t_{i+1,1}}} = -1 \quad \frac{\partial F_{J1,_1}}{\partial Q_{t_{i+2,1}}} = -1$$

$$\frac{\partial F_{J1,_2}}{\partial y_{i+1,_1}} = -1 \quad \frac{\partial F_{J1,_2}}{\partial y_i N_{i+1}} = -1$$

$$\frac{\partial F_{J1,_2}}{\partial y_{i+2,_1}} = -1 \quad \frac{\partial F_{J1,_2}}{\partial y_i N_{i+1}} = 1 \tag{54}$$

Similarly partial derivatives at junction of one upstream channel and two downstream channel and junction of series channels can be written.

Partial derivatives of boundary conditions

The partial derivatives of boundary conditions for one upstream channel and two downstream channels may be written as

$$\frac{\partial F_{BC,_1}}{\partial Q_{t_{i,1}}} = -1 \quad \frac{\partial F_{BC,_2}}{\partial y_{i+1,} N_{i+1} + 1} = -1$$

$$\frac{\partial F_{BC,_2}}{\partial y_{i+2,} N_{i+2} + 1} = -1 \tag{55}$$

Similarly partial derivatives may be written for single inlet and single outlet channel or any other combination of boundary conditions.

Solution procedure

Matrix of system of equations is formed by grouping of continuity and energy equations in reaches, junction equations and boundary conditions. Solution procedure starts with arbitrary initial estimates of $y_{i,j}^{(0)}$ and $Q_{t_{i,j}}^{(0)}$ at each and every section of the network. Reasonable initial values may be assigned by setting them equal to boundary conditions or by using the experience. Newton Raphson method is an iterative procedure and corrections in the flow rates and depths are obtained between iterations. Improved values of the flow variables at subsequent intervals are obtained by adding corrections to previous values by applying a relaxation factor

$$Q_{t_{i,J}}{}^{(new)} = Q_{t_{i,J}}{}^{(previous)} + r\Delta Q_{t_{i,j}} \tag{56}$$

$$y_{t_J}{}^{(new)} = y_{i,j}{}^{(previous)} + r\Delta y_{i,j} \tag{57}$$

where r is a relaxation factor; Numerical experimentation revealed that r = 0.5 converges well for the complex networks and for simple networks r = 1 gives the faster convergence.

Generalized application

This method is a general method which is applicable to compound channel networks, trapezoidal channel networks and as well as mixed channel networks (some channels are compound and remaining channels are trapezoidal). If the water level goes below the flood plain level for any channel i.e $y \le Z$ then assign $Z=y$ and variables A_f. B_f. R_f. P_f and their derivatives $\frac{da}{dA_f}$, $\frac{dA_f}{dy}$, $\frac{dR_f}{dy}$, $\frac{da}{dR_f}$, $\frac{dS_F}{dy}$ and $\frac{dS_F}{dR_f}$ of that channel are equal to zero.

Model Application

The developed model was applied to two different types of channel networks. First of all series of channels with compound channel cross section was modeled, and then the model was applied to the looped channel network. Secondly looped channel network was modeled. The detailed description of the model input channel characteristics, boundary conditions and model output for both channel networks are described in the next section.

Tree type channel network

Tree type channel network as shown in Fig 4 was modeled. The channel number and node number are shown in Fig 4. There are 15 channels and 16 nodes. The main channel and flood plain have side slopes 2H: 1V, bed slope is different for each channel. The flow in the entire channels is subcritical and the end condition at downstream node are y_d = 4.0 m in all the channels. The upstream discharge is Q_u = 80 m^3/sec. The remaining channel

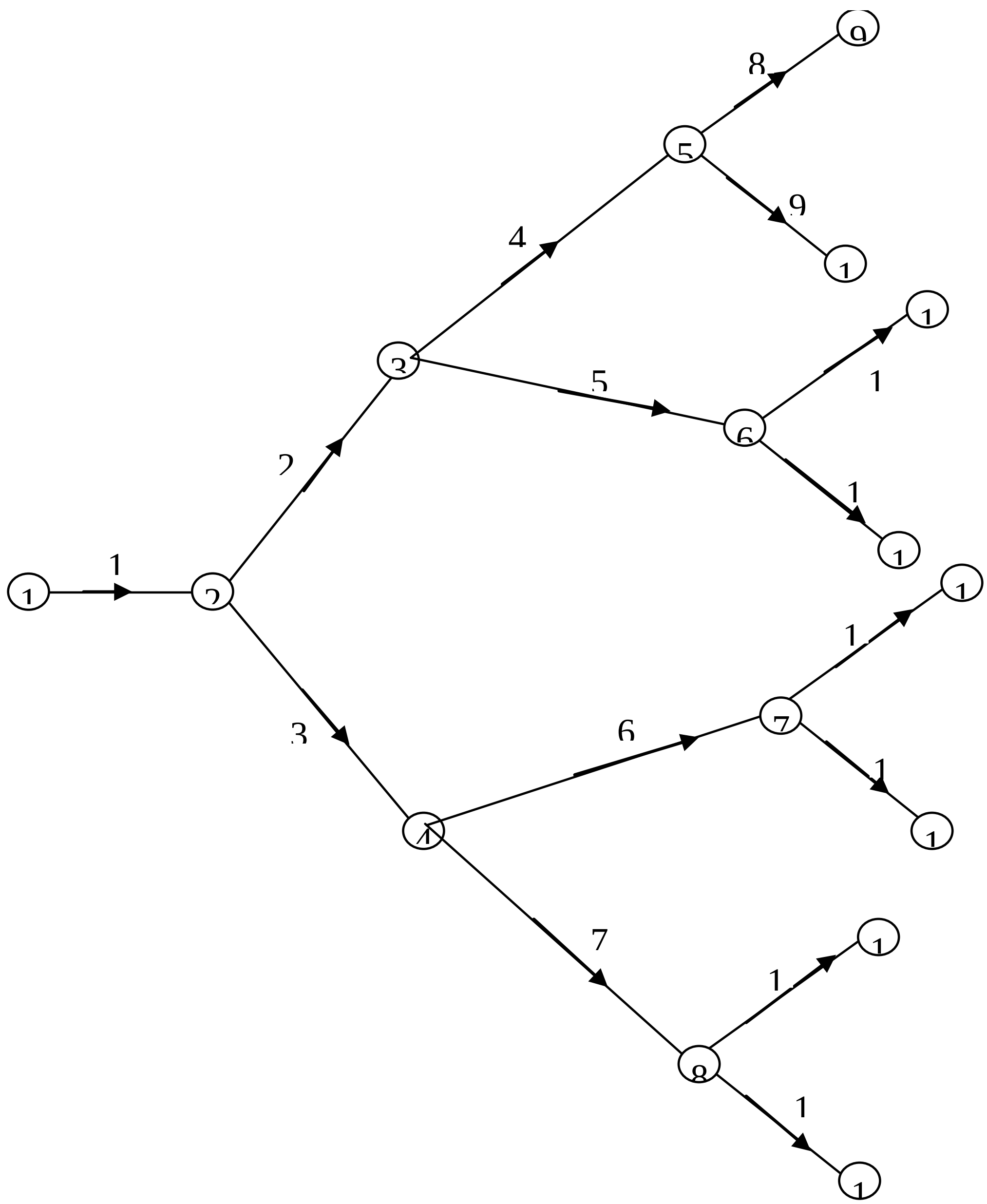


Fig 4 Tree channel network

characteristics are given in Table 1. The bottom slope of the channels is 0.0001.

In table 1, i is channel number, *D/S* is downstream, B_f is flood plain width, S_f is side slope of bank of flood plain, S_m is side slope of main channel, n_f is roughness of the flood plain, n_m is roughness of main channel, N_i is number of reaches, Q_o is initial discharge, y_d is initial depth.

The iterative procedure was started by assuming the flow depth equal to 4.0 m. A tolerance of 0.0001 for $y_{i,j}$ and $Q_{i,j}$ were specified for convergence of solution procedure. The solution was converged after 10 iterations. The computed discharges and water depth at different cross section are shown in Table 2.

Table 1 Channel characteristics of the tree type compound channel network.

i	U/S node	D/S Node	B_f(m)	Z(m)	s_f	B_m	s_m	L(m)	n_f	n_m	N_i
1	1	2	9	2.5	2	20	2	10000	0.025	0.02	5
2	2	3	8	2.5	2	10	2	8000	0.025	0.02	5
3	2	4	8	2.5	2	10	2	8000	0.025	0.02	5
4	3	5	7	2.5	2	8	2	5000	0.025	0.02	5
5	3	6	7	2.5	2	8	2	5000	0.025	0.02	5
6	4	7	7	2.5	2	8	2	5000	0.025	0.02	5
7	4	8	7	2.5	2	8	2	5000	0.025	0.02	5
8	5	9	4	2.5	2	5	2	4000	0.025	0.02	5
9	5	10	4	2.5	2	5	2	4000	0.025	0.02	5
10	6	11	4	2.5	2	5	2	4000	0.025	0.02	5
11	6	12	4	2.5	2	5	2	4000	0.025	0.02	5
12	7	13	4	2.5	2	5	2	4000	0.025	0.02	5
13	7	14	4	2.5	2	5	2	4000	0.025	0.02	5
14	8	15	4	2.5	2	5	2	4000	0.025	0.02	5
15	8	16	4	2.5	2	5	2	4000	0.025	0.02	5

Table 2 Computed water depth and discharge at different locations in tree channel network

	Section	Distance(m)	Depth (m)		Section	Distance(m)			Section	Distance(m)	Depth (m)
Channel 1	1	0	3.096	Channel 6	1	0	3.194	Channel 11	1	0	3.616
Q=80.0 m^3/sec	2	2500	3.085	Q=20.0 m^3/sec	2	1250	3.295	Q=10.0 m^3/sec	2	1000	3.711
	3	5000	3.068		3	2500	3.399		3	2000	3.807
	4	7500	3.042		4	3750	3.506		4	3000	3.903
	5	10000	3.003		5	5000	3.616		5	4000	4.000
Channel 2	1	0	3.003	Channel 7	1	0	3.194	Channel 12	1	0	3.616
Q=40.0 m^3/sec	2	2000	3.034	Q=20.0 m^3/sec	2	1250	3.295	Q=10.0 m^3/sec	2	1000	3.711
	3	4000	3.075		3	2500	3.399		3	2000	3.807
	4	6000	3.128		4	3750	3.506		4	3000	3.903
	5	8000	3.194		5	5000	3.616		5	4000	4.000
Channel 3	1	0	3.003	Channel 8	1	0	3.616	Channel 13	1	0	3.616
Q=40.0 m^3/sec	2	2000	3.034	Q=10.0 m^3/sec	2	1000	3.711	Q=10.0 m^3/sec	2	1000	3.711
	3	4000	3.075		3	2000	3.807		3	2000	3.807
	4	6000	3.128		4	3000	3.903		4	3000	3.903
	5	8000	3.194		5	4000	4.000		5	4000	4.000
Channel 4	1	0	3.194	Chennel9	1	0	3.616	Channel 14	1	0	3.616
Q=20.0 m^3/sec	2	1250	3.295	Q=10.0 m^3/sec	2	1000	3.711	Q=10.0 m^3/sec	2	1000	3.711
	3	2500	3.399		3	2000	3.807		3	2000	3.807
	4	3750	3.506		4	3000	3.903		4	3000	3.903
	5	5000	3.616		5	4000	4.000		5	4000	4.000
Channel 5	1	0	3.194	Chennel10	1	0	3.616	Channel 15	1	0	3.616
Q=20.0 m^3/sec	2	1250	3.295	Q=10.0 m^3/sec	2	1000	3.711	Q=10.0 m^3/sec	2	1000	3.711
	3	2500	3.399		3	2000	3.807		3	2000	3.807
	4	3750	3.506		4	3000	3.903		4	3000	3.903
	5	5000	3.616		5	4000	4.000		5	4000	4.000

Looped channel network

In the second case loop channel network was simulated as shown in Fig. 5. There are 10 channels and eight nodes in looped network. Similar to the tree type channel network, in looped network main channel and floodplain have side slopes 2H:1V, bed slope is different for each channel. The flow in all the channels is subcritical and the end condition at downstream node are y_d = 6.0 m and Q_u = 75.0 m^3/sec. The remaining channel characters are given in Table 3. The bottom slope of the channels is 0.0001. The bottom slope is taken constant for all the channels for simplicity but it can vary.

The iterative procedure was started by assuming the flow depth equal to 6.0 m. A tolerance of 0.0001 for $y_{i,j}$ and $Q_{i,j}$ were specified for convergence of solution procedure. The solution was converged after 10 iterations. The computed discharges and water depth at different cross section are shown in Table 4.

Comparisons with Physical Model

The simulation was also done with the physical model developed at hydraulic lab of Centre of Excellence in Water Resources Engineering. Due to space and discharge limitation the model the maximum discharge was 8.3 liter/sec. The maximum channel length was 5 meter. The channels were made of acrylic sheet. The roughness value selected in model was 0.01. The observed and computed depths are compared in Table 5. The difference between observed and computed depth varies from 2 to 5.54 % which are acceptable.

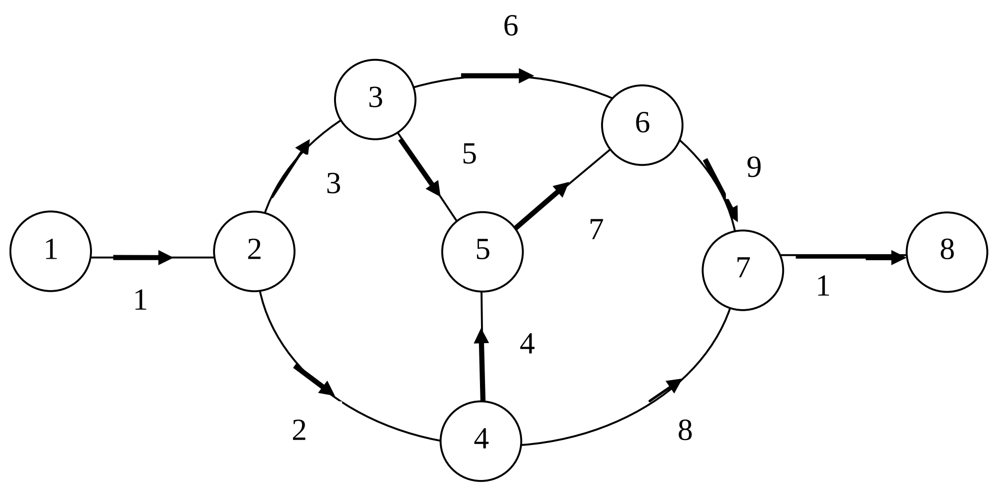


Fig 5 Looped channel network

Table 3 Channel characteristics of looped channel network.

i	U/S node	D/S Node	B_f(m)	Z(m)	s_f	B_m	s_m	L(m)	n_f	n_m	N_i
1	1	2	10	5	2	20	2	10000	0.025	0.02	20
2	2	3	6	5	2	8	2	5000	0.025	0.02	20
3	2	4	6	5	2	7	2	5000	0.025	0.02	20
4	3	5	5	5	1	5	2	5000	0.025	0.02	20
5	4	5	5	5	1	6	2	5000	0.025	0.02	20
6	4	6	6	5	1	6	2	10000	0.025	0.02	20
7	5	6	6	5	1	6.5	2	5000	0.025	0.02	20
8	3	7	7	5	2	7	2	15000	0.025	0.02	20
9	6	7	7	5	2	9	2	5000	0.025	0.02	20
10	7	8	10	5	2	15	2	10000	0.025	0.02	20

Table 4 Computed water depth and discharge at different locations in looped channel network

	Section	Distance (m)	Depth (m)		Section	Distance (m)	Depth (m)
Channel 1	1	0	3.159	Channel 6	1	0	3.754
Q=75.00 m^3/sec	2	2000	3.200	Q=20.45 m^3/sec	2	2000	3.939
	3	4000	3.251		3	4000	4.109
	4	6000	3.314		4	6000	4.293
	5	8000	3.389		5	8000	4.480
	6	10000	3.477		6	10000	4.669
Channel 2	1	0	3.477	Channel 7	1	0	4.222
Q=39.14 m^3/sec	2	1000	3.524	Q=27.79 m^3/sec	2	1000	4.310
	3	2000	3.574		3	2000	4.398
	4	3000	3.628		4	3000	4.487
	5	4000	3.685		5	4000	4.587
	6	5000	3.746		6	5000	4.669
Channel 3	1	0	3.477	Channel 8	1	0	3.746
Q=35.86 m^3/sec	2	1000	3.525	Q=26.75 m^3/sec	2	3000	3.997
	3	2000	3.577		3	6000	4.260
	4	3000	3.632		4	9000	4.534
	5	4000	3.691		5	12000	4.814
	6	5000	3.753		6	15000	5.099
Channel 4	1	0	3.746	Channel 9	1	0	4.669
Q=12.39 m^3/sec	2	1000	3.840	Q=48.24 m^3/sec	2	1000	4.753
	3	2000	3.935		3	2000	4.838
	4	3000	4.030		4	3000	4.924
	5	4000	4.126		5	4000	5.011
	6	5000	4.222		6	5000	5.099
Channel 5	1	0	3.754	Channel 10	1	0	5.099
Q=15.40 m^3/sec	2	1000	3.846	Q=75.00 m^3/sec	2	2000	5.273
	3	2000	3.939		3	4000	5.451
	4	3000	4.033		4	6000	5.632
	5	4000	4.127		5	8000	5.815
	6	5000	4.222		6	10000	6.000

Table 5 Comparison between observed and computed depth in channel network

	Section	Distance (m)	Depth Obs.	Depth (m)	Diff (%)
Main Channel Q=8.3 L/sec)	1	0	0.28	0.270	3.5
	2	0.5	0.2799	0.270	3.4
	3	1.5	0.278	0.267	4.1
	4	2	0.277	0.268	3.2
	5	2.5	0.2776	0.272	2
Branch1 Q=4.0 L/sec)	1	0	0.2	0.195	2.5
	2	0.5	0.19	0.180	5.01
	3	1.5	1.18	1.133	4.01
	4	2	0.18	0.174	3.5
	5	2.5	0.179	0.175	2.24
Branch2 Q=4.0 L/sec)	1	0	0.2	0.196	2.21
	2	0.5	0.19	0.183	3.5
	3	1.5	1.19	1.148	3.54
	4	2	0.18	0.171	5
	5	2.5	0.179	0.172	3.8
Distributry-1 Q=2.0 L/sec	1	0	0.17	0.161	5.01
	2	0.5	0.169	0.162	3.9
	3	1.5	0.17	0.162	4.6
	4	2	0.16	0.153	4.2
	5	2.5	0.15	0.145	3.5
Distributry-2 Q=2.0 L/sec	1	0	0.17	0.165	2.8
	2	0.5	0.16	0.152	4.85
	3	1.5	0.17	0.161	5.02
	4	2	0.159	0.152	4.7
	5	2.5	0.15	0.145	3.5

Conclusions

The algorithm for gradually varied flow computation was based on the solving the continuity and momentum equation by Newton-Raphson method. This proposed methodology is equally good for modeling the trapezoidal and compound channel networks. The equations and the solution algorithm are presented in detail to facilitate application of the method by the practicing hydraulic engineer. Finally the methodology is demonstrated on a tree type and looped compound open channel networks. The model can be easily applied to the draw the water surface profiles in irrigation network in which there is a main canal, branch canal, distributary and minors. The model developed in this study can be applied to any number of channels. The number of reaches and nodes can also be extended to any desired number.

Notations

z = elevation of channel bottom above datum

y = flow depth

$\propto$ = velocity weighting coefficients

Q_t = total discharge at section

g = acceleration due to gravity

A_t = total flow area

h_e = energy headloss

Δx = reach length

$\bar{S}_F$ = average friction slope

Q_f = discharge in the left flood plain and also equal to discharge in the right flood bank

Q_m = discharge in the main channel

A_f = flow area of the left flood bank also equal to flow area of right flood bank

Am = flow area of the main channel

K = Conveyance of the flow in the subelement

S_F = friction slope of the flow in the subelement

Q = discharge in subelement

K_f = conveyance of the left flood bank and also equal to discharge in the right flood bank

K_m = conveyance of the main channel

A = flow area of the subelement

R = Hydraulic radius of the subelement

n = mannings roughness coefficient

a = velocity weighting coefficients divided by square of the flow area

B_f = bottom width of the left flood bank and also equal to right flood bank

B_m = bottom width of the main channel

Z = invert level of flood plain from the channel bottom

s = side slope

P = perimeter

k = relaxation factor

Subscripts 1 and 2 denotes the section 1 and 2 respectively.

Subscripts *f* and *m* denote the flood area and main channel section respectively

Subscript *i* denotes the channel number, *j* denotes the section number and *k* denotes the equation number.

Subscript *BC* refers to boundary condition and subscript *J* refers to junctions.

References

[1] Chaudhry, M. H. (2008). *Open channel flow*, Second edition, Springer, New York NY 10013, USA.

[2] Chaudhry, M. H., and Schulte, A. (1986). “Computation of steady state, gradually varied flows in parallel channels.” *Can. J. Civ. Eng.*, 13(1), 39–45.

[3] Choi, G. W., and Molinas, A. (1993). “Simultaneous solution algorithm for channel network modeling.” *Water Resour. Res.*, 29(2), 321–328.

[4] Chow, V. T. (1959). *Open channel hydraulics*, McGraw–Hill, New York.

[5] Kutija, V. (1995). “A generalized method for the solution of flows in networks.” *J. Hydraul. Res.*, 33(4), 535–555.

[6] Mukherjee, K. N. (1985). *River basin atlas of India*, Central Board for the Prevention and Control of Water Pollution, New Delhi.

[7] Naidu, B. J., Bhallamudi, S. M., and Narasimhan, S. (1997). “GVF computation in tree-type channel networks.” *J. Hydraul. Eng.*, 123(8), 700–708.

[8] Reddy, H. P and Bhallamudi, S. M (2004). “Gradually Varied Flow Computation in Cyclic Looped Channel Networks “.*J. Irrig. Drain. Eng.*, 130(5)-424–431

[9] Reddy, H. P. (2001). “Computation of gradually varied flow in looped open channel networks.” MTech thesis, Indian Institute of Technology, Chennai, India.

[10] Schulte, A. M., and Chaudhry, M. H. (1987). “Gradually varied flows in open channel networks.” *J. Hydraul. Res.*, 25(3), 357–371.

[11] Sen, D. J., and Garg, N. K. (2002). “Efficient algorithm for gradually varied flows in channel networks,” *J. Irrig. Drain. Eng.*, 128(6),351-357.

[12] Wylie, E. B. (1972). “Water surface profiles in divided channels.” *J. Hydraul. Res.*, 10(3), 325–341.