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Development of Shear Capacity Equations for Rectangular Reinforced Concrete Beams

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Abstract

The problem of shear is not yet fully understood due to involvement of number of parameters. Designers are extra careful about shear, especially in beams and consequently higher safety factors are used in shear design. Several equations are available in literature to determine the shear capacity of beams, i.e. ACI equation, Zsutty equation and KIM & White equation. To verify the application of these equations, extensive experimental study was carried out on rectangular reinforced concrete beams without web reinforcement.

Three parameters i.e. compressive strength, percentage of tension steel and shear span to depth ratio were considered and equations were developed for the shear strength prediction. Results of the study show that the concrete shear capacity ranges from $1.7\sqrt{f_c'}$ to $1.8\sqrt{f_c'}$ before any cracking is observed. After cracking,

ρ.

Key Words: Shear span, Cracking shear strength, Steel ratio, Shear capacity, Divorcing point.

1. Introduction

Shear strength of concrete has prime importance in structural design. It is reported as ranging from 35% to 80% of compressive strength, i.e. in between tensile and compressive strength of concrete [1]. Sudden and violent mode of failure in shear requires extra care and results in employment of higher safety factors.

The research about inherent shear capacity of rectangular beams produced two different schools of thought in Europe and America. According to European researchers

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$2\sqrt{f_c'}$.

Ultimate method of design was introduced in the early sixties of last century. Research work on the ultimate flexural capacity of a section was successfully completed with in few years. To predict the flexural capacity, equations were developed, whose results were in good agreement with the actual/experimental values. Mechanism of flexural failure of a rectangular reinforced concrete beam was much simpler to understand as compared to shear failure. In fact shear failure of reinforced concrete beams is a very complex phenomenon due to involvement of too many parameters. Factors influencing the shear capacity of beams are shear span to depth ratio (a/d), tension steel ratio (ρ), compressive strength of Concrete (f_c'), size of coarse aggregate, density of concrete, use of fibers in concrete, size of beam, position and geometry of haunches, tensile strength of concrete, support conditions, clear span to depth ratio (*L*/d), number of layers of tension reinforcement, grade of tension reinforcement and end anchorage of tension reinforcement.

Research work on ultimate shear capacity of beams without shear reinforcement is still continuing. Up till now no unique solution is available for its prediction. Many equations have been developed by various researchers (6-11) based on theoretical concepts and experimental data. Each equation has its own merit and demerits. There are few equations which present very strong co-relation between dependent variable vu and independent variables fc′, ρ and a/d.

With the development of Fracture Mechanics approach and Finite Element analysis[2,3] for the inelastic behavior of reinforced concrete beams it can be hoped that in few years it will be possible to understand the true interaction of so many variables affecting the shear strength of beams.

In the present study it has been tried to find out the shear strength of rectangular reinforced beams without web reinforcement, considering three parameters only, i.e. f_c', ρ and a/d, and equations were developed for shear strength prediction.

1.1 *Modes of shear failure*

Depending upon the shear span, shear failure may be classified into three types [1] i.e. diagonal tension failure, diagonal compression failure and splitting or true shear failure (shown in Figures 1 to 3 respectively).

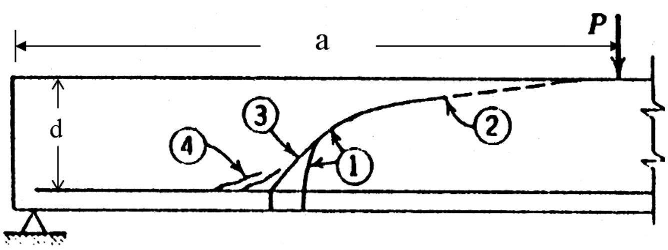


Fig.1 Formation of Diagonal Tension Cracks

Diagonal tension failure always occur when a > 2d (Figure 1), but in very limited cases it may occur for a < 2d [1]. When load and reaction are far apart then diagonal tension failure is possible. The diagonal crack starts from the last flexural crack and turns more and more inclined under shear loading. The diagonal crack encounters resistance as it moves up into the compression zone. It becomes f . With further increase in load, the tension crack gradually extends at a very flat slope until sudden failure at point 2, the subsequent arch mechanism is not capable of sustaining the load beyond cracking load [4]. Shortl

, 2, 3, and 4[1]. This type of failure is also called beam action failure.

In diagonal compression failure the specimen is able to carry additional load after formation of the first fully developed inclined tension crack [5]. This type of failure occur for shear span (a) to depth (d) ratio 1 to 2.5(d < a < 2.5d) [1]. When reaction and load are closely spaced then vertical compressive stresses under the load reduces the possibility of further tension cracking. Similarly vertical compressive stresses over the reaction also limit the bond splitting and diagonal cracking along the steel. Hence a large shear in a short length may initiate 45^{o} crack (web shear crack) across the neutral axis before a flexural crack appears. Such cracks restrict the shear resistance into a smaller un-cracked depth, thus increasing the shear stress. Hence the beam acts as a tied arch. Propagation of inclined crack reduces the compression zone. In the vicinity of load point it becomes too small to resist the compression force, and it crushes (shaded area in Figure 2). Such failure of compression zone above diagonal cracking is also called arch action [4].

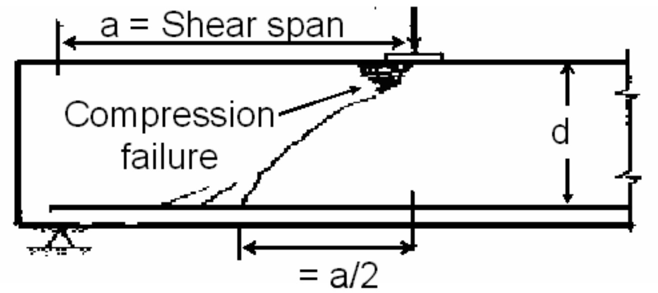


Fig.2 Shear compression failure for small span.

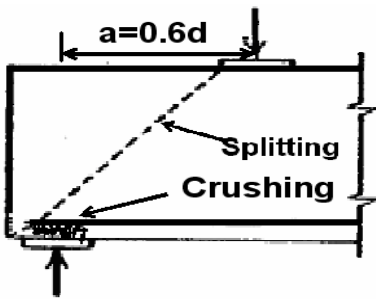


Fig. 3: True shear failure.

Splitting or true shear failure occurs when shear span is too short i.e. a < d (Figure 3). In this case shear is carried as an inclined thrust between load and reaction, which almost eliminates ordinary diagonal tension concepts. Shear strength is much higher in such cases. The final failure is splitting or it may fail in compression at the reaction.

2. Equations to predict shear capacity

Shear capacity of rectangular beams has been an area of major interest among the researchers. Based on experimental observations, different researchers have developed different equations for the prediction of shear capacity of the rectangular beams. A brief account is given below.

2.1 Zsutty's Equation

T C Zsutty [6, 7] collected the test data of about 200 beams from different responsible sources and developed equation for the prediction of shear strength of longitudinally reinforced beams. Equations were developed by combining the techniques of dimensional and statistical regression analysis. Zsutty also noted that on the v_u ~a/d graph there is a critical point (divorcing point) at a/d = 2.5, which indicate the separate modes of shear failure. For a/d > 2.5 beam fails under beam action and for a/d < 2.5 beams fails under arch action. Hence Zsutty was first to realize this and developed two different equations.

$$v_u = 60\,(fc'\rho\,\frac{d}{a})^{1/3} \qquad \text{for } a/d \geq 2.5 \qquad (1)$$

$$v_u = 150\,(fc'\rho)^{1/3}(\frac{d}{a})^{4/3} \qquad \text{for } a/d \langle\, 2.5 \qquad (2)$$

The equations (1) and (2) represent very strong relation between dependent (v_u) and three independent variables (f_c', ρ, d/a). However Zsutty fails to impose maximum and minimum limits on the variables, as ACI placed a limit of $3.5\sqrt{f_c'}$ and Placas and Regan [8] placed a limit of $12(f_c')^{1/3}$ on the maximum estimated value of ultimate shear. Moreover he used a random data of about 200 beams, where as, well organized data is required for development of equation.

2.2 Mphonde and Frantz's Equation

A G Mphonde and G C Frantz [9] in 1984 developed an equation for shear strength of rectangular reinforced beams using regression analysis.

$$v_u = 10.1\,(fc')^{1/3} + 71 \qquad (3)$$

This equation has a very limited application and is only valid for a/d = 3.6. Only one variable i.e. f_c' is considered during derivation of this equation and contribution of steel ratio and shear span to depth ratio are altogether ignored.

2.3 Bazant and Kim's equation

Z P Bazant and J K Kim [10] in 1984 analyzed diagonal shear failure of longitudinally reinforced concrete beams using recent fracture mechanics approach. In addition to size effect a rational formula for the effect of steel ratio and shear span was derived.

$$v_u = \left[\frac{10\sqrt[3]{\rho}}{(1+\frac{d}{25da})^{1/2}}\right]\left[\sqrt{fc'} + 3000\sqrt{\frac{\rho}{(a/d)^5}}\right] \qquad (4)$$

Where *da* is the Max. aggregate size

The above equation (4) has better agreement with the test data. In this equation five parameters (f_c', ρ, d/a, d and da) are correlated with ultimate shear strength of rectangular beams, especially the effect of aggregate size, which plays very important role in the shear strength.

2.4 Kim and White's Equation

In 1991 W Kim and R N White [11] proposed a hypothesis for the shear cracking mechanism for point loaded reinforced concrete beams with no web reinforcement. By adopting an approximate analytical approach they give the following relation for cracking shear strength.

$$v_{cr} = 9.4\left[\sqrt{\rho}\,(1-\sqrt{\rho})^2\left(\frac{d}{a}\right)\right]^{1/3}\sqrt{fc'}\;bd \qquad (5)$$

Equation (5) is derived purely on analytical basis. Its applicability was examined for more that 100 test beams and results showed good correlation between the predicted and measured value [11]. However single equation is developed for all values of a/d which is unjustified. Separate equations should have been developed for both sides of divorcing point i.e. a = 2.5d.

2.5 ACI code equation

ACI code [12] presented a formula for the prediction of shear cracking load in 1963, which was developed by the linear regression based on thousands of beam test results subjected to UDL only.

$$v_{cr} = 1.9\sqrt{fc'} + 2500\left(\rho\frac{V_u d}{M_u}\right) \leq 3.5\sqrt{fc'} \qquad (6)$$

This equation remains the same till the issue of 2008 ACI code. In 2008 the equation is little bit modified which is shown as below.

$$v_{cr} = 1.9\lambda\sqrt{fc'} + 2500\left(\rho\frac{V_u d}{M_u}\right) \leq 3.5\lambda\sqrt{fc'} \qquad (7)$$

Only difference is that a factor λ is introduced along with concrete strength. It defined as, "modification factor" reflecting the reduced mechanical properties of lightweight concrete. For lightweight concrete it varies from 0.75 to 0.85. For normal weight concrete it is equal to 1, hence practically for normal weight concrete there is no change in the equation (6) given in 1963 code. ACI 318-63 equation has serious imperfections. For low values of ρ, v_u and d/M, lower strength values are well predicted by this equation [6]. Two types of beam behaviors (beam and arch actions) are not separately treated in this equation [7]. This equation for shear design is conservative at a/d = 3.6, at a/d = 2.5 it gives lower bound values, and at a/d = 1.5 it under estimates even the lower bound measured shear capacity by 71% for high concrete strengths[9]. This equation is un-conservative when the ratio of the longitudinal reinforcement ρ is small [13].

3. Casting and Testing of Beams

In the present study beams were cast in steel forms with the tension reinforcement near the bottom. No stirrups (shear reinforcement) are provided in the beams. Lifting lugs were also provided for transporting the finished specimen to the test platform. Nominal size of the beams was 6x12 inches x9 feet. The concrete was compacted with an internal vibrator. Form work was removed after 48 hours. Beams were kept in the curing room for 28 days. To facilitate the tracing of cracks, the beams were distempered white prior to testing.

To study the effect of three variables (f_c', ρ & d/a) total nineteen beams were cast in four series. First series (a/d series) consisted of seven beams (designated as B1 to B7 a/d) in which all other parameters were kept constant except the parameter a/d. This series of seven beams helped to investigate the effect of a/d ratio on cracking, ultimate shear strength of beam and to determine the divorcing point. Second series (E-series) consisted of 12 beams. Ten new beams and data of two beams from a/d series are also used for analysis. This series is further subdivided into two groups. One group having a/d ratio

& ρ were also changed in a systematic order to investigate the effect of three parameters (f_c', ρ & d/a) on the beam shear strength and to develop equations. All the parameters were either increasing or decreasing the shear strength. The aim of study is to develop shear capacity equation. Third series is ρ series. Two beams were cast keeping the percentage of steel as variable and data of one of the beams from E series is also used in this series. Hence data of nineteen beams actually serves as 22 for the purpose of analysis. Properties of material used are listed and explained below.

3.1 Cement

Maple leaf cement (locally manufactured) is used for casting of specimens. It has standard consistency equal to 31%. Initial and final setting time was 120 and 180 minutes respectively. Soundness expansion measured as 5mm and fineness (%age passing ASTM sieve # 200) is 85%.

3.2 Fine Aggregate

Laurencepur sand was used as fine aggregates having loose bulk density 92.31 pcf and rodded bulk density 101.22 pcf. Fineness modulus was determined equal to 2.56 with the following gradation. Passing ASTM sieve # 4 is 99.2%, # 8 is 93.7%, # 16 is 80.7%, # 30 is 50%, # 50 is 18.1%, and # 100 was 2.7%.

3.3 Coarse Aggregate

Margalla crush with maximum size ¾" was used as coarse aggregate, having loose bulk density 85.97pcf and rodded bulk density 96.03 pcf. Aggregate impact value measured as 17.61% and aggregate crushing value 26.8%. Fineness modulus is equal to 6.77. Percentage passing ASTM sieve 1-1/2" (38.1mm) is 100%, 3/4" (19mm) is 95.15, 3/8" (9.5mm) 26.3%, sieve # 4(4.75mm) 1.2% and # 8(2.36mm) 0.1%.

3.4 Steel Reinforcement

Deformed bars meeting the requirements of ASTM A 615-80 were used as reinforcement in all beams to insure shear failure. Steel bars #4 had shown yield strength 72500 psi, ultimate strength 102800 psi and percentage elongation 13.75%, where as bars # 5 having yield strength 65700 psi, ultimate strength 102200 psi and %age elongation 18.75%.

Proportioning by weight is adopted for preparation of concrete. To vary the cylinder strength of concrete ranging from 2250 to 5250 psi mix proportions of 1:3:6, 1:2.5:5, 1:2:4, 1:1.75:3.5, 1:1.6:3.2 and 1:1.5:3 were used. Water cement ratio was varying from 0.45 to 0.80. Concrete was mixed in a horizontal pan-type mixer of 3 cft capacity. Concrete was mixed in two batches in the mixer for one beam. Cylinders of 6"x12" were cast to determine 28 days cylinder strength of concrete.

3.5 *Instrumentation and Testing Procedure*

The loading arrangements and instrumentation is shown in Figure 4.

Beams were tested as simply supported in a reaction frame and load was applied with the help of manually operated hydraulic jack. Concrete pedestals supported the specimen and the load was applied through steel loading beam. Bourdon gauge which was calibrated before testing was used to measure load. Pressure gauge and pressure cell were kept at the same level to eliminate the elevation correction to the bourdon gage. The forces at the load and reaction points were evenly distributed by steel bearing plates 3x6 inch in size, which were pasted by a thin layer of plaster of Paris.

Nine deflection gauges were employed to record deflection. The space between load points is divided into 8 equal parts. Gauges L_1 and L_2 are placed under the loads and gauge # 4 is at the center to record maximum deflection. This arrangement facilitated us to verify the theoretical curvature of beam with the observed values. Flexural cracks in critical flexural zone, flexure shear cracks and shear cracks in critical shear zone were carefully observed using magnifying glass during and after each load increment. The

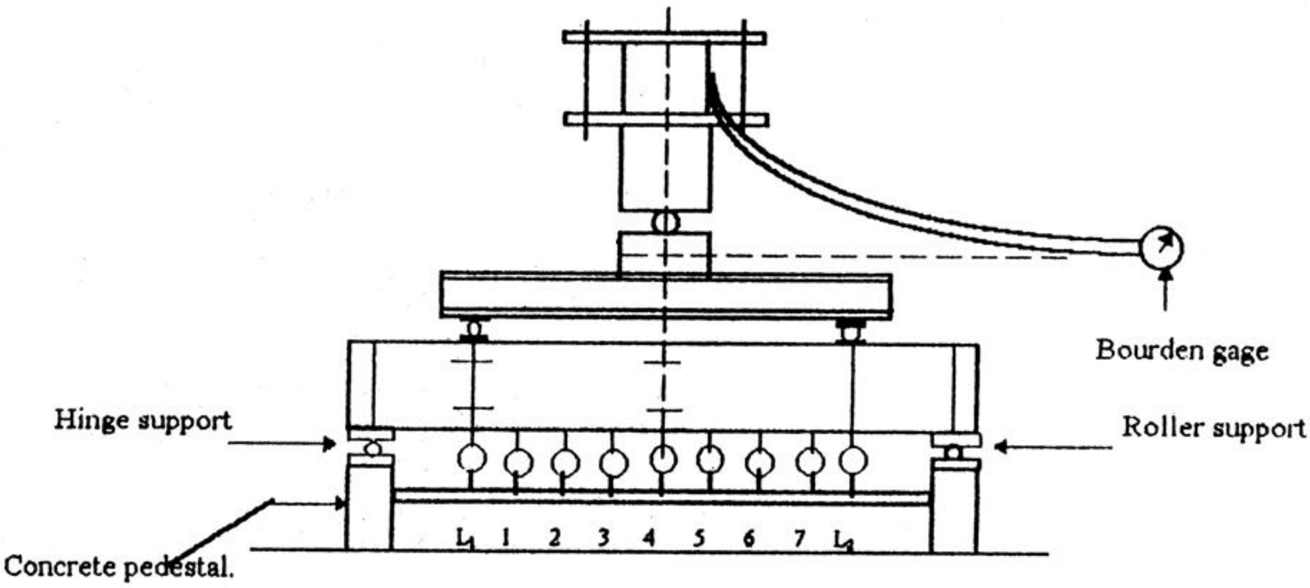


Fig. 4: Test set up

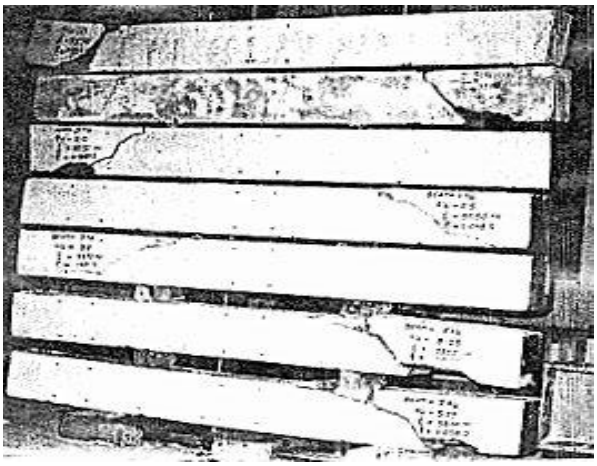


Fig. 5: Beams of a/d-series

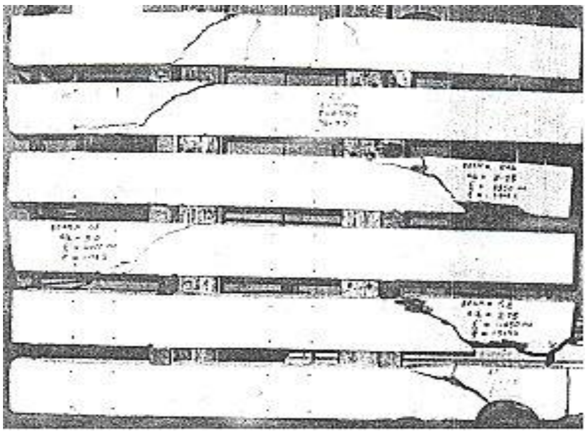


Fig. 6: E-series Beams having shear span to depth ratio ≥ 2.5

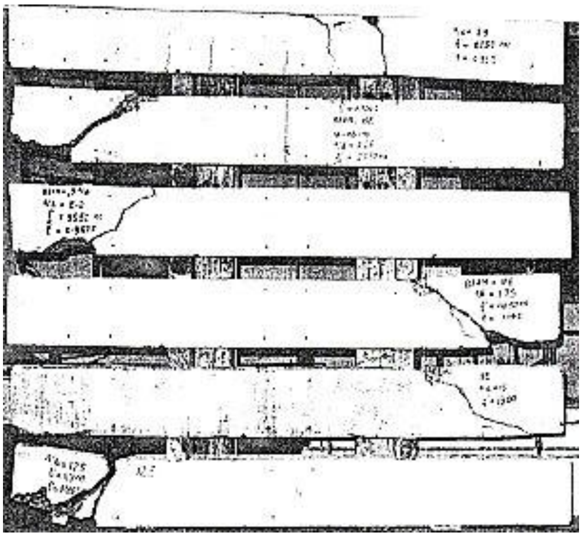
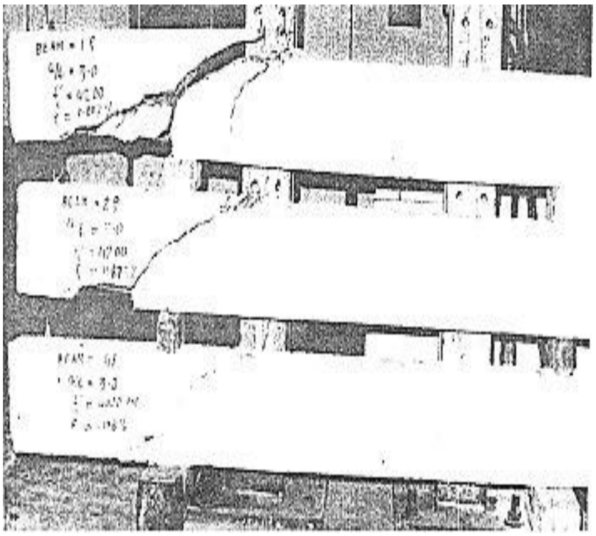


Fig. 7: E-series Beams having shear span to depth ratio < 2.5



Fig, 8: Beams of ρ-series

crack locations were marked sequently on the beams mentioning the load increments. The load at which first inclined shear crack slightly crosses the mid depth of beam in the critical shear zone was designated as "cracking shear load". After observing the cracking shear load, deflection gauges were removed to avoid damage and beams were loaded to failure. Failed beams are shown in Figures 5-8.

4. Test Results

Summary of test results of RCC beams are given in Table 1. The shear strength and relative beam strength calculations are presented in Table 2. Graphical representation of data of Table 2 is also given in Figs. 9 and 10. Both graph show clear change of slope at a/d value equal to 2.5(divorcing point). From Fig: 10 it is evident that the relative beam strength (ratio of ultimate moment to the flexural moment capacity of the beam) is the lowest at divorcing point. These figures justify the Zsuttys' stance of developing separate equation for both sides of divorcing point.

5. Development of Equations.

The best available form of equations is that presented by Zsutty. Though ACI code had not recognized the importance of divorcing point, but the authenticity of code is very hard to challenge. Hence shear capacity equations are developed in both ACI and Zsutty's patterns.

Table 1: Detailed summary of Test Results

Sr.#	Beam	d (in)	b (in)	ρ (%)	a/d	f (psi)	v_{cr} (psi)	v_u (psi)
1.	B1 a/d	10.18	6.6	0.952	1.00	3550	257.99	630.96
2.	B2 a/d	10.18	6.3	0.998	1.50	3550	210.12	454.75
3.	B3 a/d	10.18	6.5	0.967	2.00	3550	174.01	306.16
4.	B4 a/d	10.18	6.0	1.048	2.50	3550	154.63	203.85
5.	B5 a/d	10.28	6.1	1.021	3.00	3550	141.89	183.61
6.	B6 a/d	10.18	6.0	1.048	3.50	3550	127.75	182.28
7.	B7 a/d	10.28	6.0	1.021	3.75	3550	122.42	169.32
8.	B1E	10.18	6.0	0.524	3.75	2600	79.027	127.75
9.	B2E	10.26	6.0	0.926	3.50	3000	111.24	144.54
10	B3E=B6 a/d	10.18	6.0	1.048	3.5	3550	127.75	182.28
11.	B4E	10.25	6.0	1.138	3.00	4000	161.950	209.561
12.	B5E	10.28	6.0	1.345	2.75	4650	174.33	251.62
13.	B6E	10.18	6.1	1.546	2.50	5250	186.834	265.226
14.	B7E	10.28	6.1	0.51	2.50	2250	109.204	171.477
15.	B8E	10.26	6.1	0.926	2.25	2550	124.68	222.43
16.	B9E=B3 a/d	10.18	6.5	0.967	2.00	3550	162.5	306.16
17.	B10E	10.45	6.0	1.116	1.75	4650	198.58	433.28
18.	B11E	10.28	6.3	1.281	1.50	4300	254.31	581.96
19.	B12E	10.38	6.3	1.468	1.25	4700	295.41	721.59
20.	B1 ρ	10.26	6.1	0.607	3.00	4200	104.8	147.06
21.	B2 ρ	10.36	6.3	0.873	3.00	4200	115.69	153.69
22.	B3 ρ = B4E	10.25	6.0	1.138	3.00	4000	161.95	209.56

Table 2: Comparison of Relative beam Strength

Sr. #	Beam	a/d	v_{cr} psi	v_u psi	v_{cr} / v	$M_u = V_u.a$ (K-in)	M_{fl} (K-in)	M_u/M_{fl}
1.	Bl a/d	1.00	257.99	640.96	0.41	431.56	382.30	0.95
2.	B2 a/d	1.50	210.12	454.75	0.46	445.35	377.65	1.00
3.	B3 a/d	2.00	174.01	306.16	0.57	412.47	381.74	0.90
4.	B4 a/d	2.50	154.63	203.85	0.76	316.88	378.13	0.66
5.	B5 a/d	3.00	141.89	183.61	0.77	355.09	380.33	0.75
6.	B6 a/d	3.50	127.75	182.28	0.70	368.31	378.13	0.79
7.	B7 a/d	3.75	122.42	169.32	0.72	402.62	385.59	0.86

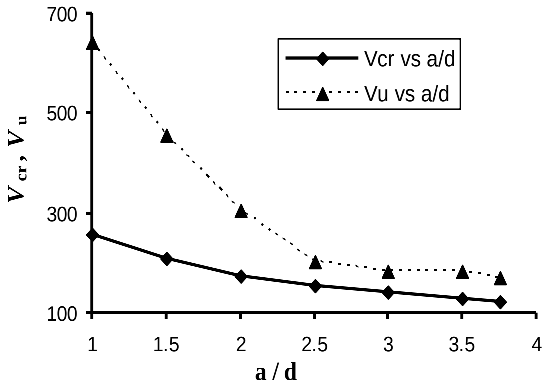


Fig. 9: Shear strength vs (a/d)

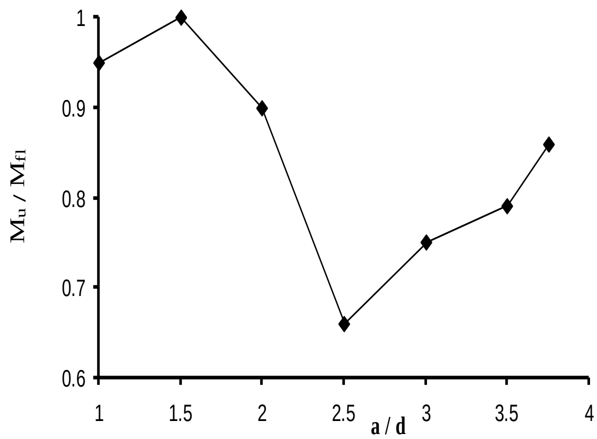


Fig. 10: Relative beam strength

Statistical method of least squares was used. Linear regression generates the following equations in ACI pattern.

for a/d ≥ 2.5

$$v_{cr} = 1.02\sqrt{fc'} + 21260\rho(\frac{d}{a}) \quad (8)$$

$$v_u = 1.54\sqrt{fc'} + 26770\rho(\frac{d}{a}) \quad (9)$$

for a/d < 2.5

$$v_{cr} = 1.36\sqrt{fc'} + 18050\rho(\frac{d}{a}) \quad (10)$$

$$v_u = -0.042\sqrt{fc'} + 64660\rho(\frac{d}{a}) \approx 64660\ \rho(\frac{d}{a}) \quad (11)$$

For all values of a/d

$$v_{cr} = 1.12\sqrt{fc'} + 197000\rho(\frac{d}{a}) \quad (12)$$

$$v_u = -0.0091\sqrt{fc'} + 61100\rho(\frac{d}{a}) \approx 61100\ \rho(\frac{d}{a}) \quad (13)$$

Multiple regression generates the following equations in Zsutty's pattern.

for a/d ≥ 2.5

$$v_{cr} = 3254.22\ (fc')^{0.024}(\frac{d}{a})^{0.6}(\rho)^{0.59} \quad (14)$$

$$v_u = 1340.59\ (fc')^{0.106}(\frac{d}{a})^{0.5}(\rho)^{0.5} \quad (15)$$

for a/d < 2.5

$$v_{cr} = 12.6(fc')^{0.48}(\frac{d}{a})^{0.64}(\rho)^{0.19} \quad (16)$$

$$v_u = 23.98(fc')^{0.53}(\frac{d}{a})^{1.23}(\rho)^{0.21} \quad (17)$$

The developed equations can help to estimate the shear capacity of beams. The Zsutty's equations only gives ultimate shear strength and ACI code equation only gives cracking strength, where as presented equations helps in estimating both the strengths. Moreover ACI equation does not recognize the importance of divorcing point where as these equations considered the effect of divorcing point. However these equations have following limitations;

5.1 Limitations

Developed equations (8 to 17)
values ranging from 2200 to 5200 psi. Equations are applicable for steel ratio ρ form ρ_{min} to ρ_{max} as defined by ACI Code. Equations are applicable for all a/d values ranging from 1 to 3.75. Equations are applicable for vibrated

concrete only, with maximum size of aggregate as 3/4”. Equations are applicable for beams reinforced with grade 60 deformed bars. Equations are applicable to two point loaded beams, but can be used for UDL by replacing (d/a) with (Vd/M).

6. Validity of developed equations

The comparison of developed equations with the existing equations /experimental values is presented in Figures 11 to 14. Equations were developed in two patterns, i.e. ACI and Zsutty. Figure 11 shows that equation developed according to ACI pattern for all values of a/d is slightly conservative to actually observed cracking shear. Equations developed for both sides of divorcing point are not in total agreement with the actual shear. For a/d ≥ 2.5 it agrees with actual but for a/d <2.5 the developed equation drastically under estimate the cracking shear. Since ACI code do not recognize the concept of divorcing point hence developing independent equations for both sides of divorcing point is un-necessary. It is also evident that ACI code equation is quite conservative in estimating the cracking shear.

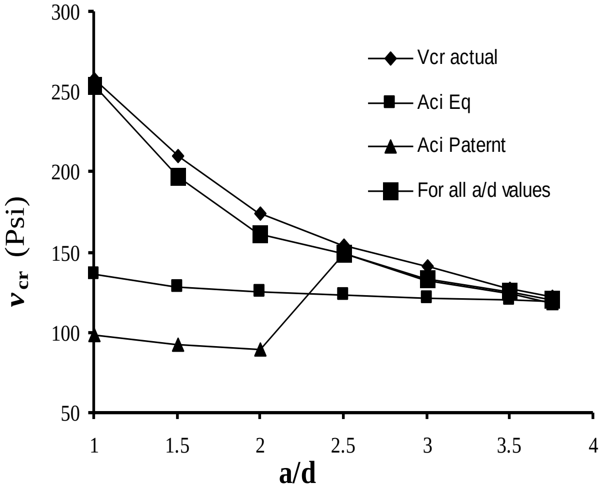


Fig. 11: Comparison of Cracking Shear (ACI)

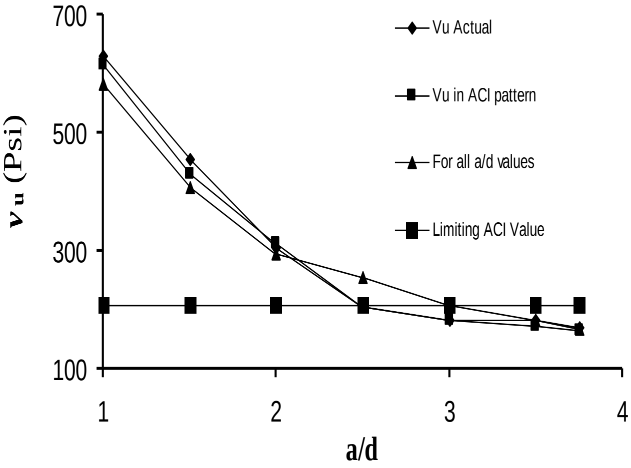


Fig. 12: Comparison of Ultimate Shear (ACI)

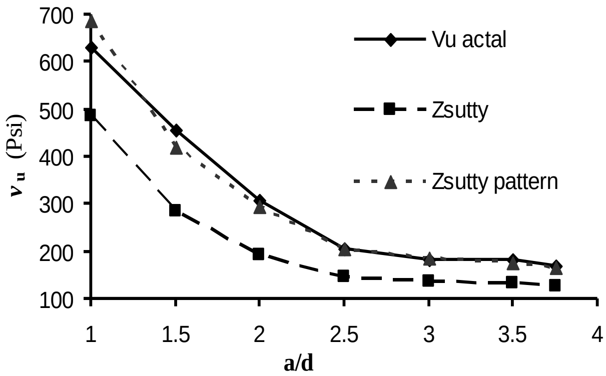


Fig. 13: Comparison of Ultimate Shear (Zsutty)

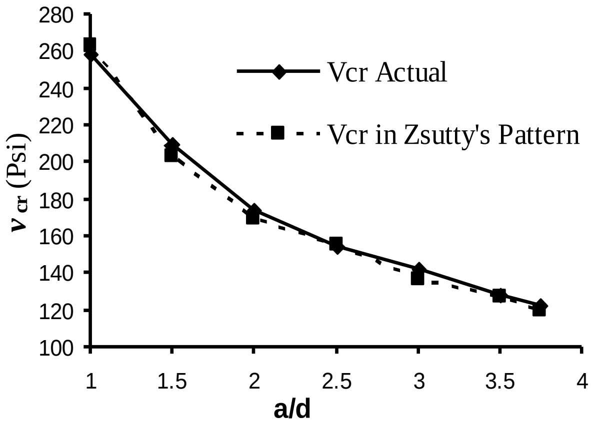


Fig.14: Comparison of Cracking Shear (Zsutty)

ACI code had not formulated any equation for ultimate shear; however a maximum limit of 3.5 $\sqrt{f_c'}$ is specified. Treating this value as ultimate shear (represented by a straight line in Figure 12), equations are developed for ultimate shear capacity and are graphically represented in Figure 12. Both equations, i.e. for all values of a/d and different relations for two sides of divorcing point are in close agreement to actual value, whereas ACI limiting criteria gives very low values for a/d < 2.5. From equations (11) & (13) it is evident that concrete crushing strength have negative impact on ultimate shear capacity, which is totally illogical, that means ACI pattern equations are not suitable for estimating ultimate shear capacity.

On the other hand Zsutty only developed relations for ultimate shear. Figure 13 shows that relation developed in Zsutty’s pattern is in very close agreement with the actual shear strength; where as Zsutty’s equation is conservative in estimating the ultimate shear capacity. For cracking shear developed equation (graphically shown in Figure 14) is also in very close agreement with the observed values.

2.2 Conclusions

ACI pattern equation was developed using method of least squares (linear regression). This method is not suitable for developing correlation between four variables i.e. *v*, d/a, f_c' & ρ. That is why equation seems to have imperfections and presented a poor correlation for a/d < 2.5. According to ACI contribution of f_c' is about 80 to 90 % of the total shear before any cracking is observed, which is against the Kani's as well as our experimental research.

Residual shear strength of a beam after shear cracking remains constant as a/d is decreased upto 2.5, then it increases rapidly as a/d is further decreased below 2.5 showing minimum capacity at a/d = 2.5.

Cracking shear capacity should be evaluated according to ACI pattern equation (12) for all values of a/d (without any mention of divorcing point), because this equation itself takes care of divorcing point as its slope automatically changes at this point.

Ultimate shear capacity should be estimated using equations (15) & (17) with due consideration of divorcing point.

Both ACI code and Zsutty's equations give conservative value especially for a/d < 2.5. Beam design may be more economical if shear capacity supplied by equations (12), (15) and (17) is kept in view.

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